

CUET UG (Applied Maths) - 02 June 2025 Shift 1

Common Maths

Question1

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$$

is equal to:

1. $\frac{1}{2} \log |e^{2x} - e^{-2x}| + C$, where C is an arbitrary constant.
2. $\log |e^{2x} - e^{-2x}| + C$, where C is an arbitrary constant.
3. $\frac{1}{2} \log |e^{2x} + e^{-2x}| + C$, where C is an arbitrary constant.
4. $\log |e^{2x} + e^{-2x}| + C$, where C is an arbitrary constant.

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

1. Choose a substitution:

Let u be the denominator:

$$u = e^{2x} + e^{-2x}$$

2. Find the differential du :

Take the derivative of u with respect to x :

$$\frac{du}{dx} = \frac{d}{dx}(e^{2x}) + \frac{d}{dx}(e^{-2x})$$

Using the chain rule, $\frac{d}{dx}(e^{kx}) = ke^{kx}$:

$$\frac{du}{dx} = (2e^{2x}) + (-2e^{-2x}) = 2e^{2x} - 2e^{-2x} = 2(e^{2x} - e^{-2x})$$

Therefore, the differential du is:

$$du = 2(e^{2x} - e^{-2x})dx$$

This means the numerator, $(e^{2x} - e^{-2x})dx$, is equal to $\frac{1}{2}du$:

$$(e^{2x} - e^{-2x})dx = \frac{1}{2}du$$

3. Substitute u and du into the integral:

$$I = \int \frac{1}{u} \left(\frac{1}{2} du \right) = \frac{1}{2} \int \frac{1}{u} du$$

4. Integrate with respect to u :

The integral of $\frac{1}{u}$ is $\log |u|$:

$$I = \frac{1}{2} \log |u| + C$$

where C is the arbitrary constant of integration.

5. Substitute back for x :

Replace u with $e^{2x} + e^{-2x}$:

$$I = \frac{1}{2} \log |e^{2x} + e^{-2x}| + C$$

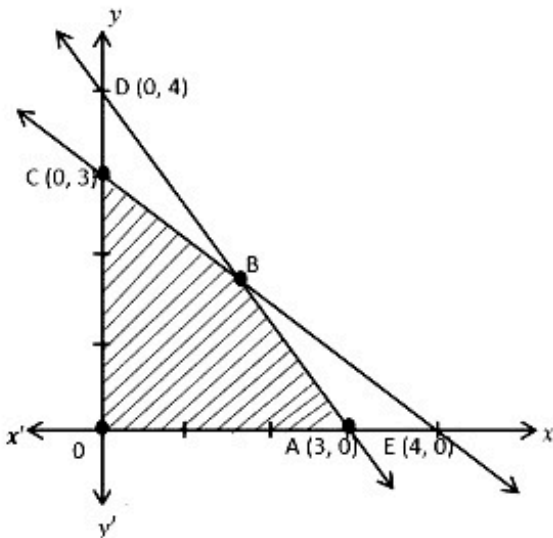
Since $e^{2x} + e^{-2x}$ is always positive, the absolute value is technically not required, but including it is mathematically correct for the general form $\int \frac{1}{u} du =$

$$I = \frac{1}{2} \log(e^{2x} + e^{-2x}) + C$$

[The correct option is 3.]

Question 2

Linear inequalities corresponding to the shaded feasible region OABCO in the given figure are



1. $4x + 3y \leq 12, 3x + 4y \leq 12, x \geq 0, y \geq 0$
2. $4x + 3y \geq 12, 3x + 4y \geq 12, x \geq 0, y \geq 0$
3. $4x + 3y \leq 12, 3x + 4y \geq 12, x \geq 0, y \geq 0$
4. $4x + 3y \geq 12, 3x + 4y \leq 12, x \geq 0, y \geq 0$

Options:

A.

1

B.

2

C.
3
D.
4

Answer: A

Solution:

1. Axes (Non-Negativity)

The region is in the first quadrant:

$x \geq 0, \quad y \geq 0$

2. Line 1: Through D(0, 4) and A(3, 0)

Intercepts: x -intercept $a = 3$, y -intercept $b = 4$.

Equation (Intercept Form): $\frac{x}{3} + \frac{y}{4} = 1 \implies 4x + 3y = 12$

Inequality: Since the region includes the origin (0, 0), the constraint is:

$4x + 3y \leq 12$

3. Line 2: Through C(0, 3) and E(4, 0)

Intercepts: x -intercept $a = 4$, y -intercept $b = 3$.

Equation (Intercept Form): $\frac{x}{4} + \frac{y}{3} = 1 \implies 3x + 4y = 12$

Inequality: Since the region includes the origin (0, 0), the constraint is:

$3x + 4y \leq 12$

Complete System of Constraints

$x \geq 0, \quad y \geq 0, \quad 4x + 3y \leq 12, \quad 3x + 4y \leq 12$

[The correct option is 1.]

Question3

Match List-I with List-II

List-I	List-II
(A) The degree of differential equation $\frac{d^3y}{dx^3} = e^{\frac{dy}{dx}}$	(I) 2
(B) The order of differential equation $\left(\frac{dy}{dx}\right)^2 + \frac{d^3y}{dx^3} = 0$	(II) 4
(C) The sum of order and degree of differential equation $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^5 = x$	(III) not defined
(D) The number of arbitrary constants in the general solution of a differential equation of order 2	(IV) 3

Choose the correct answer from the options given below:

1. (A) - (I), (B) - (IV), (C) - (II), (D) - (III)
2. (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

3. (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

4. (A) - (IV), (B) - (I), (C) - (II), (D) - (III)

Options:

A.

1

B.

2

C.

3

D.

4

Answer: C

Solution:

The **order** of a differential equation is the **order of the highest derivative** present in the equation.

The **degree** of a differential equation is the **power of the highest-order derivative** after the equation has been rationalized (i.e., cleared of any fractional powers of the derivatives).

A. The degree of differential equation $\frac{d^3y}{dx^3} = e^{\frac{dy}{dx}}$ is not defined (due to $e^{\frac{dy}{dx}}$ term) $\rightarrow$ (III) not defined

B. The order of differential equation $\left(\frac{dy}{dx}\right)^2 + \frac{d^3y}{dx^3} = 0$ is 3 $\rightarrow$ (IV) 3

C. The sum of order and degree of differential equation $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^5 = x$ is $(3 + 1 = 4) \rightarrow$ (II) 4

D. The number of arbitrary constants in the general solution of a differential equation of order 2 is 2 $\rightarrow$ (I) 2

The final match is: (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

[The correct option is 3.]

Question4

$$\text{If } \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} A \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

then matrix 'A' is

1. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 1 \\ 3 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Solving the Matrix Equation for A

The given equation is $BAC = D$, where:

$$B = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To solve for A , we pre-multiply by B^{-1} and post-multiply by C^{-1} :

$$A = B^{-1}DC^{-1}$$

1. Calculate Inverses

$$\text{For a } 2 \times 2 \text{ matrix } M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad M^{-1} = \frac{1}{|M|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Inverse of B

$$|B| = 3(1) - 1(2) = 1.$$

$$B^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

Inverse of C

$$|C| = 2(1) - 1(1) = 1.$$

$$C^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

2. Compute $B^{-1}D$

$$B^{-1}D = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + (-1)(0) & 1(1) + (-1)(1) \\ -2(1) + 3(0) & -2(1) + 3(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

3. Compute $A = (B^{-1}D)C^{-1}$

$$A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1(1) + 0(-1) & 1(-1) + 0(2) \\ -2(1) + 1(-1) & -2(-1) + 1(2) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

[The correct option is 3.]

Question5

For the function $f(x) = -2x^3 + 3x^2 + 36x - 10$, which of the following is/are true?

- (A) f is increasing in $(-\infty, -2)$
- (B) f is increasing in $(-2, 3)$
- (C) f is decreasing in $(-\infty, -2)$
- (D) f is decreasing in $(3, \infty)$

Choose the correct answer from the options given below:

1. (B) and (D) only
2. (B), (C) and (D) only
3. (A) and (D) only
4. (B) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Analysis of Monotonicity for $f(x) = -2x^3 + 3x^2 + 36x - 10$

Function: $f(x) = -2x^3 + 3x^2 + 36x - 10$

1. Derivative and Critical Points:

$$f'(x) = -6x^2 + 6x + 36$$

Set $f'(x) = 0$:

$$-6x^2 + 6x + 36 = 0$$

$$-6(x^2 - x - 6) = 0 \implies -6(x - 3)(x + 2) = 0$$

Critical points are $x = -2$ and $x = 3$.

2. Check Intervals ($f'(x)$ sign):

Interval $(-\infty, -2)$: $f'(-3) = -6((-3)^2 - (-3) - 6) = -6(9 + 3 - 6) = -36 < 0 \implies$ **Decreasing (C is TRUE)**

Interval $(-2, 3)$: $f'(0) = -6(0 - 0 - 6) = 36 > 0 \implies$ **Increasing (B is TRUE)**

Interval $(3, \infty)$: $f'(4) = -6(4^2 - 4 - 6) = -6(16 - 10) = -36 < 0 \implies$ **Decreasing (D is TRUE)**

3. Conclusion:

Statements (B), (C), and (D) are TRUE.

[The correct option is 2.]

Question6

If $y = \frac{1}{\sqrt[3]{1-x^3}}$, then $\frac{dy}{dx}$ is equal to:

1. $x(1-x^3)^{-\frac{4}{3}}$
2. $x^2(1-x^3)^{-\frac{4}{3}}$
3. $(1-x^3)^{-\frac{4}{3}}$
4. $-x^2(1-x^3)^{-\frac{4}{3}}$

Options:

- A. 1
- B. 2

C. 3

D. 4

Answer: B

Solution:

Function: $y = \frac{1}{\sqrt[3]{1-x^3}}$

1. Rewrite y using exponent notation:

$$y = (1 - x^3)^{-\frac{1}{3}}$$

2. Differentiate $\frac{dy}{dx}$ using the Chain Rule $\frac{d}{dx}[u^n] = nu^{n-1} \cdot \frac{du}{dx}$:

Let $u = (1 - x^3)$. Then $\frac{du}{dx} = -3x^2$.

$$\frac{dy}{dx} = -\frac{1}{3}(1 - x^3)^{-\frac{1}{3}-1} \cdot \frac{d}{dx}(1 - x^3)$$

$$\frac{dy}{dx} = -\frac{1}{3}(1 - x^3)^{-\frac{4}{3}} \cdot (-3x^2)$$

3. Simplify the expression:

$$\frac{dy}{dx} = \left(-\frac{1}{3}\right)(-3x^2)(1 - x^3)^{-\frac{4}{3}}$$

$$\frac{dy}{dx} = x^2(1 - x^3)^{-\frac{4}{3}}$$

[The correct option is 2.]

Question7

If x is real, the minimum value of $x^2 - 8x + 20$ is:

1. 4

2. 2

3. 0

4. - 1

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Function: $f(x) = x^2 - 8x + 20$

1. First Derivative and Critical Point:

$$f'(x) = 2x - 8$$

$$f'(x) = 0$$

$$\implies 2x = 8$$

$$\implies x = 4 \quad (\text{Critical Point})$$

2. Second Derivative Test (Confirmation):

$$f''(x) = \frac{d}{dx}(2x - 8) = 2$$

Since $f''(4) = 2 > 0$ (positive), the function has a **Local Minimum** at $x = 4$.

3. Minimum Value:

$$f(4) = (4)^2 - 8(4) + 20 = 16 - 32 + 20 = 4$$

The minimum value is **4**.

[The correct option is 1.]

Question8

If A is a square matrix of order 3 and $|A| = 5$, then the value of $|-AA^T|$ is:

1. 25
2. - 25
3. 5
4. - 5

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Given: $|A| = 5$, Order $n = 3$.

Required: $|-AA^T|$

$$= (-1)^3 |AA^T|$$

$$= -1 \cdot |A||A^T|$$

$$= -1 \cdot (5)(5)$$

$$= -25$$

Answer: -25

Question9

Consider an LPP: Maximise $Z = 50x + 15y$ subjected to constraints

$$x + y \leq 60,$$

$$5x + y \leq 100,$$

$$x, y \geq 0.$$

If the maximum value of Z occurs at $x = a$ and $y = \beta$, then the value of $a + \beta$ is:

1. 10

2. 60

3. 50

4. 40

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

LPP: Maximize $Z = 50x + 15y$

Constraints:

1. $x + y \leq 60$

2. $5x + y \leq 100$

3. $x \geq 0, \quad y \geq 0$

1. Find Vertices of Feasible Region:

The corner points are the intersection of the boundary lines ($x + y = 60$ and $5x + y = 100$) and the axes.

Intersection of Axes: $(0, 0)$

Line 1 intersects axes: $(60, 0)$ and $(0, 60)$

Line 2 intersects axes: $(20, 0)$ and $(0, 100)$

2. Find Intersection of $x + y = 60$ and $5x + y = 100$:

Subtracting the first equation from the second:

$$(5x + y) - (x + y) = 100 - 60$$

$$4x = 40 \implies x = 10$$

Substitute $x = 10$ into $x + y = 60$:

$$10 + y = 60$$

$$\implies y = 50$$

The vertex is $(10, 50)$.

3. Evaluate Z at Corner Points:

The corner points of the feasible region are $(0, 0)$, $(20, 0)$, $(0, 60)$, and $(10, 50)$.

$$Z(0, 0) = 50(0) + 15(0) = 0$$

$$Z(20, 0) = 50(20) + 15(0) = 1000$$

$$Z(0, 60) = 50(0) + 15(60) = 900$$

$$Z(10, 50) = 50(10) + 15(50) = 500 + 750 = \mathbf{1250} \quad (\text{Maximum})$$

4. Final Value:

The maximum value occurs at $\alpha = 10$ and $\beta = 50$.

$$\alpha + \beta = 10 + 50 = \mathbf{60}$$

[The correct option is 2.]

Question10

If the matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible, then the value of λ is:

1. 0
2. -1
3. 1
4. $\frac{1}{2}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Condition for Non-Invertibility:

A matrix A is not invertible if and only if its determinant is zero, i.e., $|A| = 0$.

1. Calculate the Determinant $|A|$:

$$A = \begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$$

Expand along the second row (R_2) for simplicity (due to the zero element):

$$|A| = -\lambda \begin{vmatrix} -1 & 3 \\ 1 & 4 \end{vmatrix} + 0 \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} - 7 \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}$$

$$|A| = -\lambda[(-1)(4) - (3)(1)] + 0 - 7[(2)(1) - (-1)(-1)]$$

$$|A| = -\lambda[-4 - 3] - 7[2 - 1]$$

$$|A| = -\lambda(-7) - 7(1)$$

$$|A| = 7\lambda - 7$$

2. Set $|A| = 0$ and Solve for λ :

$$7\lambda - 7 = 0$$

$$7\lambda = 7$$

$$\lambda = 1$$

[The correct option is 3.]

Question11

If the matrix $\begin{bmatrix} 3 & 2a & -5 \\ 4 & 0 & b \\ -5 & 3 & 10 \end{bmatrix}$ is symmetric, then the value of $5a + 2b$ is:

1. 6
2. 10
3. 16
4. 20

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Condition for a Symmetric Matrix:

A square matrix A is symmetric if $A = A^T$, which means the elements must satisfy $\mathbf{a}_{ij} = \mathbf{a}_{ji}$.

The matrix is:

$$A = \begin{bmatrix} 3 & 2a & -5 \\ 4 & 0 & b \\ -5 & 3 & 10 \end{bmatrix}$$

1. Apply the symmetry condition to find a and b :

Element $\mathbf{a}_{12}$ must equal $\mathbf{a}_{21}$:

$$2a = 4$$

$$\implies \mathbf{a = 2}$$

Element $\mathbf{a}_{23}$ must equal $\mathbf{a}_{32}$:

$$b = 3$$

Element $a_{13}(-5)$ equals $a_{31}(-5)$, which is consistent.

2. Calculate the required value $5a + 2b$:

Substitute the values of $a = 2$ and $b = 3$:

$$5a + 2b = 5(2) + 2(3)$$

$$5a + 2b = 10 + 6$$

$$5a + 2b = \mathbf{16}$$

[The correct option is 3.]

Question12

The solution of the differential equation $y \, dx + (x - y^2) \, dy = 0$ is:

1. $3xy + x^3 = C$: C is an arbitrary constant
2. $3xy + y^3 = C$: C is an arbitrary constant
3. $3xy - y^3 = C$: C is an arbitrary constant
4. $3xy - x^3 = C$: C is an arbitrary constant

Options:

- A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Differential Equation (DE): $y \, dx + (x - y^2) \, dy = 0$

1. Rearrange into a Linear DE form $\left(\frac{dx}{dy} + P(y)x = Q(y)\right)$:

Divide by dy :

$$y \frac{dx}{dy} + x - y^2 = 0$$

$$y \frac{dx}{dy} + x = y^2$$

$$\frac{dx}{dy} + \left(\frac{1}{y}\right)x = y$$

This is a linear DE in x , where $P(y) = \frac{1}{y}$ and $Q(y) = y$.

2. Find the Integrating Factor (IF):

$$\text{IF} = e^{\int P(y) \, dy} = e^{\int \frac{1}{y} \, dy} = e^{\ln |y|} = y$$

(We assume $y > 0$ for simplicity, so $\text{IF} = y$.)

3. Find the Solution:

The general solution is $x \cdot (\text{IF}) = \int Q(y) \cdot (\text{IF}) \, dy + C$.

$$x \cdot y = \int y \cdot y \, dy + C_1$$

$$xy = \int y^2 \, dy + C_1$$

$$xy = \frac{y^3}{3} + C_1 \quad (\text{Let } C_1 \text{ be the constant})$$

4. Rearrange to Match Options:

Multiply the entire equation by 3:

$$3xy = y^3 + 3C_1$$

$$3xy - y^3 = 3C_1$$

Let $C = 3C_1$ (where C is the new arbitrary constant):

$$3xy - y^3 = C$$

[The correct option is 3.]

Question13

The value of $\int_0^1 \log_e \left(\frac{1}{x} - 1\right) dx$ is:

1. 1

2. 0

3. $\frac{3}{2}$

4. $\log 2$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Integral: $I = \int_0^1 \log_e \left(\frac{1}{x} - 1 \right) dx = \int_0^1 \log \left(\frac{1-x}{x} \right) dx$

Property: Use the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$. Here $a = 1$.

$$I = \int_0^1 \log \left(\frac{1-(1-x)}{1-x} \right) dx$$

$$I = \int_0^1 \log \left(\frac{x}{1-x} \right) dx$$

Logarithm Property: Use $\log(A/B) = -\log(B/A)$.

$$I = \int_0^1 -\log \left(\frac{1-x}{x} \right) dx$$

$$I = - \int_0^1 \log \left(\frac{1-x}{x} \right) dx$$

$$I = -I$$

Solve for I:

$$I = -I$$

$$2I = 0$$

$$I = 0$$

[The correct option is 2.]

Question14

Area of the region bounded by $y = x^2$ and the line $y = 16$ is

1. $\frac{256}{3}$ square units
2. $\frac{128}{3}$ square units
3. $\frac{64}{3}$ square units
4. $\frac{32}{3}$ square units

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

Area Calculation

The area of the region bounded by $y = x^2$ and the line $y = 16$ is:

$$\begin{aligned} A &= \int_{-4}^4 (16 - x^2) dx \\ &= 2 \int_0^4 (16 - x^2) dx \\ &= 2 \left[16x - \frac{x^3}{3} \right]_0^4 \\ &= 2 \left(\left(16(4) - \frac{4^3}{3} \right) - (0) \right) \\ &= 2 \left(64 - \frac{64}{3} \right) \\ &= 2 \left(\frac{192 - 64}{3} \right) \\ &= 2 \left(\frac{128}{3} \right) \\ &= \frac{256}{3} \text{ square units} \end{aligned}$$

[The correct option is 1.]

Question15

The probability distribution of a random variable x is, $P(x) = \frac{k}{2^x}$, $x = 0, 1, 2, 3$. Then

Match List-I with List-II

List-I	List-II
(A) k	(I) $2/15$
(B) $P(x=1)$	(II) $1/5$
(C) $P(1)$	(III) $8/15$
(D) $P(x \geq 2)$	(IV) $4/15$

Choose the correct answer from the options given below:

1. (A) - (III), (B) - (II), (C) - (I), (D) - (IV)
2. (A) - (IV), (B) - (II), (C) - (III), (D) - (I)
3. (A) - (IV), (B) - (III), (C) - (I), (D) - (II)
4. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

Probability Distribution Matching

The probability distribution is $P(x) = \frac{k}{2^x}$ for $x \in \{0, 1, 2, 3\}$.

Calculations

(A) Finding k

Using $\sum P(x) = 1$:

$$k \left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} \right) = 1$$

$$k \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \right) = 1$$

$$k \left(\frac{8+4+2+1}{8} \right) = 1$$

$$k \left(\frac{15}{8} \right) = 1 \implies k = \frac{8}{15} \rightarrow \text{(III)}$$

(B) Finding $P(x=1)$

$$\text{With } k = \frac{8}{15}, P(x) = \frac{8}{15 \cdot 2^x}.$$

$$P(x=1) = \frac{8}{15 \cdot 2^1} = \frac{4}{15} \rightarrow \text{(IV)}$$

(C) Finding $P(1 < x < 3)$

$$P(1 < x < 3) = P(x=2):$$

$$P(x=2) = \frac{8}{15 \cdot 2^2} = \frac{8}{60} = \frac{2}{15} \rightarrow \text{(I)}$$

(D) Finding $P(x \geq 2)$

$$P(x \geq 2) = P(x=2) + P(x=3):$$

$$P(x \geq 2) = \frac{2}{15} + \frac{8}{15 \cdot 8}$$

$$P(x \geq 2) = \frac{2}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5} \rightarrow \text{(II)}$$

Applied Mathematics

Question1

Calculate Three yearly moving averages for the following data

Year	2016	2017	2018	2019	2020	2021
Production (thousand tonnes)	200	220	231	254	202	243

1.217, 229 , 233 , 218

2.217, 235 , 229 , 233

3.235, 229 , 233 , 218

4. 235, 229 , 233 , 218

Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

A 3-Year Moving Average is calculated as the sum of 3 consecutive years divided by 3, centered at the middle year.

1. Calculate the 3-Year Moving Totals (MT) and Averages (MA):

For 2017 (using 2016, 2017, 2018):

$$MT = 200 + 220 + 231 = 651$$

$$MA = \frac{651}{3} = 217$$

For 2018 (using 2017, 2018, 2019):

$$MT = 220 + 231 + 254 = 705$$

$$MA = \frac{705}{3} = 235$$

For 2019 (using 2018, 2019, 2020):

$$MT = 231 + 254 + 202 = 687$$

$$MA = \frac{687}{3} = 229$$

For 2020 (using 2019, 2020, 2021):

$$MT = 254 + 202 + 243 = 699$$

$$MA = \frac{699}{3} = 233$$

2. The sequence of 3-yearly moving averages is:

217, 235, 229, 233

This corresponds to Option 2.

Question2

A machine costing Rs.50,000 has a useful life of 4 years .The estimated scrap value is Rs 10,000 . The rate of depreciation per annum is:

1. 20%
2. 30%
3. 25%
4. 15%

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

1. Calculate Total Depreciable Value (TDV):

$$\text{TDV} = \text{Original Cost} - \text{Scrap Value}$$

$$\text{TDV} = 50,000 - 10,000 = \text{Rs } 40,000$$

2. Calculate Annual Depreciation (AD):

$$\text{AD} = \frac{\text{TDV}}{\text{Useful Life}}$$

$$\text{AD} = \frac{40,000}{4} = \text{Rs } 10,000$$

3. Calculate Rate of Depreciation (R):

$$R = \frac{\text{AD}}{\text{Original Cost}} \times 100$$

$$R = \frac{10,000}{50,000} \times 100$$

$$R = \frac{1}{5} \times 100 = 20\%$$

The rate of depreciation per annum is 20% (Option 1).

Question3

For the linear programming problem,

Minimize (Z) = 60x + 30y

subject to: $2x - y \geq -5$; $3x + y \geq 3$; $2x - 3y \leq 12$; $x, y \geq 0$

the optimal value of z is

- 1. 360
- 2. 150
- 3. 60
- 4. No Minimum value of Z

Options:

- A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

$$\text{Minimize } Z = 60x + 30y$$

$$\text{Constraints: } (L1) \ 2x - y \geq -5 \implies y \leq 2x + 5$$

$$(L2) \ 3x + y \geq 3$$

$$(L3) \ 2x - 3y \leq 12$$

$$x \geq 0, y \geq 0$$

1. Find the Corner Points of the Feasible Region in the First Quadrant:

$$\text{Intersection } L2 \text{ and } x = 0 \implies (0, 3)$$

$$\text{Intersection } L2 \text{ and } y = 0 \implies (1, 0)$$

$$\text{Intersection } L1 \text{ and } x = 0 \implies (0, 5)$$

$$\text{Intersection } L3 \text{ and } y = 0 \implies (6, 0)$$

The effective corner points defining the boundary in the first quadrant are: $(0, 5)$, $(0, 3)$, $(1, 0)$, $(6, 0)$.

2. Evaluate the Objective Function $Z = 60x + 30y$ at the corner points:

$$Z(0, 5) = 60(0) + 30(5) = 150$$

$$Z(0, 3) = 60(0) + 30(3) = 90$$

$$Z(1, 0) = 60(1) + 30(0) = 60$$

$$Z(6, 0) = 60(6) + 30(0) = 360$$

3. Conclusion:

The minimum value of Z is 60, which occurs at $(1, 0)$.

The minimum value of Z is 60 (Option 3).

Question4

If $A = \begin{bmatrix} 0 & x^2 - 6 & -3 \\ -x & 0 & -8 \\ x^2 - 2x & 8 & 0 \end{bmatrix}$ is a skew symmetric matrix, then the value(s) of x is/ are -

- (A) 3
- (B) -3
- (C) -2
- (D) -1

Choose the **correct** answer from the options given below:

- 1. (A) and (C) only
- 2. (A), (C) and (D) only
- 3. (A) only
- 4. (B), (C) and (D) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

A matrix A is skew symmetric if $a_{ij} = -a_{ji}$.

The given matrix is $A = \begin{bmatrix} 0 & x^2 - 6 & -3 \\ -x & 0 & -8 \\ x^2 - 2x & 8 & 0 \end{bmatrix}$

1. Apply the condition to element a_{12} and a_{21} (i.e., $a_{12} = -a_{21}$):

$$x^2 - 6 = -(-x)$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3 \text{ or } x = -2 \quad (C1)$$

2. Apply the condition to element a_{13} and a_{31} (i.e., $a_{13} = -a_{31}$):

$$-3 = -(x^2 - 2x)$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \text{ or } x = -1 \quad (C2)$$

3. Apply the condition to element a_{23} and a_{32} (i.e., $a_{23} = -a_{32}$):

$$-8 = -(8) \implies -8 = -8 \quad (\text{Satisfied for all } x)$$

4. Find the common solution satisfying (C1) and (C2):

$$x \in \{3, -2\} \cap \{3, -1\} \implies x = 3$$

The only value of x is 3 (Option 3).

Question5

If CAGR stands for Compound Annual Growth Rate, F.V stands for final value of an investment, P.V stands for present value of an investment and n is the number of years then

1. $\text{CAGR} = \left(\frac{\text{F.V}}{\text{P.V}} \right)^{\frac{1}{n}} - 1$

2. $\text{CAGR} = \left(\frac{\text{P.V}}{\text{F.V}} \right)^{\frac{1}{n}} - 1$

3. $\text{CAGR} = \left(\frac{\text{F.V}}{\text{P.V}} \right)^n - 1$

4. $\text{CAGR} = \left(\frac{\text{P.V}}{\text{F.V}} \right)^n - 1$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

The Compound Interest Formula is:

$$\text{F.V} = \text{P.V}(1 + r)^n$$

Where $r = \text{CAGR}$:

$$\text{F.V} = \text{P.V}(1 + \text{CAGR})^n$$

1. Isolate the growth factor $(1 + \text{CAGR})^n$:

$$\frac{\text{F.V}}{\text{P.V}} = (1 + \text{CAGR})^n$$

2. Isolate the annual growth factor $(1 + \text{CAGR})$ by taking the n -th root:

$$\left(\frac{\text{F.V}}{\text{P.V}} \right)^{\frac{1}{n}} = 1 + \text{CAGR}$$

3. Isolate CAGR:

$$\text{CAGR} = \left(\frac{\text{F.V}}{\text{P.V}} \right)^{\frac{1}{n}} - 1$$

The correct formula is Option 1.

Question6

If maximum value of $f(x) = 2x^3 + 3x^2 - 6ax + 10$ occurs at $x = -3$, then the value of a is _____

1. 1

2. 5

3. 6

4. 3

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

The function is $f(x) = 2x^3 + 3x^2 - 6ax + 10$.

1. Find the First Derivative $f'(x)$:

$$f'(x) = \frac{d}{dx}(2x^3 + 3x^2 - 6ax + 10)$$

$$f'(x) = 6x^2 + 6x - 6a$$

2. Apply the condition for a maximum: $f'(-3) = 0$.

$$f'(-3) = 6(-3)^2 + 6(-3) - 6a = 0$$

3. Solve for a :

$$6(9) - 18 - 6a = 0$$

$$54 - 18 - 6a = 0$$

$$36 - 6a = 0$$

$$6a = 36$$

$$a = 6$$

The value of a is 6 (Option 3).

Question 7

If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, then $|A \operatorname{adj} A|$ is

1. 3
2. 6
3. 9
4. 12

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

1. Calculate the determinant of A :

$$|A| = \det A = (1)(3) - (2)(0) = 3$$

2. Use the matrix property $A(\operatorname{adj} A) = (\det A)I$:

$$|A \operatorname{adj} A| = |(\det A)I|$$

3. Use the determinant property $|kA| = k^n|A|$, where $n = 2$ and $k = \det A$:

$$|(\det A)I| = (\det A)^2|I|$$

Since $|I| = 1$:

$$|A \operatorname{adj} A| = (\det A)^2$$

4. Substitute the value of $|A|$:

$$|A \operatorname{adj} A| = (3)^2 = 9$$

The correct option is 9 (Option 3).

Question8

A machine costing Rs 3,00,000 will have its scrap value of Rs 50,000. The company at present plans to put Rs 36,650 per annum at the end of each year in a sinking fund at the rate 5% per annum for the replacement of the machine after its useful life. Suppose the new machine will cost Rs 4,00,000 at that time, then the useful life (approx.) of the machine is:

[Given: $(1.4775)^{\frac{1}{8}} \approx 1.05$]

1. 6.5 yrs
2. 8.5 yrs
3. 7 yrs
4. 8 yrs

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

1. Calculate Required Future Value (A):

$$A = \text{Cost of New Machine} - \text{Scrap Value}$$

$$A = 4,00,000 - 50,000 = ₹3,50,000$$

2. Use Future Value of an Annuity Formula:

$$A = P \left[\frac{(1+i)^n - 1}{i} \right]$$

Substitute $A = 3,50,000$, $P = 36,650$, and $i = 0.05$:

$$3,50,000 = 36,650 \left[\frac{(1.05)^n - 1}{0.05} \right]$$

3. Isolate $(1.05)^n$:

$$\frac{3,50,000}{36,650} \cdot 0.05 \approx (1.05)^n - 1$$

$$9.5498 \cdot 0.05 \approx (1.05)^n - 1$$

$$0.47749 \approx (1.05)^n - 1$$

$$1.47749 \approx (1.05)^n$$

4. Use the given information to find n :

$$\text{Given } (1.4775)^{1/8} \approx 1.05.$$

$$\text{Raising to the power of 8 gives } 1.4775 \approx (1.05)^8.$$

Comparing with the derived equation, we find $n \approx 8$.

The useful life is approximately 8 yrs (Option 4).

Question9

If the sum and difference of squares of mean and variance of a Binomial distribution is $\frac{225}{256}$ and $\frac{63}{256}$ respectively, the

$P(X \geq 2)$ is:

1. $\frac{81}{256}$

2. $\frac{9}{16}$

3. $\frac{5}{32}$

4. $\frac{9}{256}$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Let mean = μ and variance = σ^2

$$\mu^2 + \sigma^2 = \frac{225}{256}$$

$$\mu^2 - \sigma^2 = \frac{63}{256}$$

$$2\mu^2 = \frac{288}{256}$$

$$\mu^2 = \frac{144}{256}$$

$$\mu = \frac{12}{16} = \frac{3}{4}$$

$$2\sigma^2 = \frac{162}{256}$$

$$\sigma^2 = \frac{81}{256}$$

For binomial distribution, $\mu = np$ and $\sigma^2 = npq$

$$\frac{npq}{np} = q = \frac{81}{144}$$

$$q = \frac{9}{16}$$

$$p = \frac{7}{16}$$

$$n = \frac{\mu}{p} = \frac{3/4}{7/16} = 2$$

$$P(X \geq 2) = P(X = 2)$$

$$P(X = 2) = \left(\frac{7}{16}\right)^2 = \frac{49}{256}$$

$$P(X \geq 2) = 1 - (P(X = 0) + P(X = 1))$$

$$= 1 - \left(\left(\frac{9}{16}\right)^2 + 2 \times \frac{7}{16} \times \frac{9}{16} \right) = \frac{81}{256}$$

Therefore, option 1 is correct

Question10

The effective rate per annum equivalent to a nominal rate of 8% compounded semi-annually is

1. 8.24%
2. 8.33%
3. 8.16%
4. 8.06%

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Formula for Effective Rate (EAR): $\text{EAR} = \left(1 + \frac{r}{n}\right)^n - 1$

Given: Nominal rate $r = 8\% = 0.08$

Compounding semi-annually: $n = 2$

1. Substitute the values:

$$\text{EAR} = \left(1 + \frac{0.08}{2}\right)^2 - 1$$

$$\text{EAR} = (1 + 0.04)^2 - 1$$

$$\text{EAR} = (1.04)^2 - 1$$

2. Calculate the value:

$$\text{EAR} = 1.0816 - 1$$

$$\text{EAR} = 0.0816$$

3. Convert to Percentage:

$$\text{Effective Rate} = 0.0816 \cdot 100\% = 8.16\%$$

The correct option is 8.16% (Option 3).

Question11

If A is a square matrix such that $A^2 = A$ and I is the identity matrix of the same order as A then $(I + 2A)^3 - 6A$ is equal to

1. $I + 26A$
2. $20A$
3. $I + 20A$
4. $26A$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Given the idempotent property: $A^2 = A$.

This implies that for $n \geq 1$, $A^n = A$.

1. Expand $(I + 2A)^3$ using the Binomial Theorem (since I and A commute):

$$(I + 2A)^3 = I^3 + 3(I^2)(2A) + 3(I)(2A)^2 + (2A)^3$$

2. Simplify the terms using $A^2 = A$ and $I^n = I$:

$$3(I^2)(2A) = 6A$$

$$3(I)(2A)^2 = 3(4A^2) = 12A \quad (\text{since } A^2 = A)$$

$$(2A)^3 = 8A^3 = 8A \quad (\text{since } A^3 = A)$$

3. Substitute back and sum:

$$(I + 2A)^3 = I + 6A + 12A + 8A$$

$$(I + 2A)^3 = I + 26A$$

4. Evaluate the final expression $(I + 2A)^3 - 6A$:

$$(I + 2A)^3 - 6A = (I + 26A) - 6A$$

$$(I + 2A)^3 - 6A = I + (26 - 6)A$$

$$(I + 2A)^3 - 6A = I + 20A$$

The correct option is $I + 20A$ (Option 3).

Question 12

If $x_1, x_2, x_3, \dots, x_n$ are n observations in a sample then which of following is/are TRUE?

- (A) The mean $\bar{x}$ has n degree of freedom
- (B) The mean $\bar{x}$ has $(n - 1)$ degree of freedom
- (C) The standard deviation of the sample has $(n - 1)$ degree of freedom
- (D) The standard deviation of the sample has n degree of freedom

Choose the **correct** answer from the options given below:

- 1. (A) and (C) only
- 2. (A) and (D) only
- 3. (B) and (C) only
- 4. (B) and (D) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

The degrees of freedom (DOF) is the number of values that are free to vary.

1. Analysis of the Sample Mean ($\bar{x}$):

The calculation of the mean itself does not impose constraints on the n observations.

Statement (A): The mean $\bar{x}$ has n degree of freedom. (TRUE)

Statement (B): The mean $\bar{x}$ has $(n - 1)$ degree of freedom. (FALSE)

2. Analysis of the Sample Standard Deviation (s):

The standard deviation is calculated using the sample mean $\bar{x}$.

The constraint $\sum_{i=1}^n (x_i - \bar{x}) = 0$ fixes one observation.

Statement (C): The standard deviation of the sample has $(n - 1)$ degree of freedom. (TRUE)

Statement (D): The standard deviation of the sample has n degree of freedom. (FALSE)

The correct statements are (A) and (C) only (Option 1).

Question13

A and B are two independent events. The probability that both events A and B occur is $\frac{1}{6}$ and the probability that neither of them occur is $\frac{1}{3}$. If $P(A) = x, P(B) = y$ then the value of $x + y$ is.

1. $\frac{1}{6}$
2. $\frac{5}{6}$
3. $\frac{2}{6}$
4. $\frac{4}{6}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Given: $P(A) = x, P(B) = y$, and A, B are independent.

1. Set up the equations:

Condition 1: $P(A \cap B) = P(A) \cdot P(B)$

$$\frac{1}{6} = xy \quad (1)$$

Condition 2: $P(A' \cap B') = P(A') \cdot P(B')$

$$\frac{1}{3} = (1 - x)(1 - y) \quad (2)$$

2. Solve for $x + y$:

Expand (2):

$$\frac{1}{3} = 1 - x - y + xy$$

Rearrange to isolate $x + y$:

$$x + y = 1 + xy - \frac{1}{3}$$

Substitute $xy = \frac{1}{6}$ from (1):

$$x + y = 1 + \frac{1}{6} - \frac{1}{3}$$

Find a common denominator (6):

$$x + y = \frac{6}{6} + \frac{1}{6} - \frac{2}{6}$$

$$x + y = \frac{5}{6}$$

The value of $x + y$ is $\frac{5}{6}$ (Option 2).

Question14

The area bounded by the x -axis and the parabola $y = 3x - x^2$ is:

1. $\frac{27}{2}$ Sq.unit
2. $\frac{9}{2}$ Sq.unit
3. $\frac{15}{2}$ Sq.unit
4. $\frac{11}{2}$ Sq.unit

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

The parabola is $y = 3x - x^2$.

1. Find the limits of integration (intersection with x -axis, $y = 0$):

$$3x - x^2 = 0$$

$$x(3 - x) = 0$$

The limits are $x = 0$ and $x = 3$.

2. Set up the Definite Integral for Area (A):

$$A = \int_0^3 (3x - x^2) dx$$

3. Evaluate the Integral:

$$A = \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3$$

$$A = \left(\frac{3(3)^2}{2} - \frac{(3)^3}{3} \right) - \left(\frac{3(0)^2}{2} - \frac{(0)^3}{3} \right)$$

$$A = \left(\frac{27}{2} - \frac{27}{3} \right) - (0)$$

Simplify the fractions:

$$A = \frac{27}{2} - 9$$

$$A = \frac{27}{2} - \frac{18}{2}$$

$$A = \frac{9}{2}$$

The area is $\frac{9}{2}$ Sq.unit (Option 2).

Question15

A pair of dice is thrown until the sum of numbers appeared is a perfect square or a non-perfect square sum appeared five times in succession. If random variable X denotes the number of non perfect square sums appeared. then $P(X > 0)$ is

1. $\frac{7}{36}$
2. $\frac{29}{36}$
3. 1
4. $\frac{1}{2}$

Options:

- A. 1
- B. 2
- C. 3

D. 4

Answer: B

Solution:

Total Outcomes: $N = 36$

1. Identify Perfect Square Sums (Success, S):

Sum 4 : $(1, 3), (2, 2), (3, 1)$ (3 outcomes)

Sum 9 : $(3, 6), (4, 5), (5, 4), (6, 3)$ (4 outcomes)

Total Success Outcomes: $N_S = 3 + 4 = 7$

2. Calculate Probability of Success (p) and Failure (q):

$$p = P(S) = \frac{7}{36}$$

$$q = P(F) = 1 - p = 1 - \frac{7}{36} = \frac{29}{36}$$

3. Analyze $P(X > 0)$:

X is the number of non-perfect square sums (Failures).

$$P(X > 0) = 1 - P(X = 0)$$

4. Calculate $P(X = 0)$:

$X = 0$ means the first throw was a Success (Perfect Square), stopping the trial.

$$P(X = 0) = P(S) = p = \frac{7}{36}$$

5. Calculate the Final Probability:

$$P(X > 0) = 1 - \frac{7}{36} = \frac{29}{36}$$

The probability $P(X > 0)$ is $\frac{29}{36}$ (Option 2).

Question16

$\int e^{(x \log 5)} e^x dx$, is:

Where c is the constant of integration.

1. $\frac{5^x}{\log 5} e^x + C$
2. $5^x e^x + C$
3. $(5e)^x \log(5e) + C$
4. $\frac{(5e)^x}{\log(5e)} + C$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

The given integral is $I = \int e^{(x \log 5)} e^x dx$

1. Simplify the first term using $n \log a = \log a^n$ and $e^{\log a} = a$:

$$e^{(x \log 5)} = e^{(\log 5^x)} = 5^x$$

The integral becomes:

$$I = \int 5^x e^x dx$$

2. Combine the exponential terms using $a^m b^m = (ab)^m$:

$$I = \int (5e)^x dx$$

3. Integrate using the formula $\int a^x dx = \frac{a^x}{\log a} + C$:

$$I = \frac{(5e)^x}{\log(5e)} + C$$

The correct option is 4.

Question17

The maximum value of Z for the linear programming problem maximize $Z = x + y$ subject to the constraints $x + 4y \leq 8, 2x + 3y \leq 12, 3x + y \leq 9, x \geq 0, y \geq 0$ is:

1. $3\frac{10}{11}$
2. $3\frac{9}{11}$
3. $4\frac{10}{11}$
4. $4\frac{9}{11}$

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: A

Solution:

$$\text{Maximize } Z = x + y$$

$$\text{Constraints: } (L1) \ x + 4y \leq 8, (L2) \ 2x + 3y \leq 12, (L3) \ 3x + y \leq 9, \ x \geq 0, y \geq 0$$

1. Find Corner Points (x_0, y_0) :

$$\text{Intersection of } x = 0 \text{ and } y = 0 \implies (0, 0)$$

$$\text{Intersection of } L1 \text{ and } x = 0 \implies 4y = 8 \implies (0, 2)$$

$$\text{Intersection of } L3 \text{ and } y = 0 \implies 3x = 9 \implies (3, 0)$$

Intersection of $L1$ and $L3$:

$$x + 4y = 8 \text{ and } 3x + y = 9$$

$$\text{Multiply } L3 \text{ by 4: } 12x + 4y = 36$$

$$\text{Subtract } L1: (12x + 4y) - (x + 4y) = 36 - 8$$

$$11x = 28 \implies x = \frac{28}{11}$$

$$\text{Substitute } x \text{ into } L3: 3\left(\frac{28}{11}\right) + y = 9 \implies y = 9 - \frac{84}{11} = \frac{15}{11}$$

$$\text{Corner Point 4: } \left(\frac{28}{11}, \frac{15}{11}\right)$$

2. Evaluate $Z = x + y$ at the corner points:

$$Z(0,0) = 0$$

$$Z(0,2) = 2$$

$$Z(3,0) = 3$$

$$Z\left(\frac{28}{11}, \frac{15}{11}\right) = \frac{28}{11} + \frac{15}{11} = \frac{43}{11}$$

Convert $\frac{43}{11}$ to mixed fraction:

$$\frac{43}{11} = 3\frac{10}{11}$$

The maximum value of Z is $3\frac{10}{11}$ (Option 1).

Question 18

Solution of the inequality $\frac{2x+3}{4x-5} \geq 0$ is

1. $\left(-\frac{3}{2}, \frac{5}{4}\right)$
2. $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{5}{4}, \infty\right)$
3. $\left[-\frac{3}{2}, \frac{5}{4}\right)$
4. $\left(-\infty, -\frac{3}{2}\right] \cup \left(\frac{5}{4}, \infty\right)$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

We are solving $\frac{2x+3}{4x-5} \geq 0$.

1. Find Critical Points:

$$\text{Numerator: } 2x + 3 = 0 \implies x = -\frac{3}{2}$$

$$\text{Denominator: } 4x - 5 = 0 \implies x = \frac{5}{4}$$

2. Determine Endpoint Inclusion:

Since $\geq$ is used, $x = -\frac{3}{2}$ is included.

Since the denominator cannot be zero, $x = \frac{5}{4}$ is excluded.

3. Check value for intervals :

Test intervals $\left(-\infty, -\frac{3}{2}\right), \left(-\frac{3}{2}, \frac{5}{4}\right), \left(\frac{5}{4}, \infty\right)$.

Interval $\left(-\infty, -\frac{3}{2}\right)$ (Test $x = -2$): $\frac{(-)}{(-)} = (+) \geq 0$ (Satisfied)

Interval $\left(-\frac{3}{2}, \frac{5}{4}\right)$ (Test $x = 0$): $\frac{(+)}{(-)} = (-) < 0$ (Not Satisfied)

Interval $\left(\frac{5}{4}, \infty\right)$ (Test $x = 2$): $\frac{(+)}{(+)} = (+) \geq 0$ (Satisfied)

4. Write the Solution Set:

The solution is the union of the satisfied intervals, applying endpoint rules:

$$\left(-\infty, -\frac{3}{2}\right] \cup \left(\frac{5}{4}, \infty\right)$$

The correct option is 4.

Question 19

The probability distribution of a random discrete variable is given

X	-1	0	1	2	3
P(X)	0.1	p	0.3	q	r

If it is known that $P(X=1)$ is the mean of $P(X=0)$ and $P(X=2)$.

Then the value of r is:

- 0
- 0.1
- 0.2
- 0.3

Options:

- 1
- 2
- 3
- 4

Answer: A

Solution:

The probabilities are: $P(X = -1) = 0.1, P(X = 0) = p, P(X = 1) = 0.3, P(X = 2) = q, P(X = 3) = r$.

1. Apply the rule that the sum of all probabilities is 1:

$$\sum P(X) = 0.1 + p + 0.3 + q + r = 1$$

$$p + q + r = 1 - 0.4$$

$$p + q + r = 0.6 \quad (1)$$

2. Apply the given condition: $P(X = 1)$ is the mean of $P(X = 0)$ and $P(X = 2)$:

$$P(X = 1) = \frac{P(X = 0) + P(X = 2)}{2}$$

$$0.3 = \frac{p + q}{2}$$

$$p + q = 0.3 \times 2$$

$$p + q = 0.6 \quad (2)$$

3. Substitute (2) into (1) to solve for r :

$$(0.6) + r = 0.6$$

$$r = 0.6 - 0.6$$

$$r = 0$$

The value of r is 0 (Option 1).

Question20

Data available for profit (₹ Thousands) of a company as

Year	2004	2005	2006	2007	2008	2009	2010
Profit(000)	114	130	126	144	138	156	164

Based on the above data using least square method the trend value for the year ‘2007’ is

- 1. 147.64
- 2. 138.86
- 3. 130.57
- 4. 131.22

Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

The trend line is given by $Y = a + bX$.

Since $n = 7$ (odd), we set the origin at 2007, making $X = 0$ for 2007.

The coded values for X are $\{-3, -2, -1, 0, 1, 2, 3\}$.

1. Calculate the required sums from the data:

$$n = 7$$

$$\sum Y = 114 + 130 + 126 + 144 + 138 + 156 + 164 = 972$$

$$\sum X = -3 - 2 - 1 + 0 + 1 + 2 + 3 = 0$$

$$\sum XY = -342 - 260 - 126 + 0 + 138 + 312 + 492 = 214$$

$$\sum X^2 = 9 + 4 + 1 + 0 + 1 + 4 + 9 = 28$$

2. Calculate the constant a :

$$a = \frac{\sum Y}{n} = \frac{972}{7} \approx 138.857$$

3. Calculate the constant b :

$$b = \frac{\sum XY}{\sum X^2} = \frac{214}{28} \approx 7.6428$$

4. Find the Trend Value for 2007:

The year 2007 corresponds to the coded value $X = 0$.

$$Y_{2007} = a + b(0) = a$$

$$Y_{2007} = 138.857 \approx 138.86$$

The trend value for the year 2007 is 138.86 (Option 2).

Question 21

If $y = e^{\frac{1}{2} \log(1 + \tan^2 x)}$, then $\frac{d^2 y}{dx^2}$ is equal to:

1. $\sec x$
2. $\frac{1}{2} \left(\frac{1}{1 + \tan^2 x} \right) e^{\log(1 + \tan^2 x)}$
3. $\sec x (\sec^2 x + \tan^2 x)$
4. $\sec x \tan x$

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: C

Solution:

$$\text{Given } y = e^{\frac{1}{2} \log(1 + \tan^2 x)}$$

1. Simplify the expression for y :

$$\text{Use } n \log a = \log a^n:$$

$$y = e^{\log((1 + \tan^2 x)^{1/2})}$$

$$\text{Use } 1 + \tan^2 x = \sec^2 x:$$

$$y = e^{\log((\sec^2 x)^{1/2})}$$

Simplify the exponent:

$$y = e^{\log(\sec x)}$$

$$\text{Use } e^{\log a} = a:$$

$$y = \sec x$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sec x)$$

$$\frac{dy}{dx} = \sec x \tan x$$

2. Calculate the Second Derivative ($\frac{d^2y}{dx^2}$):

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\sec x \tan x)$$

Apply the Product Rule: $\frac{d}{dx}(uv) = u'v + uv'$

$$\frac{d^2y}{dx^2} = \left(\frac{d}{dx} \sec x \right) \tan x + \sec x \left(\frac{d}{dx} \tan x \right)$$

$$\frac{d^2y}{dx^2} = (\sec x \tan x) \cdot \tan x + \sec x \cdot (\sec^2 x)$$

$$\frac{d^2y}{dx^2} = \sec x \tan^2 x + \sec^3 x$$

Factor out $\sec x$:

$$\frac{d^2y}{dx^2} = \sec x (\tan^2 x + \sec^2 x)$$

The result is $\sec x (\sec^2 x + \tan^2 x)$ (Option 3).

Question22

Which of the given values of x and y make the following pair of matrices equal?

$$\begin{bmatrix} 2x-1 & 4 \\ y-1 & 3+2x \end{bmatrix} \text{ and } \begin{bmatrix} 0 & y-2 \\ 5 & 4 \end{bmatrix}$$

1. $x = \frac{1}{4}, y = 6$
2. $x = \frac{1}{2}, y = 6$
3. $x = 6, y = \frac{1}{2}$
4. $x = 2, y = 6$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

For two matrices to be equal, their corresponding elements must be equal.

1. Equate element a_{11} (First row, first column):

$$2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

2. Equate element a_{21} (Second row, first column):

$$y - 1 = 5$$

$$y = 6$$

3. Verify element a_{22} (Second row, second column):

$$3 + 2x = 3 + 2 \cdot \frac{1}{2} = 3 + 1 = 4$$

This is equal to the right side value of 4.

The values are $x = \frac{1}{2}$ and $y = 6$ (Option 2).

Question23

If under pure competition demand and supply functions are given by $p = \sqrt{10 - x}$ and $p = \frac{1}{2}(x - 2)$ respectively, where p is price per unit and x is quantity, then the consumer surplus is:

1. $\frac{2}{3} [11 - 10\sqrt{10}]$
2. $\frac{10}{3} [1 - 2\sqrt{10}]$
3. $\frac{2}{3} [10\sqrt{10} - 26]$
4. $\frac{2}{3} [11 + 10\sqrt{10}]$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Consumer Surplus (CS) is calculated as $\int_0^{x_0} (p_D - p_0)dx$ after finding equilibrium (x_0, p_0) .

1. Find Equilibrium Point (x_0, p_0) :

$$\sqrt{10-x} = \frac{1}{2}(x-2)$$

$$4(10-x) = (x-2)^2$$

$$x^2 - 36 = 0 \implies x_0 = 6$$

$$p_0 = \frac{1}{2}(6-2) = 2$$

2. Calculate CS:

$$CS = \int_0^6 (\sqrt{10-x} - 2) dx$$

$$CS = \left[-\frac{2}{3}(10-x)^{3/2} - 2x \right]_0^6$$

$$CS = \left(-\frac{2}{3}(4)^{3/2} - 12 \right) - \left(-\frac{2}{3}(10)^{3/2} - 0 \right)$$

$$CS = -\frac{16}{3} - \frac{36}{3} + \frac{20\sqrt{10}}{3}$$

$$CS = \frac{20\sqrt{10} - 52}{3}$$

$$CS = \frac{2}{3}[10\sqrt{10} - 26]$$

The consumer surplus is $\frac{2}{3}[10\sqrt{10} - 26]$ (Option 3).

Question24

The ratio of the speeds of a motor boat and that of the current of water is 26:4. The boat goes certain distance against the current in 6 hrs. The time taken by the boat to come back is _____

1. 3 hrs 48 minutes
2. 4 hrs 24 minutes
3. 4 hrs 42 minutes
4. 3 hrs 56 minutes

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

1. Define Speeds:

Let Boat Speed $S_B = 26k$ and Current Speed $S_C = 4k$.

$$S_{Up} = S_B - S_C = 26k - 4k = 22k$$

$$S_{Down} = S_B + S_C = 26k + 4k = 30k$$

2. Calculate Distance (D):

$$T_{Up} = 6 \text{ hrs}$$

$$D = S_{Up} \cdot T_{Up} = 22k \cdot 6 = 132k$$

3. Calculate Downstream Time (T_{Down}):

$$T_{Down} = \frac{D}{S_{Down}} = \frac{132k}{30k}$$

$$T_{Down} = \frac{132}{30} = 4.4 \text{ hrs}$$

4. Convert to Hours and Minutes:

$$T_{Down} = 4 \text{ hrs} + 0.4 \cdot 60 \text{ min}$$

$$T_{Down} = 4 \text{ hrs } 24 \text{ minutes}$$

Question25

In a kilometer race P runs at 10 m/sec, Q runs at 5 m/sec then which of the following statement(s) is/ are correct ?

- (A) P wins the race in 100 sec
- (B) Q wins the race in 100 sec
- (C) P defeats Q by 500 m
- (D) Q defeats P by 500 m

Choose the **correct** answer from the options given below:

- 1. (A) only
- 2. (A) and (C) only
- 3. (B) and (D) only
- 4. (D) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Total Race Distance $D = 1 \text{ km} = 1000 \text{ m}$

Speed of P (S_P) = 10 m/sec

Speed of Q (S_Q) = 5 m/sec

1. Time taken by P (T_P):

$$T_P = \frac{D}{S_P} = \frac{1000}{10} = 100 \text{ sec}$$

Statement (A) is CORRECT.

2. Time taken by Q (T_Q):

$$T_Q = \frac{D}{S_Q} = \frac{1000}{5} = 200 \text{ sec}$$

Statement (B) is INCORRECT.

3. Calculate Margin:

P wins as $T_P < T_Q$.

Distance covered by Q when P finishes (at $T_P = 100 \text{ sec}$):

$$\text{Distance}_Q = S_Q \cdot T_P = 5 \cdot 100 = 500 \text{ m}$$

$$\text{Margin} = \text{Total Distance} - \text{Distance}_Q = 1000 - 500 = 500 \text{ m}$$

P defeats Q by 500 m.

Statement (C) is CORRECT.

Statement (D) is INCORRECT.

The correct statements are (A) and (C) (Option 2).

Question26

The unit's digit of 2^{2025} is

1. 1
2. 2
3. 3
4. 4

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

1. Analyze the pattern of the unit's digits of powers of 2:

The unit's digit cycle is 2, 4, 8, 6, which has a length of 4.

2. Determine the remainder of the exponent divided by the cycle length:

We calculate $2025 \div 4$.

$$2025 = 4 \cdot 506 + 1$$

The remainder is 1.

3. Identify the Unit's Digit:

The unit's digit is the same as the 1st element in the cycle.

The 1st element is 2.

The unit's digit of 2^{2025} is 2 (Option 2).

Question27

Three pipes A, B & C fill a tank in 3 hours working simultaneously. The pipe C is twice as faster as B and B is twice as fast as A. The time taken by pipe A alone to fill the tank is:

1. 27 hours
2. 81 hours
3. 12 hours
4. 21 hours

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

The efficiency of a pipe is its rate of filling the tank.

Let the rate of pipe A be R_A

Pipe B is twice as fast as A, so its rate is $R_B = 2R_A$

Pipe C is twice as fast as B, so its rate is $R_C = 2R_B = 4R_A$

2. Calculate Combined Efficiency (R_{Total}):

$$R_{\text{Total}} = R_A + 2R_A + 4R_A = 7R_A$$

3. Calculate Total Work (Capacity, W):

Total Work is equal to the combined rate multiplied by the total time.

$$W = R_{\text{Total}} \times \text{Time}_{\text{Total}}$$

$$W = 7R_A \times 3 = 21R_A$$

4. Calculate Time taken by Pipe A alone (T_A):

Time taken by A is the Total Work divided by A's rate.

$$T_A = \frac{W}{R_A}$$

$$T_A = \frac{21R_A}{R_A} = 21 \text{ hours}$$

The time taken by pipe A alone is 21 hours (Option 4).

Question28

The value of $\int_{-a}^a f(x)dx$ where $f(x) = \frac{7^x}{1+7^x}$, is:

1. a
2. $2a$
3. $\frac{a}{2}$
4. $7a$

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: A

Solution:

Let I be the given integral:

$$I = \int_{-a}^a \frac{7^x}{1+7^x} dx \quad (1)$$

1. Apply the property $\int_{-a}^a f(x)dx = \int_{-a}^a f(-x)dx$:

$$I = \int_{-a}^a \frac{7^{-x}}{1+7^{-x}} dx$$

Simplify by multiplying by 7^x :

$$I = \int_{-a}^a \frac{1}{1+7^x} dx$$

2. Add (1) and (2):

$$2I = \int_{-a}^a \left(\frac{7^x}{1+7^x} + \frac{1}{1+7^x} \right) dx$$

$$2I = \int_{-a}^a \frac{7^x + 1}{1+7^x} dx$$

$$2I = \int_{-a}^a 1 dx$$

3. Evaluate the integral:

$$2I = [x]_{-a}^a = a - (-a) = 2a$$

$$I = a$$

The value of the integral is a (Option 1).

Question29

Mameesh took a loan of Rs 9,00,800 from bank at an interest rate of 6% per annum for 10 years. If she has to pay the loan back with the help of equal monthly installments (EMI). Then, the EMI using reduced balance method is (approx):

(Given: $(1.005)^{-120} = 0.5496$)

1. Rs 12,000
2. Rs 9,000
3. Rs 10,000
4. Rs 9,820

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Principal Loan Amount (P) = Rs 9,00,800

Annual Interest Rate (r) = 6% = 0.06

Time Period in years (t) = 10

Compounding Frequency (n) = 12 (monthly)

1. Calculate Monthly Interest Rate (i) and Total Payments (N):

$$i = \frac{r}{n} = \frac{0.06}{12} = 0.005$$

$$N = t \times n = 10 \times 12 = 120$$

2. Use the EMI Formula:

$$\text{EMI} = P \cdot \frac{i}{1 - (1 + i)^{-N}}$$

3. Substitute values and the given factor $(1.005)^{-120} = 0.5496$:

$$EMI = 9,00,800 \cdot \frac{0.005}{1 - 0.5496}$$

$$EMI = 9,00,800 \cdot \frac{0.005}{0.4504}$$

$$EMI \approx 9,00,800 \cdot 0.01110124$$

$$EMI \approx 9,999.88 \text{ Rs}$$

The approximate EMI is Rs 10,000 (Option 3).

Question30

Match List-1 with List-2

List-1	List-2
A fire in a factory causing production delay for some time is	Secular trend
Technological progress is	Seasonal trend
The rise in prices before big festival is example of	Irregular trend
Rise and fall of share market is	Cyclic trend

Choose the Correct answer from the options given below :-

- 1. (A) – (III), (B) – (II), (C) – (IV), (D) – (I)
- 2. (A) – (III), (B) – (I), (C) – (II), (D) – (IV)
- 3. (A) – (II), (B) – (I), (C) – (IV), (D) – (III)
- 4. (A) – (II), (B) – (IV), (C) – (I), (D) – (III)

Options:

- A.
1
- B.
2
- C.
3
- D.
4

Answer: B

Solution:

1. Analysis of List-1 Items and Matching to List-2 Components:

(A) A fire in a factory (Sudden, unpredictable event) $\implies$ Irregular trend (III)

(B) Technological progress (Long-term, smooth movement) $\implies$ Secular trend (I)

(C) Rise in prices before festival (Regular, within-year variation) $\implies$ Seasonal trend (II)

(D) Rise and fall of share market (Long-term, non-periodic oscillation) $\implies$ Cyclic trend (IV)

2. The correct matching is:

(A) - (III), (B) - (I), (C) - (II), (D) - (IV)

The correct option is 2.

Question31

The amount of money needed to ensure for a prize of *Rs* 5000 at the beginning of each year indefinitely if money is worth 5% compounded annually is:

1. 1, 07, 500
2. 1, 10, 000
3. 1, 02, 500
4. 1, 05, 000

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

Since the prize is paid at the beginning of each year indefinitely, we use the Present Value of Perpetuity Due formula:

$$PV = \text{Payment} + \frac{\text{Payment}}{\text{Interest Rate}}$$

1. Identify parameters:

Payment (A) = Rs 5,000

Interest Rate (i) = 5% = 0.05

2. Calculate Present Value (PV):

$$PV = 5,000 + \frac{5,000}{0.05}$$

$$PV = 5,000 + 1,00,000$$

$$PV = 1,05,000$$

The amount needed is Rs 1,05,000 (Option 4).

Question 32

For 95% confidence interval for a population mean reported to be 132 to 142 with standard deviation $\sigma = 17.85$ then the sample size used in this case, is:

[Given that: $Z_{.0125} = 1.96$]

1. 36
2. 49
3. 64
4. 81

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: B

Solution:

The Margin of Error (E) is half the width of the interval:

$$E = \frac{142 - 132}{2} = \frac{10}{2} = 5$$

The formula for Margin of Error is:

$$E = Z \frac{\sigma}{\sqrt{n}}$$

Rearranging to solve for sample size (n):

$$n = \left(\frac{Z\sigma}{E} \right)^2$$

Substitute known values ($Z = 1.96$, $\sigma = 17.85$, $E = 5$):

$$n = \left(\frac{1.96 \times 17.85}{5} \right)^2$$

$$n = \left(\frac{34.986}{5} \right)^2$$

$$n = (6.9972)^2$$

$$n \approx 48.96$$

Question 33

400 g of apple vinegar has 40% apple juice in it. The amount of apple juice, which should be added to make it 60% in apple vinegar, is:

1. 100 g
2. 200 g
3. 150 g
4. 170 g

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

1. Initial amount of apple juice:

$$\text{Initial Juice} = 400 \times \frac{40}{100} = 160 \text{ g}$$

2. Define final state:

Let x be the amount of juice added.

$$\text{Final Juice} = 160 + x$$

$$\text{Final Total Mixture} = 400 + x$$

$$\text{Target Concentration} = 60\% = 0.60$$

3. Set up the equation:

$$160 + x = 0.60 \times (400 + x)$$

4. Solve for x :

$$160 + x = 240 + 0.60x$$

$$x - 0.60x = 240 - 160$$

$$0.40x = 80$$

$$x = \frac{80}{0.40}$$

$$x = 200 \text{ g}$$

The amount of apple juice to be added is 200 g (Option 2).

Question 34

A random sample of size 16 has 53 as mean. The sum of the squares of the deviations taken from mean is 150. If the population mean is 56 then the value of t -test statistic is:

1. -3.79
2. -5.79
3. -4.79
4. -2.79

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

Given:

Sample size $n = 16$

Sample mean $\bar{x} = 53$

Population mean $\mu = 56$

Sum of squares $\sum (x_i - \bar{x})^2 = 150$

1. Calculate Sample Standard Deviation (s):

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

$$s = \sqrt{\frac{150}{16 - 1}} = \sqrt{\frac{150}{15}} = \sqrt{10}$$

$$s \approx 3.162$$

2. Calculate the t -test statistic:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{53 - 56}{3.162/\sqrt{16}}$$

$$t = \frac{-3}{3.162/4}$$

$$t \approx \frac{-3}{0.7905}$$

$$t \approx -3.79$$

The value of the t -test statistic is approximately -3.79 (Option 1).

Question35

If $a_{ij} = i + 3j$, then the matrix of order 2 with elements as a_{ij} is

1. $\begin{bmatrix} 4 & 7 \\ 5 & 12 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 7 \\ 5 & 12 \end{bmatrix}$

3. $\begin{bmatrix} 4 & 7 \\ 5 & 8 \end{bmatrix}$

4. $\begin{bmatrix} 3 & 7 \\ 5 & 14 \end{bmatrix}$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

The element a_{ij} is defined by $a_{ij} = i + 3j$ for a 2×2 matrix.

1. Calculate a_{11} (i=1, j=1):

$$a_{11} = 1 + 3(1) = 4$$

2. Calculate a_{12} (i=1, j=2):

$$a_{12} = 1 + 3(2) = 7$$

3. Calculate a_{21} (i=2, j=1):

$$a_{21} = 2 + 3(1) = 5$$

4. Calculate a_{22} (i=2, j=2):

$$a_{22} = 2 + 3(2) = 8$$

The resulting matrix is:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 5 & 8 \end{bmatrix}$$

The correct matrix is Option 3.
