

Question1

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$$

is equal to:

1. $\frac{1}{2} \log |e^{2x} - e^{-2x}| + C$, where C is an arbitrary constant.
2. $\log |e^{2x} - e^{-2x}| + C$, where C is an arbitrary constant.
3. $\frac{1}{2} \log |e^{2x} + e^{-2x}| + C$, where C is an arbitrary constant.
4. $\log |e^{2x} + e^{-2x}| + C$, where C is an arbitrary constant.

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

1. Choose a substitution:

Let u be the denominator:

$$u = e^{2x} + e^{-2x}$$

2. Find the differential du :

Take the derivative of u with respect to x :

$$\frac{du}{dx} = \frac{d}{dx}(e^{2x}) + \frac{d}{dx}(e^{-2x})$$

Using the chain rule, $\frac{d}{dx}(e^{kx}) = ke^{kx}$:

$$\frac{du}{dx} = (2e^{2x}) + (-2e^{-2x}) = 2e^{2x} - 2e^{-2x} = 2(e^{2x} - e^{-2x})$$

Therefore, the differential du is:

$$du = 2(e^{2x} - e^{-2x})dx$$

This means the numerator, $(e^{2x} - e^{-2x})dx$, is equal to $\frac{1}{2}du$:

$$(e^{2x} - e^{-2x})dx = \frac{1}{2}du$$

3. Substitute u and du into the integral:

$$I = \int \frac{1}{u} \left(\frac{1}{2} du \right) = \frac{1}{2} \int \frac{1}{u} du$$

4. Integrate with respect to u :

The integral of $\frac{1}{u}$ is $\log |u|$:

$$I = \frac{1}{2} \log |u| + C$$

where C is the arbitrary constant of integration.

5. Substitute back for x :

Replace u with $e^{2x} + e^{-2x}$:

$$I = \frac{1}{2} \log |e^{2x} + e^{-2x}| + C$$

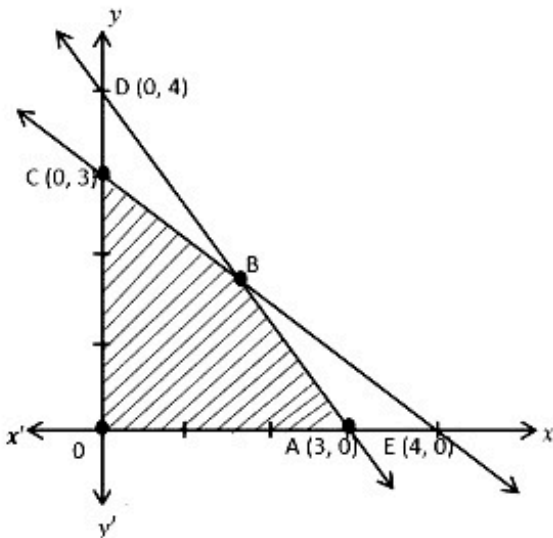
Since $e^{2x} + e^{-2x}$ is always positive, the absolute value is technically not required, but including it is mathematically correct for the general form $\int \frac{1}{u} du =$

$$I = \frac{1}{2} \log(e^{2x} + e^{-2x}) + C$$

[The correct option is 3.]

Question 2

Linear inequalities corresponding to the shaded feasible region OABCO in the given figure are



1. $4x + 3y \leq 12, 3x + 4y \leq 12, x \geq 0, y \geq 0$
2. $4x + 3y \geq 12, 3x + 4y \geq 12, x \geq 0, y \geq 0$
3. $4x + 3y \leq 12, 3x + 4y \geq 12, x \geq 0, y \geq 0$
4. $4x + 3y \geq 12, 3x + 4y \leq 12, x \geq 0, y \geq 0$

Options:

A.

1

B.

2

C.
3
D.
4

Answer: A

Solution:

1. Axes (Non-Negativity)

The region is in the first quadrant:

$x \geq 0, \quad y \geq 0$

2. Line 1: Through D(0, 4) and A(3, 0)

Intercepts: x -intercept $a = 3$, y -intercept $b = 4$.

Equation (Intercept Form): $\frac{x}{3} + \frac{y}{4} = 1 \implies 4x + 3y = 12$

Inequality: Since the region includes the origin (0, 0), the constraint is:

$4x + 3y \leq 12$

3. Line 2: Through C(0, 3) and E(4, 0)

Intercepts: x -intercept $a = 4$, y -intercept $b = 3$.

Equation (Intercept Form): $\frac{x}{4} + \frac{y}{3} = 1 \implies 3x + 4y = 12$

Inequality: Since the region includes the origin (0, 0), the constraint is:

$3x + 4y \leq 12$

Complete System of Constraints

$x \geq 0, \quad y \geq 0, \quad 4x + 3y \leq 12, \quad 3x + 4y \leq 12$

[The correct option is 1.]

Question3

Match List-I with List-II

List-I	List-II
(A) The degree of differential equation $\frac{d^3y}{dx^3} = e^{\frac{dy}{dx}}$	(I) 2
(B) The order of differential equation $\left(\frac{dy}{dx}\right)^2 + \frac{d^3y}{dx^3} = 0$	(II) 4
(C) The sum of order and degree of differential equation $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^5 = x$	(III) not defined
(D) The number of arbitrary constants in the general solution of a differential equation of order 2	(IV) 3

Choose the correct answer from the options given below:

1. (A) - (I), (B) - (IV), (C) - (II), (D) - (III)
2. (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

3. (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

4. (A) - (IV), (B) - (I), (C) - (II), (D) - (III)

Options:

A.

1

B.

2

C.

3

D.

4

Answer: C

Solution:

The **order** of a differential equation is the **order of the highest derivative** present in the equation.

The **degree** of a differential equation is the **power of the highest-order derivative** after the equation has been rationalized (i.e., cleared of any fractional powers of the derivatives).

A. The degree of differential equation $\frac{d^3y}{dx^3} = e^{\frac{dy}{dx}}$ is not defined (due to $e^{\frac{dy}{dx}}$ term) $\rightarrow$ (III) not defined

B. The order of differential equation $\left(\frac{dy}{dx}\right)^2 + \frac{d^3y}{dx^3} = 0$ is 3 $\rightarrow$ (IV) 3

C. The sum of order and degree of differential equation $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^5 = x$ is $(3 + 1 = 4) \rightarrow$ (II) 4

D. The number of arbitrary constants in the general solution of a differential equation of order 2 is 2 $\rightarrow$ (I) 2

The final match is: (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

[The correct option is 3.]

Question4

$$\text{If } \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} A \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

then matrix 'A' is

1. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 1 \\ 3 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Solving the Matrix Equation for A

The given equation is $BAC = D$, where:

$$B = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To solve for A , we pre-multiply by B^{-1} and post-multiply by C^{-1} :

$$A = B^{-1}DC^{-1}$$

1. Calculate Inverses

$$\text{For a } 2 \times 2 \text{ matrix } M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad M^{-1} = \frac{1}{|M|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Inverse of B

$$|B| = 3(1) - 1(2) = 1.$$

$$B^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

Inverse of C

$$|C| = 2(1) - 1(1) = 1.$$

$$C^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

2. Compute $B^{-1}D$

$$B^{-1}D = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + (-1)(0) & 1(1) + (-1)(1) \\ -2(1) + 3(0) & -2(1) + 3(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

3. Compute $A = (B^{-1}D)C^{-1}$

$$A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1(1) + 0(-1) & 1(-1) + 0(2) \\ -2(1) + 1(-1) & -2(-1) + 1(2) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

[The correct option is 3.]

Question5

For the function $f(x) = -2x^3 + 3x^2 + 36x - 10$, which of the following is/are true?

- (A) f is increasing in $(-\infty, -2)$
- (B) f is increasing in $(-2, 3)$
- (C) f is decreasing in $(-\infty, -2)$
- (D) f is decreasing in $(3, \infty)$

Choose the correct answer from the options given below:

1. (B) and (D) only
2. (B), (C) and (D) only
3. (A) and (D) only
4. (B) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Analysis of Monotonicity for $f(x) = -2x^3 + 3x^2 + 36x - 10$

Function: $f(x) = -2x^3 + 3x^2 + 36x - 10$

1. Derivative and Critical Points:

$$f'(x) = -6x^2 + 6x + 36$$

Set $f'(x) = 0$:

$$-6x^2 + 6x + 36 = 0$$

$$-6(x^2 - x - 6) = 0 \implies -6(x - 3)(x + 2) = 0$$

Critical points are $x = -2$ and $x = 3$.

2. Check Intervals ($f'(x)$ sign):

Interval $(-\infty, -2)$: $f'(-3) = -6((-3)^2 - (-3) - 6) = -6(9 + 3 - 6) = -36 < 0 \implies$ **Decreasing (C is TRUE)**

Interval $(-2, 3)$: $f'(0) = -6(0 - 0 - 6) = 36 > 0 \implies$ **Increasing (B is TRUE)**

Interval $(3, \infty)$: $f'(4) = -6(4^2 - 4 - 6) = -6(16 - 10) = -36 < 0 \implies$ **Decreasing (D is TRUE)**

3. Conclusion:

Statements (B), (C), and (D) are TRUE.

[The correct option is 2.]

Question6

If $y = \frac{1}{\sqrt[3]{1-x^3}}$, then $\frac{dy}{dx}$ is equal to:

1. $x(1-x^3)^{-\frac{4}{3}}$
2. $x^2(1-x^3)^{-\frac{4}{3}}$
3. $(1-x^3)^{-\frac{4}{3}}$
4. $-x^2(1-x^3)^{-\frac{4}{3}}$

Options:

- A. 1
- B. 2

C. 3

D. 4

Answer: B

Solution:

Function: $y = \frac{1}{\sqrt[3]{1-x^3}}$

1. Rewrite y using exponent notation:

$$y = (1 - x^3)^{-\frac{1}{3}}$$

2. Differentiate $\frac{dy}{dx}$ using the Chain Rule $\frac{d}{dx}[u^n] = nu^{n-1} \cdot \frac{du}{dx}$:

Let $u = (1 - x^3)$. Then $\frac{du}{dx} = -3x^2$.

$$\frac{dy}{dx} = -\frac{1}{3}(1 - x^3)^{-\frac{1}{3}-1} \cdot \frac{d}{dx}(1 - x^3)$$

$$\frac{dy}{dx} = -\frac{1}{3}(1 - x^3)^{-\frac{4}{3}} \cdot (-3x^2)$$

3. Simplify the expression:

$$\frac{dy}{dx} = \left(-\frac{1}{3}\right)(-3x^2)(1 - x^3)^{-\frac{4}{3}}$$

$$\frac{dy}{dx} = x^2(1 - x^3)^{-\frac{4}{3}}$$

[The correct option is 2.]

Question7

If x is real, the minimum value of $x^2 - 8x + 20$ is:

1. 4

2. 2

3. 0

4. - 1

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Function: $f(x) = x^2 - 8x + 20$

1. First Derivative and Critical Point:

$$f'(x) = 2x - 8$$

$$f'(x) = 0$$

$$\implies 2x = 8$$

$$\implies x = 4 \quad (\text{Critical Point})$$

2. Second Derivative Test (Confirmation):

$$f''(x) = \frac{d}{dx}(2x - 8) = 2$$

Since $f''(4) = 2 > 0$ (positive), the function has a **Local Minimum** at $x = 4$.

3. Minimum Value:

$$f(4) = (4)^2 - 8(4) + 20 = 16 - 32 + 20 = 4$$

The minimum value is **4**.

[The correct option is 1.]

Question8

If A is a square matrix of order 3 and $|A| = 5$, then the value of $|-AA^T|$ is:

1. 25
2. - 25
3. 5
4. - 5

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Given: $|A| = 5$, Order $n = 3$.

Required: $|-AA^T|$

$$= (-1)^3 |AA^T|$$

$$= -1 \cdot |A||A^T|$$

$$= -1 \cdot (5)(5)$$

$$= -25$$

Answer: -25

Question9

Consider an LPP: Maximise $Z = 50x + 15y$ subjected to constraints

$$x + y \leq 60,$$

$$5x + y \leq 100,$$

$$x, y \geq 0.$$

If the maximum value of Z occurs at $x = a$ and $y = \beta$, then the value of $a + \beta$ is:

$$1. 10$$

$$2. 60$$

$$3. 50$$

$$4. 40$$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

LPP: Maximize $Z = 50x + 15y$

Constraints:

$$1. x + y \leq 60$$

$$2. 5x + y \leq 100$$

$$3. x \geq 0, \quad y \geq 0$$

1. Find Vertices of Feasible Region:

The corner points are the intersection of the boundary lines ($x + y = 60$ and $5x + y = 100$) and the axes.

Intersection of Axes: $(0, 0)$

Line 1 intersects axes: $(60, 0)$ and $(0, 60)$

Line 2 intersects axes: $(20, 0)$ and $(0, 100)$

2. Find Intersection of $x + y = 60$ and $5x + y = 100$:

Subtracting the first equation from the second:

$$(5x + y) - (x + y) = 100 - 60$$

$$4x = 40 \implies x = 10$$

Substitute $x = 10$ into $x + y = 60$:

$$10 + y = 60$$

$$\implies y = 50$$

The vertex is $(10, 50)$.

3. Evaluate Z at Corner Points:

The corner points of the feasible region are $(0, 0)$, $(20, 0)$, $(0, 60)$, and $(10, 50)$.

$$Z(0, 0) = 50(0) + 15(0) = 0$$

$$Z(20, 0) = 50(20) + 15(0) = 1000$$

$$Z(0, 60) = 50(0) + 15(60) = 900$$

$$Z(10, 50) = 50(10) + 15(50) = 500 + 750 = \mathbf{1250} \quad (\text{Maximum})$$

4. Final Value:

The maximum value occurs at $\alpha = 10$ and $\beta = 50$.

$$\alpha + \beta = 10 + 50 = \mathbf{60}$$

[The correct option is 2.]

Question10

If the matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible, then the value of λ is:

1. 0
2. -1
3. 1
4. $\frac{1}{2}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Condition for Non-Invertibility:

A matrix A is not invertible if and only if its determinant is zero, i.e., $|A| = 0$.

1. Calculate the Determinant $|A|$:

$$A = \begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$$

Expand along the second row (R_2) for simplicity (due to the zero element):

$$|A| = -\lambda \begin{vmatrix} -1 & 3 \\ 1 & 4 \end{vmatrix} + 0 \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} - 7 \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}$$

$$|A| = -\lambda[(-1)(4) - (3)(1)] + 0 - 7[(2)(1) - (-1)(-1)]$$

$$|A| = -\lambda[-4 - 3] - 7[2 - 1]$$

$$|A| = -\lambda(-7) - 7(1)$$

$$|A| = 7\lambda - 7$$

2. Set $|A| = 0$ and Solve for λ :

$$7\lambda - 7 = 0$$

$$7\lambda = 7$$

$$\lambda = 1$$

[The correct option is 3.]

Question11

If the matrix $\begin{bmatrix} 3 & 2a & -5 \\ 4 & 0 & b \\ -5 & 3 & 10 \end{bmatrix}$ is symmetric, then the value of $5a + 2b$ is:

1. 6
2. 10
3. 16
4. 20

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Condition for a Symmetric Matrix:

A square matrix A is symmetric if $A = A^T$, which means the elements must satisfy $\mathbf{a}_{ij} = \mathbf{a}_{ji}$.

The matrix is:

$$A = \begin{bmatrix} 3 & 2a & -5 \\ 4 & 0 & b \\ -5 & 3 & 10 \end{bmatrix}$$

1. Apply the symmetry condition to find a and b :

Element $\mathbf{a}_{12}$ must equal $\mathbf{a}_{21}$:

$$2a = 4$$

$$\implies \mathbf{a = 2}$$

Element $\mathbf{a}_{23}$ must equal $\mathbf{a}_{32}$:

$$b = 3$$

Element $a_{13}(-5)$ equals $a_{31}(-5)$, which is consistent.

2. Calculate the required value $5a + 2b$:

Substitute the values of $a = 2$ and $b = 3$:

$$5a + 2b = 5(2) + 2(3)$$

$$5a + 2b = 10 + 6$$

$$5a + 2b = \mathbf{16}$$

[The correct option is 3.]

Question12

The solution of the differential equation $y \, dx + (x - y^2) \, dy = 0$ is:

1. $3xy + x^3 = C$: C is an arbitrary constant
2. $3xy + y^3 = C$: C is an arbitrary constant
3. $3xy - y^3 = C$: C is an arbitrary constant
4. $3xy - x^3 = C$: C is an arbitrary constant

Options:

- A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Differential Equation (DE): $y \, dx + (x - y^2) \, dy = 0$

1. Rearrange into a Linear DE form $\left(\frac{dx}{dy} + P(y)x = Q(y)\right)$:

Divide by dy :

$$y \frac{dx}{dy} + x - y^2 = 0$$

$$y \frac{dx}{dy} + x = y^2$$

$$\frac{dx}{dy} + \left(\frac{1}{y}\right)x = y$$

This is a linear DE in x , where $P(y) = \frac{1}{y}$ and $Q(y) = y$.

2. Find the Integrating Factor (IF):

$$\text{IF} = e^{\int P(y) \, dy} = e^{\int \frac{1}{y} \, dy} = e^{\ln |y|} = y$$

(We assume $y > 0$ for simplicity, so $\text{IF} = y$.)

3. Find the Solution:

The general solution is $x \cdot (\text{IF}) = \int Q(y) \cdot (\text{IF}) \, dy + C$.

$$x \cdot y = \int y \cdot y \, dy + C_1$$

$$xy = \int y^2 \, dy + C_1$$

$$xy = \frac{y^3}{3} + C_1 \quad (\text{Let } C_1 \text{ be the constant})$$

4. Rearrange to Match Options:

Multiply the entire equation by 3:

$$3xy = y^3 + 3C_1$$

$$3xy - y^3 = 3C_1$$

Let $C = 3C_1$ (where C is the new arbitrary constant):

$$3xy - y^3 = C$$

[The correct option is 3.]

Question13

The value of $\int_0^1 \log_e \left(\frac{1}{x} - 1\right) dx$ is:

1. 1

2. 0

3. $\frac{3}{2}$

4. $\log 2$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Integral: $I = \int_0^1 \log_e \left(\frac{1}{x} - 1 \right) dx = \int_0^1 \log \left(\frac{1-x}{x} \right) dx$

Property: Use the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$. Here $a = 1$.

$$I = \int_0^1 \log \left(\frac{1-(1-x)}{1-x} \right) dx$$

$$I = \int_0^1 \log \left(\frac{x}{1-x} \right) dx$$

Logarithm Property: Use $\log(A/B) = -\log(B/A)$.

$$I = \int_0^1 -\log \left(\frac{1-x}{x} \right) dx$$

$$I = - \int_0^1 \log \left(\frac{1-x}{x} \right) dx$$

$$I = -I$$

Solve for I:

$$I = -I$$

$$2I = 0$$

$$I = 0$$

[The correct option is 2.]

Question14

Area of the region bounded by $y = x^2$ and the line $y = 16$ is

1. $\frac{256}{3}$ square units
2. $\frac{128}{3}$ square units
3. $\frac{64}{3}$ square units
4. $\frac{32}{3}$ square units

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

Area Calculation

The area of the region bounded by $y = x^2$ and the line $y = 16$ is:

$$\begin{aligned} A &= \int_{-4}^4 (16 - x^2) dx \\ &= 2 \int_0^4 (16 - x^2) dx \\ &= 2 \left[16x - \frac{x^3}{3} \right]_0^4 \\ &= 2 \left(\left(16(4) - \frac{4^3}{3} \right) - (0) \right) \\ &= 2 \left(64 - \frac{64}{3} \right) \\ &= 2 \left(\frac{192 - 64}{3} \right) \\ &= 2 \left(\frac{128}{3} \right) \\ &= \frac{256}{3} \text{ square units} \end{aligned}$$

[The correct option is 1.]

Question15

The probability distribution of a random variable x is, $P(x) = \frac{k}{2^x}$, $x = 0, 1, 2, 3$. Then

Match List-I with List-II

List-I	List-II
(A) k	(I) 2/15
(B) $P(x=1)$	(II) 1/5
(C) $P(1)$	(III) 8/15
(D) $P(x \geq 2)$	(IV) 4/15

Choose the correct answer from the options given below:

1. (A) - (III), (B) - (II), (C) - (I), (D) - (IV)
2. (A) - (IV), (B) - (II), (C) - (III), (D) - (I)
3. (A) - (IV), (B) - (III), (C) - (I), (D) - (II)
4. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

Probability Distribution Matching

The probability distribution is $P(x) = \frac{k}{2^x}$ for $x \in \{0, 1, 2, 3\}$.

Calculations

(A) Finding k

Using $\sum P(x) = 1$:

$$k \left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} \right) = 1$$

$$k \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \right) = 1$$

$$k \left(\frac{8+4+2+1}{8} \right) = 1$$

$$k \left(\frac{15}{8} \right) = 1 \implies k = \frac{8}{15} \rightarrow \text{(III)}$$

(B) Finding $P(x = 1)$

$$\text{With } k = \frac{8}{15}, P(x) = \frac{8}{15 \cdot 2^x}.$$

$$P(x = 1) = \frac{8}{15 \cdot 2^1} = \frac{4}{15} \rightarrow \text{(IV)}$$

(C) Finding $P(1 < x < 3)$

$$P(1 < x < 3) = P(x = 2):$$

$$P(x = 2) = \frac{8}{15 \cdot 2^2} = \frac{8}{60} = \frac{2}{15} \rightarrow \text{(I)}$$

(D) Finding $P(x \geq 2)$

$$P(x \geq 2) = P(x = 2) + P(x = 3):$$

$$P(x \geq 2) = \frac{2}{15} + \frac{8}{15 \cdot 8}$$

$$P(x \geq 2) = \frac{2}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5} \rightarrow \text{(II)}$$

Core Mathematics

Question1

If A is a skew-symmetric matrix of order 5, then $|adj A|$ is equal to

1. -1
2. 0
3. 5
4. -5

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Determinant of Adjoint of a Skew-Symmetric Matrix

1. Properties of Skew-Symmetric Matrices

A matrix A is skew-symmetric if $A^T = -A$.

The determinant of a skew-symmetric matrix of odd order (n) is always zero.

Given A is of order $n = 5$ (odd order), we have:

$$|A| = 0$$

2. Property of Adjugate Determinant

For any square matrix A of order n , the determinant of its adjugate (adjoint) is given by:

$$|\text{adj } A| = |A|^{n-1}$$

3. Calculation

Given $n = 5$ and $|A| = 0$:

$$|\text{adj } A| = |A|^{5-1} = |A|^4$$

Substitute $|A| = 0$:

$$|\text{adj } A| = (0)^4 = 0$$

[The correct option is 2.]

Question2

Relation R on the set $A = \{1, 2, \dots, 15\}$ defined as $R = \{(x, y) : y - 4x = 0\}$ is

1. Reflexive and transitive
2. Neither Reflexive nor symmetric
3. Reflexive but not symmetric
4. Not Reflexive but symmetric

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Properties of Relation $R = \{(x, y) : y - 4x = 0\}$

The set is $A = \{1, 2, \dots, 15\}$.

The relation is $R = \{(x, y) \in A \times A : y = 4x\}$.

The elements of R are:

$$x = 1 \implies y = 4 \implies (1, 4) \in R$$

$$x = 2 \implies y = 8 \implies (2, 8) \in R$$

$$x = 3 \implies y = 12 \implies (3, 12) \in R$$

$$x = 4 \implies y = 16 \notin A \implies \text{no more elements.}$$

$$R = \{(1, 4), (2, 8), (3, 12)\}.$$

1. Reflexivity

A relation is reflexive if $(x, x) \in R$ for all $x \in A$.

$$\text{Condition for } (x, x) \in R \text{ is } x = 4x \implies 3x = 0 \implies x = 0.$$

Since $0 \notin A$ and $(1, 1) \notin R$, R is Not Reflexive.

2. Symmetry

A relation is symmetric if whenever $(x, y) \in R$, then $(y, x) \in R$.

We have $(1, 4) \in R$.

For symmetry, $(4, 1)$ must be in R , but $1 \neq 4(4)$, so $(4, 1) \notin R$.

Therefore, R is Not Symmetric.

3. Transitivity (Optional Check)

Since there is no pair (a, b) and (b, c) in R , the condition for transitivity is vacuously true.

Therefore, R is Transitive.

4. Conclusion

R is not Reflexive and not Symmetric.

[The correct option is 2.]

Question3

Let $\vec{a}$ and $\vec{b}$ be unit vectors. If $\sqrt{3}\vec{a} - \vec{b}$ is a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is

1. $\frac{\pi}{6}$

2. $\frac{\pi}{4}$

3. $\frac{\pi}{3}$

4. $\frac{\pi}{2}$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Angle Between Unit Vectors

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

Given that $\vec{a}$ and $\vec{b}$ are unit vectors:

$$|\vec{a}| = 1, \quad |\vec{b}| = 1$$

Given that $|\sqrt{3}\vec{a} - \vec{b}|$ is a unit vector:

$$|\sqrt{3}\vec{a} - \vec{b}| = 1$$

1. Square the Magnitude Condition

Square both sides of the condition:

$$|\sqrt{3}\vec{a} - \vec{b}|^2 = 1^2$$

Using the property $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$:

$$(\sqrt{3}\vec{a} - \vec{b}) \cdot (\sqrt{3}\vec{a} - \vec{b}) = 1$$

$$(\sqrt{3}\vec{a} \cdot \sqrt{3}\vec{a}) - 2(\sqrt{3}\vec{a} \cdot \vec{b}) + (\vec{b} \cdot \vec{b}) = 1$$

$$3|\vec{a}|^2 - 2\sqrt{3}(\vec{a} \cdot \vec{b}) + |\vec{b}|^2 = 1$$

2. Substitute Known Values

Substitute $|\vec{a}| = 1$, $|\vec{b}| = 1$, and $\vec{a} \cdot \vec{b} = \cos \theta$:

$$3(1)^2 - 2\sqrt{3}(\cos \theta) + (1)^2 = 1$$

$$3 - 2\sqrt{3} \cos \theta + 1 = 1$$

$$4 - 2\sqrt{3} \cos \theta = 1$$

3. Solve for $\cos \theta$

$$2\sqrt{3} \cos \theta = 4 - 1$$

$$2\sqrt{3} \cos \theta = 3$$

$$\cos \theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Since $0 \leq \theta \leq \pi$, the angle is:

$$\theta = \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

[The correct option is 1.]

Question4

A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ respectively.

1. $\frac{3}{4}$

2. 1

3. $\frac{1}{3}$

4. $\frac{1}{6}$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Probability of a Problem being Solved

Let $P(A)$, $P(B)$, and $P(C)$ be the probabilities that students A, B, and C solve the problem, respectively.

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3}, \quad P(C) = \frac{1}{4}$$

The events are independent. The problem is solved if at least one student solves it, which is $P(A \cup B \cup C)$.

It is easier to calculate the probability that the problem is not solved and subtract it from 1.

$$P(\text{problem not solved}) = P(A' \cap B' \cap C')$$

$$\text{Since the events are independent, } P(A' \cap B' \cap C') = P(A')P(B')P(C').$$

1. Calculate Probabilities of Not Solving

$$P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(C') = 1 - P(C) = 1 - \frac{1}{4} = \frac{3}{4}$$

2. Calculate $P(\text{not solved})$

$$P(A' \cap B' \cap C') = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = \frac{6}{24} = \frac{1}{4}$$

3. Calculate $P(\text{solved})$

$$P(\text{solved}) = 1 - P(\text{not solved}) = 1 - \frac{1}{4} = \frac{3}{4}$$

[The correct option is 1.]

Question5

If x, y and z are real numbers such that $x + y + z = 0$, then the value of

$$\begin{vmatrix} 3x & -x + y & -x + z \\ x - y & 3y & z - y \\ x - z & y - z & 3z \end{vmatrix}$$

is

1. $3(xy + yz + zx)$

2. $xy + yz + zx$

3. 0

4. 1

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Determinant Simplification with $x + y + z = 0$

The given determinant is Δ :

$$\Delta = \begin{vmatrix} 3x & -x + y & -x + z \\ x - y & 3y & z - y \\ x - z & y - z & 3z \end{vmatrix}$$

Given the condition $x + y + z = 0$.

Apply Column Operation $C_1 \rightarrow C_1 + C_2 + C_3$

$$C_1 \text{ element} = (3x) + (-x + y) + (-x + z) = 3x - 2x + y + z = x + y + z$$

Since $x + y + z = 0$, the first column of the new determinant is all zeros:

$$\Delta = \begin{vmatrix} x + y + z & -x + y & -x + z \\ x + y + z & 3y & z - y \\ x + y + z & y - z & 3z \end{vmatrix}$$

Substitute $x + y + z = 0$:

$$\Delta = \begin{vmatrix} 0 & -x + y & -x + z \\ 0 & 3y & z - y \\ 0 & y - z & 3z \end{vmatrix}$$

Since one entire column (C_1) of the determinant is zero, the value of the determinant is **0**.

[The correct option is 3.]

Question6

The domain of $y = \cos^{-1}(x^2 - 4)$ is

1. $[0, \pi]$
2. $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$
3. $[-\sqrt{5}, -\sqrt{3}] \cap [-\sqrt{5}, \sqrt{3}]$
4. $[-1, 1]$

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: B

Solution:

Domain of $y = \cos^{-1}(x^2 - 4)$

1. Condition for Domain

The domain of the inverse cosine function $y = \cos^{-1}(u)$ requires that the argument u satisfies $-1 \leq u \leq 1$.

For $y = \cos^{-1}(x^2 - 4)$, the argument is $u = x^2 - 4$, so we must have:

$$-1 \leq x^2 - 4 \leq 1$$

2. Solve the Inequality

$$(i) \ x^2 - 4 \geq -1$$

$$x^2 \geq -1 + 4 \Rightarrow x^2 \geq 3$$

This holds for $x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

$$(ii) \ x^2 - 4 \leq 1$$

$$x^2 \leq 1 + 4 \Rightarrow x^2 \leq 5$$

This holds for $x \in [-\sqrt{5}, \sqrt{5}]$

3. Find the Intersection

The domain is the intersection of these two solution sets:

$$\text{Domain} = ((-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)) \cap [-\sqrt{5}, \sqrt{5}]$$

The intersection gives:

$$x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$$

[The correct option is 2.]

Question7

The integrating factor of the differential equation $\frac{dy}{dx} = x + xy$ is:

1. $e^{-\frac{x}{2}}$

2. $e^{-\frac{x^2}{2}}$

3. $e^{\frac{x^2}{2}}$

4. $e^{\frac{x}{2}}$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

Integrating Factor of a Differential Equation

1. Standard Form

The given differential equation is:

$$\frac{dy}{dx} = x + xy$$

Rearrange it into the standard linear differential equation form $\frac{dy}{dx} + P(x)y = Q(x)$:

$$\frac{dy}{dx} - xy = x$$

By comparison, $P(x) = -x$ and $Q(x) = x$.

2. Calculate the Integrating Factor (I.F.)

The integrating factor is calculated as:

$$\text{I.F.} = e^{\int P(x) dx}$$

Substitute $P(x) = -x$:

$$\text{I.F.} = e^{\int (-x) dx}$$

Perform the integration:

$$\int (-x) dx = -\frac{x^2}{2}$$

Therefore, the integrating factor is:

$$\text{I.F.} = e^{-x^2/2}$$

[The correct option is 2.]

Question8

The area (in sq. units) of the region bounded by the line $2y + x = 8$,

the x-axis and the lines $x = 2$ and $x = 4$ is

1. 19

2. 5

3. 2

4. $\frac{3}{2}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Integrating Factor of a Differential Equation

1. Standard Form

The given differential equation is:

$$\frac{dy}{dx} = x + xy$$

Rearrange it into the standard linear differential equation form $\frac{dy}{dx} + P(x)y = Q(x)$:

$$\frac{dy}{dx} - xy = x$$

By comparison, $P(x) = -x$ and $Q(x) = x$.

2. Calculate the Integrating Factor (I.F.)

The integrating factor is calculated as:

$$\text{I.F.} = e^{\int P(x) dx}$$

Substitute $P(x) = -x$:

$$\text{I.F.} = e^{\int (-x) dx}$$

Perform the integration:

$$\int (-x) dx = -\frac{x^2}{2}$$

Therefore, the integrating factor is:

$$\text{I.F.} = e^{-x^2/2}$$

[The correct option is 2.]

Question9

In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both.

One student is chosen at random. The probability that she fails in Physics if she has failed in Mathematics is

- 1. $\frac{2}{5}$
- 2. $\frac{1}{5}$
- 3. $\frac{9}{10}$
- 4. $\frac{3}{10}$

Options:

- A. 1
- B. 2
- C. 3

D. 4

Answer: A

Solution:

Conditional Probability

Let P be the event that a student fails in Physics, and M be the event that a student fails in Mathematics.

1. Given Probabilities

$$P(P) = 30\% = 0.30$$

$$P(M) = 25\% = 0.25$$

$$P(P \cap M) = 10\% = 0.10 \quad (\text{Failing in both})$$

2. Conditional Probability Formula

We need to find the probability that she fails in Physics given she has failed in Mathematics, which is $P(P|M)$.

$$P(P|M) = \frac{P(P \cap M)}{P(M)}$$

3. Calculation

Substitute the given values:

$$P(P|M) = \frac{0.10}{0.25} = \frac{10}{25}$$

Simplify the fraction:

$$P(P|M) = \frac{2}{5}$$

[The correct option is 1.]

Question10

An urn contains 5 red and 5 black balls. A ball is drawn at random, its color is noted and is returned to

Moreover, 2 additional balls of the same color are put in the urn and then a ball is drawn at random.

The probability that the second drawn ball is red, is:

1. $\frac{5}{12}$

2. $\frac{1}{2}$

3. $\frac{1}{3}$

4. $\frac{1}{4}$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

Urn Problem: Probability of Second Ball being Red

Let R_1 be the event that the first ball drawn is red, and B_1 be the event that the first ball drawn is black.

Let R_2 be the event that the second ball drawn is red.

We want to find $P(R_2)$.

1. Initial Probabilities and Conditions

Initial Urn: 5 Red (R), 5 Black (B). Total = 10 balls.

$$P(R_1) = \frac{5}{10} = \frac{1}{2}$$

$$P(B_1) = \frac{5}{10} = \frac{1}{2}$$

The ball is returned, and 2 additional balls of the same color are added.

2. Conditional Probabilities for R_2

We use the Law of Total Probability: $P(R_2) = P(R_2|R_1)P(R_1) + P(R_2|B_1)P(B_1)$

Case 1: R_1 occurs (First ball is Red)

The red ball is returned, and 2 additional red balls are added.

New Urn: $5 + 2 = 7$ R, 5 B. Total = 12 balls.

$$P(R_2|R_1) = \frac{7}{12}$$

Case 2: B_1 occurs (First ball is Black)

The black ball is returned, and 2 additional black balls are added.

New Urn: 5 R, $5 + 2 = 7$ B. Total = 12 balls.

$$P(R_2|B_1) = \frac{5}{12}$$

3. Calculate $P(R_2)$

$$P(R_2) = P(R_2|R_1)P(R_1) + P(R_2|B_1)P(B_1)$$

$$P(R_2) = \left(\frac{7}{12}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{12}\right)\left(\frac{1}{2}\right)$$

$$P(R_2) = \frac{1}{2}\left(\frac{7}{12} + \frac{5}{12}\right)$$

$$P(R_2) = \frac{1}{2}\left(\frac{12}{12}\right)$$

$$P(R_2) = \frac{1}{2}(1)$$

$$P(R_2) = \frac{1}{2}$$

[The correct option is 2.]

Question11

If A(3, 2), B(1, -1) and C(2, 1) are three vertices of a parallelogram ABCD, then its area (in sq. units) is

1. 1

2. $\frac{1}{2}$

3. 5

4. $\frac{5}{2}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Urn Problem: Probability of Second Ball being Red

Let R_1 be the event that the first ball drawn is red, and B_1 be the event that the first ball drawn is black.

Let R_2 be the event that the second ball drawn is red.

We want to find $P(R_2)$.

1. Initial Probabilities and Conditions

Initial Urn: 5 Red (R), 5 Black (B). Total = 10 balls.

$$P(R_1) = \frac{5}{10} = \frac{1}{2}$$

$$P(B_1) = \frac{5}{10} = \frac{1}{2}$$

The ball is returned, and 2 additional balls of the same color are added.

2. Conditional Probabilities for R_2

We use the Law of Total Probability: $P(R_2) = P(R_2|R_1)P(R_1) + P(R_2|B_1)P(B_1)$

Case 1: R_1 occurs (First ball is Red)

The red ball is returned, and 2 additional red balls are added.

New Urn: 5 + 2 = 7 R, 5 B. Total = 12 balls.

$$P(R_2|R_1) = \frac{7}{12}$$

Case 2: B_1 occurs (First ball is Black)

The black ball is returned, and 2 additional black balls are added.

New Urn: 5 R, 5 + 2 = 7 B. Total = 12 balls.

$$P(R_2|B_1) = \frac{5}{12}$$

3. Calculate $P(R_2)$

$$P(R_2) = P(R_2|R_1)P(R_1) + P(R_2|B_1)P(B_1)$$

$$P(R_2) = \left(\frac{7}{12}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{12}\right)\left(\frac{1}{2}\right)$$

$$P(R_2) = \frac{1}{2}\left(\frac{7}{12} + \frac{5}{12}\right)$$

$$P(R_2) = \frac{1}{2}\left(\frac{12}{12}\right)$$

$$P(R_2) = \frac{1}{2}(1)$$

$$P(R_2) = \frac{1}{2}$$

[The correct option is 2.]

Question12

Which of the following statements are true?

(A) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then x, y, z are called direction ratios of $\vec{r}$.

(B) For any two vectors $\vec{a}$ and $\vec{b}$, $\vec{a} + \vec{b} = \vec{b} + \vec{a}$.

(C) $\vec{a} \perp \vec{b}$ if and only if $\vec{a} \times \vec{b} = \vec{0}$.

(D) Projection of $\vec{b}$ on $\vec{a}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$.

Choose the **correct** answer from the options given below:

1. (B) and (C) only
2. (B), (C) and (D) only
3. (C) and (D) only
4. (A) and (B) only

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: D

Solution:

True Statements about Vectors

1. Analysis of Statements

(A) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then x, y, z are called direction ratios of $\vec{r}$.

This is *TRUE*. The components of a vector are its direction ratios.

(B) For any two vectors $\vec{a}$ and $\vec{b}$, $\vec{a} + \vec{b} = \vec{b} + \vec{a}$.

This is *TRUE*. Vector addition is commutative.

(C) $\vec{a} \perp \vec{b}$ if and only if $\vec{a} \times \vec{b} = \vec{0}$.

This is *FALSE*.

$\vec{a} \perp \vec{b}$ if and only if $\vec{a} \cdot \vec{b} = 0$.

$\vec{a} \times \vec{b} = \vec{0}$ if and only if $\vec{a}$ and $\vec{b}$ are *parallel* ($\vec{a} \parallel \vec{b}$).

(D) Projection of $\vec{b}$ on $\vec{a}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$.

This is *FALSE*.

The scalar projection (component) of $\vec{b}$ on $\vec{a}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$.

The vector projection of $\vec{b}$ on $\vec{a}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$.

2. Conclusion

The only correct statements are (A) and (B).

[The correct option is 4.]

Question13

If $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ and $A^4 = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$, then the value of k is:

1. 81
2. 27
3. 12
4. 36

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Problem Statement

We are given a matrix A and its fourth power A^4 :

$$A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \quad \text{and} \quad A^4 = \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$$

The task is to find the value of the constant k .

Matrix Powers Calculation

We calculate the powers of A to identify a pattern for A^n .

Calculate A^2

$$A^2 = A \cdot A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 0 \cdot 3 & 1 \cdot 0 + 0 \cdot 1 \\ 3 \cdot 1 + 1 \cdot 3 & 3 \cdot 0 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 \cdot 3 & 1 \end{pmatrix}$$

Calculate A^3

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 0 \cdot 3 & 1 \cdot 0 + 0 \cdot 1 \\ 6 \cdot 1 + 1 \cdot 3 & 6 \cdot 0 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 9 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3 \cdot 3 & 1 \end{pmatrix}$$

General Pattern

Observing the results, we deduce the general form for the n -th power of A :

$$A^n = \begin{pmatrix} 1 & 0 \\ n \cdot 3 & 1 \end{pmatrix}$$

Finding the Value of k

Using the pattern for $n = 4$:

$$A^4 = \begin{pmatrix} 1 & 0 \\ 4 \cdot 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 12 & 1 \end{pmatrix}$$

Comparing this result with the given expression for A^4 :

$$\begin{pmatrix} 1 & 0 \\ 12 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$$

By matrix equality, the corresponding elements must be equal:

$$k = 12$$

Conclusion

[The correct option is 3.]

Question14

$\int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$ is equal to

(1) $\frac{5\sqrt{2}}{3}$

(2) $\frac{4\sqrt{2}}{3}$

(3) $\frac{4\sqrt{3}}{3}$

$$(4) \frac{\sqrt{2}}{3}$$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

Evaluation of $\int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$

1. Rationalize the Denominator

Multiply the numerator and denominator by the conjugate of the denominator, $\sqrt{1+x} + \sqrt{x}$:

$$\begin{aligned} \frac{1}{\sqrt{1+x} - \sqrt{x}} &= \frac{1}{\sqrt{1+x} - \sqrt{x}} \cdot \frac{\sqrt{1+x} + \sqrt{x}}{\sqrt{1+x} + \sqrt{x}} \\ &= \frac{\sqrt{1+x} + \sqrt{x}}{(\sqrt{1+x})^2 - (\sqrt{x})^2} \\ &= \frac{\sqrt{1+x} + \sqrt{x}}{(1+x) - x} \\ &= \frac{\sqrt{1+x} + \sqrt{x}}{1} \end{aligned}$$

2. Evaluate the Integral

The integral becomes:

$$I = \int_0^1 \left((1+x)^{1/2} + x^{1/2} \right) dx$$

Integrate term by term:

$$\begin{aligned} I &= \left[\frac{(1+x)^{3/2}}{3/2} + \frac{x^{3/2}}{3/2} \right]_0^1 \\ &= \frac{2}{3} \left[(1+x)^{3/2} + x^{3/2} \right]_0^1 \end{aligned}$$

Apply the limits:

$$\begin{aligned} I &= \frac{2}{3} \left[\left((1+1)^{3/2} + 1^{3/2} \right) - \left((1+0)^{3/2} + 0^{3/2} \right) \right] \\ &= \frac{2}{3} \left[\left(2^{3/2} + 1 \right) - (1+0) \right] \\ &= \frac{2}{3} [2\sqrt{2}] \\ I &= \frac{4\sqrt{2}}{3} \end{aligned}$$

Question15

The area (in sq. units) of a triangle formed by vertices O, A, and B where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} =$

(1) $3\sqrt{5}$

(2) $4\sqrt{5}$

(3) $6\sqrt{5}$

$$(4) 2\sqrt{5}$$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Area of Triangle OAB using Cross Product

The area of a triangle formed by vectors $\vec{a}$ and $\vec{b}$ originating from the same vertex is given by:

$$\text{Area} = \frac{1}{2} |\vec{a} \times \vec{b}|$$

$$\text{Given } \vec{a} = \vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{b} = \vec{OB} = -3\hat{i} - 2\hat{j} + \hat{k}.$$

1. Calculate the Cross Product $\vec{OA} \times \vec{OB}$

$$\vec{OA} \times \vec{OB} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{bmatrix}$$

$$= \hat{i}((2)(1) - (3)(-2)) - \hat{j}((1)(1) - (3)(-3)) + \hat{k}((1)(-2) - (2)(-3))$$

$$= \hat{i}(2 + 6) - \hat{j}(1 + 9) + \hat{k}(-2 + 6)$$

$$= 8\hat{i} - 10\hat{j} + 4\hat{k}$$

2. Calculate the Magnitude $|\vec{OA} \times \vec{OB}|$

$$|\vec{OA} \times \vec{OB}| = \sqrt{8^2 + (-10)^2 + 4^2}$$

$$= \sqrt{64 + 100 + 16}$$

$$= \sqrt{180}$$

$$\text{Simplify the square root: } \sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5}.$$

3. Calculate the Area

$$\text{Area} = \frac{1}{2} |\vec{OA} \times \vec{OB}| = \frac{1}{2} (6\sqrt{5}) = 3\sqrt{5} \text{ sq. units}$$

[The correct option is 1.]

Question 16

$$\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx \text{ is equal to}$$

$$(1) \frac{1}{2} \log_e 2 - 1$$

$$(2) 2 \log_e 2$$

$$(3) 2 \log_e 2 - 2$$

$$(4) 4 \log_e 2$$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

$$\text{Evaluation of } \int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$$

The integral is over a symmetric interval $[-1, 1]$. We split the integrand $f(x)$ into odd and even functions.

$$f(x) = \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} = \frac{x^3}{x^2 + 2|x| + 1} + \frac{|x| + 1}{x^2 + 2|x| + 1}$$

$$\text{Let } f_1(x) = \frac{x^3}{x^2 + 2|x| + 1} \text{ and } f_2(x) = \frac{|x| + 1}{x^2 + 2|x| + 1}.$$

$$f_1(-x) = \frac{(-x)^3}{(-x)^2 + 2|-x| + 1} = \frac{-x^3}{x^2 + 2|x| + 1} = -f_1(x)$$

Since $f_1(x)$ is an odd function, its integral over a symmetric interval is zero:

$$\int_{-1}^1 f_1(x) dx = \int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx = 0$$

$$f_2(-x) = \frac{|-x| + 1}{(-x)^2 + 2|-x| + 1} = \frac{|x| + 1}{x^2 + 2|x| + 1} = f_2(x)$$

Since $f_2(x)$ is an even function, the integral simplifies:

$$\int_{-1}^1 f_2(x) dx = 2 \int_0^1 f_2(x) dx$$

For $x \in [0, 1]$, $|x| = x$.

$$f_2(x) = \frac{x + 1}{x^2 + 2x + 1} = \frac{x + 1}{(x + 1)^2} = \frac{1}{x + 1}$$

The total integral I is $0 + 2 \int_0^1 \frac{1}{x + 1} dx$:

$$\begin{aligned} I &= 2 \int_0^1 \frac{1}{x + 1} dx \\ &= 2 [\log_e |x + 1|]_0^1 \\ &= 2 (\log_e (1 + 1) - \log_e (0 + 1)) \\ &= 2 (\log_e 2 - \log_e 1) \\ &= 2 (\log_e 2 - 0) \\ &= 2 \log_e 2 \end{aligned}$$

[The correct option is 2.]

Question 17

If $x = a(\cos t + \log \tan \frac{t}{2})$, $y = a \sin t$, then value of $\frac{dy}{dx}$ at $t = \pi/4$ is

$$(1) 0$$

$$(2) \ 1$$

$$(3) \ -1$$

$$(4) \ 2$$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

Derivative of Parametric Equations at $t = \frac{\pi}{4}$

Given the parametric equations:

$$x = a \left(\cos t + \log \tan \frac{t}{2} \right), \quad y = a \sin t$$

Calculate $\frac{dy}{dt}$

$$\frac{dy}{dt} = \frac{d}{dt}(a \sin t) = a \cos t$$

Calculate $\frac{dx}{dt}$

$$\begin{aligned} \frac{dx}{dt} &= a \frac{d}{dt} \left(\cos t + \log \left(\tan \frac{t}{2} \right) \right) \\ &= a \left(-\sin t + \frac{1}{\tan(t/2)} \cdot \sec^2(t/2) \cdot \frac{1}{2} \right) \\ &= a \left(-\sin t + \frac{\cos(t/2)}{\sin(t/2)} \cdot \frac{1}{\cos^2(t/2)} \cdot \frac{1}{2} \right) \\ &= a \left(-\sin t + \frac{1}{2 \sin(t/2) \cos(t/2)} \right) \end{aligned}$$

Using the double angle identity $\sin t = 2 \sin(t/2) \cos(t/2)$:

$$\frac{dx}{dt} = a \left(-\sin t + \frac{1}{\sin t} \right)$$

Combine the terms:

$$\begin{aligned} \frac{dx}{dt} &= a \left(\frac{1 - \sin^2 t}{\sin t} \right) \\ &= a \left(\frac{\cos^2 t}{\sin t} \right) \end{aligned}$$

Calculate $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a \cos t}{a \left(\frac{\cos^2 t}{\sin t} \right)} = \frac{\cos t \sin t}{\cos^2 t} = \frac{\sin t}{\cos t} = \tan t$$

Evaluate at $t = \frac{\pi}{4}$

$$\left. \frac{dy}{dx} \right|_{t=\pi/4} = \tan \left(\frac{\pi}{4} \right) = 1$$

[The correct option is 2.]

The area (in sq. units) of the region bounded by the curve $y = \cos x$ between $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$ and the x-axis is

- (1) 0
- (2) 3
- (3) 2
- (4) 1

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Area Bounded by $y = \cos x$

The area A is bounded by the curve $y = \cos x$ and the x -axis from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

Set up the Integral

The curve $y = \cos x$ is non-negative (above the x -axis) in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

The area is given by the definite integral:

$$A = \int_{-\pi/2}^{\pi/2} \cos x \, dx$$

Use Symmetry

Since $f(x) = \cos x$ is an even function ($\cos(-x) = \cos x$), we can simplify the integral:

$$A = 2 \int_0^{\pi/2} \cos x \, dx$$

Evaluate the Integral

$$\begin{aligned} A &= 2[\sin x]_0^{\pi/2} \\ &= 2\left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right) \\ &= 2(1 - 0) \\ &= 2 \text{ sq. units} \end{aligned}$$

[The correct option is 3.]

Question19

If the direction ratios of two lines are a, b, c and $(b - c), (c - a), (a - b)$ respectively, then the angle bet

- (1) $\frac{\pi}{4}$
- (2) $\frac{\pi}{3}$
- (3) $\frac{\pi}{2}$

$$(4) \frac{2\pi}{3}$$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Angle Between Lines from Direction Ratios

Let the direction ratios of the two lines be L_1 and L_2 .

L_1 direction ratios: $a_1 = a$, $b_1 = b$, $c_1 = c$.

L_2 direction ratios: $a_2 = b - c$, $b_2 = c - a$, $c_2 = a - b$.

The angle θ between the lines is given by:

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

If $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$, the lines are perpendicular, and $\theta = \pi/2$.

Calculate the Numerator

$$N = a_1 a_2 + b_1 b_2 + c_1 c_2$$

$$= a(b - c) + b(c - a) + c(a - b)$$

$$= ab - ac + bc - ba + ca - cb$$

$$= (ab - ba) + (-ac + ca) + (bc - cb)$$

$$= 0 + 0 + 0$$

$$= 0$$

Since $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$, the angle between the lines is $\theta = \arccos(0)$.

$$\theta = \frac{\pi}{2}$$

[The correct option is 3.]

Question20

If $y = \sin^{-1} x$, then $(1 - x^2) \frac{d^2 y}{dx^2}$ is equal to

$$(1) -x \frac{dy}{dx}$$

$$(2) x \frac{dy}{dx}$$

$$(3) \frac{y}{x} \frac{dy}{dx}$$

$$(4) \frac{-y}{x} \frac{dy}{dx}$$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Second Derivative of $y = \sin^{-1} x$

Given $y = \sin^{-1} x$.

1. First Derivative $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Square both sides:

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{1-x^2}$$

Rearrange to clear the fraction:

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 1 \quad (*)$$

2. Second Derivative $\frac{d^2y}{dx^2}$

Differentiate equation (*) with respect to x using the product rule and chain rule:

$$\frac{d}{dx} \left[(1-x^2) \left(\frac{dy}{dx}\right)^2 \right] = \frac{d}{dx} (1)$$

$$\left[\frac{d}{dx} (1-x^2) \right] \left(\frac{dy}{dx}\right)^2 + (1-x^2) \left[\frac{d}{dx} \left(\frac{dy}{dx}\right)^2 \right] = 0$$

$$(-2x) \left(\frac{dy}{dx}\right)^2 + (1-x^2) \left(2 \frac{dy}{dx} \frac{d^2y}{dx^2} \right) = 0$$

3. Simplify and Isolate

Divide the entire equation by $2 \frac{dy}{dx}$ (since $\frac{dy}{dx} \neq 0$):

$$-x \left(\frac{dy}{dx}\right) + (1-x^2) \frac{d^2y}{dx^2} = 0$$

Rearrange to find the required expression:

$$(1-x^2) \frac{d^2y}{dx^2} = x \frac{dy}{dx}$$

[The correct option is 2.]

Question21

For any events A and B of a sample space S , which of the following statements are TRUE?

(A) $P(S|B) = 1$

(B) $P(A \cap B) = P(A) + P(B) + P(A \cup B)$

(C) $P(\bar{A}|B) = 1 - P(A|B)$

(D) $P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$

Choose the correct answer from the options given below:

1. (A) and (B) only
2. (A) , (C) and (D) only
3. (B) and (C) only
4. (C) and (D) only

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: B

Solution:

Analysis of Probability Statements

(A) $P(S|B) = 1$

This is TRUE. Since B is always a subset of the sample space S ($B \subseteq S$), the event S is guaranteed to occur when B occurs.

$$P(S|B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

(B) $P(A \cap B) = P(A) + P(B) + P(A \cup B)$

This is FALSE. The correct formula is $P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

(C) $P(\bar{A}|B) = 1 - P(A|B)$

This is TRUE. This is the complementary property for conditional probability.

$$P(\bar{A}|B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = 1 - \frac{P(A \cap B)}{P(B)} = 1 - P(A|B)$$

(D) $P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$

This is TRUE. This is the definition of conditional probability.

Hence, the correct statements are (A), (C), and (D).

[The correct option is 2.]

Question22

The foot of the perpendicular drawn from the point $(1, 6, 3)$ to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is:

1. $(-1, -1, 1)$
2. $(1, 2, 3)$
3. $(1, 3, 5)$
4. $(1, -3, 5)$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Foot of the Perpendicular

The given line L is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Let the general point on the line be M and set the ratio equal to λ :

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

The coordinates of M are: $M = (\lambda, 2\lambda + 1, 3\lambda + 2)$

The given point is $P = (1, 6, 3)$. M is the foot of the perpendicular from P .

Direction Ratios:

The direction ratios of the line L (denominator values) are 1, 2, 3.

The direction ratios of the segment PM are:

$$PM = (\lambda - 1), (2\lambda + 1 - 6), (3\lambda + 2 - 3)$$

$$PM = (\lambda - 1), (2\lambda - 5), (3\lambda - 1)$$

Condition for Perpendicularity:

Since PM is perpendicular to L , the dot product of their direction ratios must be zero:

$$1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$$

$$\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$$

$$14\lambda - 14 = 0 \Rightarrow \lambda = 1$$

Coordinates of M :

$$x = 1, \quad y = 2(1) + 1 = 3, \quad z = 3(1) + 2 = 5$$

Hence, the foot of the perpendicular is $M = (1, 3, 5)$.

[The correct option is 3.]

Question23

If the function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{if } x \neq 0 \\ \frac{3k}{2}, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is:

- 1. $\frac{2}{3}$
- 2. 4
- 3. 2
- 4. 9

Options:

- A. 1
- B. 2

C. 3

D. 4

Answer: C

Solution:

Continuity of $f(x)$ at $x = 0$

For a function $f(x)$ to be continuous at $x = 0$, the following condition must hold:

$$\lim_{x \rightarrow 0} f(x) = f(0)$$

1. Determine $f(0)$

From the definition of the function: $f(0) = \frac{3k}{2}$

2. Calculate $\lim_{x \rightarrow 0} f(x)$

$$\text{For } x \neq 0, f(x) = \frac{\sin 3x}{x}.$$

We use the standard limit $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$:

$$\begin{aligned}\lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{3x} \cdot 3 \right) \\ &= 1 \cdot 3 = 3\end{aligned}$$

3. Equate the Limit and Function Value

For continuity, the limit must equal $f(0)$:

$$\lim_{x \rightarrow 0} f(x) = f(0)$$

$$3 = \frac{3k}{2}$$

$$3k = 6 \Rightarrow k = 2$$

[The correct option is 3.]

Question24

The value of $\int_{-\frac{\pi}{2}}^{\pi/2} (x^5 + x^3 \cos x) dx$ is:

1. 0

2. -1

3. π

4. 1

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

The integral is over a symmetric interval $[-\pi/2, \pi/2]$. We evaluate the integral by analyzing the parity of the integrand $f(x) = x^5 + x^3 \cos x$.

1. Check the Parity of $f(x)$

A function $f(x)$ is odd if $f(-x) = -f(x)$.

$$f(-x) = (-x)^5 + (-x)^3 \cos(-x)$$

Since $(-x)^n = -x^n$ for odd n , and $\cos(-x) = \cos x$:

$$f(-x) = -x^5 + (-x^3) \cos x$$

$$f(-x) = -(x^5 + x^3 \cos x) = -f(x)$$

Since $f(-x) = -f(x)$, the integrand $f(x)$ is an \textit{odd function}.

2. Apply the Property of Odd Functions

For any odd function $f(x)$, its definite integral over a symmetric interval $[-a, a]$ is zero:

$$\int_{-a}^a f(x) dx = 0$$

Here, $a = \pi/2$. Therefore:

$$\int_{-\pi/2}^{\pi/2} (x^5 + x^3 \cos x) dx = 0$$

[The correct option is 1.]

Question25

A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2 + 1}$

is (where $\mathbb{R}$ is the set of real numbers):

1. one-one but not onto
2. onto but not one-one
3. neither one-one nor onto
4. both one-one and onto

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

$$\textbf{Function Classification: } f(x) = \frac{x}{x^2 + 1}$$

The function is $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2 + 1}$.

1. Check for One-One (Injective)

A function is one-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Counterexample for One-One:

$$f(2) = \frac{2}{2^2 + 1} = \frac{2}{5}$$

$$f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2 + 1} = \frac{\frac{1}{2}}{\frac{1}{4} + 1} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$$

Since $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$, the function is \textit{not one-one}.

2. Check for Onto (Surjective)

A function is onto if the range equals the codomain $\mathbb{R}$.

Set $y = f(x)$ and solve for x :

$$y = \frac{x}{x^2 + 1}$$

$$y(x^2 + 1) = x \implies yx^2 - x + y = 0$$

For x to be real (domain $\mathbb{R}$), the discriminant D must satisfy $D \geq 0$.

$$D = (-1)^2 - 4(y)(y) = 1 - 4y^2$$

$$1 - 4y^2 \geq 0 \implies 4y^2 \leq 1 \implies y^2 \leq \frac{1}{4}$$

$$|y| \leq \frac{1}{2} \implies -\frac{1}{2} \leq y \leq \frac{1}{2}$$

Thus, the range of $f(x)$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Since the range $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is not equal to $\mathbb{R}$, the function is \textit{not onto}.

3. Conclusion

The function is \textit{neither one-one nor onto}.

[The correct option is 3.]

Question 26

The matrix $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ is a:

(A) Diagonal matrix

(B) Scalar matrix

(C) Square matrix

(D) Symmetric matrix

Choose the correct answer from the options given below:

1. (A), (B) and (C) only
2. (B) and (C) only
3. (C) and (D) only
4. (A) and (D) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Classification of Matrix A

The given matrix is:

$$A = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{pmatrix}$$

(A) Diagonal matrix: This is FALSE. A diagonal matrix requires all non-diagonal elements to be zero. Here, $a_{13} = 5$ and $a_{31} = 5$ are non-zero.

(B) Scalar matrix: This is FALSE. A scalar matrix is a diagonal matrix with equal diagonal elements. A is not diagonal and its diagonal elements $(0, 5, 0)$ are not equal.

(C) Square matrix: This is TRUE. A has order 3×3 .

(D) Symmetric matrix: This is TRUE. A matrix A is symmetric if $A^T = A$.

$$A^T = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{pmatrix} = A$$

Since $A = A^T$, the matrix is symmetric.

The only correct statements are (C) and (D).

[The correct option is 3.]

Question 27

If A is an invertible matrix of order 3, then

Match List-I with List-II

List-I	List-II
(A) $ \text{adj } A $	(I) $8 A $
(B) $ A(\text{adj } A) $	(II) $ A ^2$
(C) $ 2A $	(III) $\frac{1}{ A }$
(D) $ A^{-1} $	(IV) $ A ^3$

Choose the correct answer from the options given below:

- (A) - (II), (B) - (IV), (C) - (I), (D) - (III)
- (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
- (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

Options:

A.

1

B.

2

C.

3

D.

4

Answer: A

Solution:

Given A is an invertible matrix of order $n = 3$.

We use the following general properties for an $n \times n$ matrix A :

$$|\text{adj}A| = |A|^{n-1}$$

$$|A(\text{adj}A)| = |A||\text{adj}A| = |A| \cdot |A|^{n-1} = |A|^n$$

$$|kA| = k^n |A|$$

$$|A^{-1}| = \frac{1}{|A|}$$

For $n = 3$:

(A) $|\text{adj}A|$: Substitute $n = 3$ into the first property:

$$|\text{adj}A| = |A|^{3-1} = |A|^2$$

(A) matches (II) $|A|^2$

(B) $|A(\text{adj}A)|$: Substitute $n = 3$ into the second property:

$$|A(\text{adj}A)| = |A|^3$$

(B) matches (IV) $|A|^3$

(C) $|2A|$: Substitute $n = 3$ and $k = 2$ into the third property:

$$|2A| = 2^3 |A| = 8|A|$$

(C) matches (I) $8|A|$

(D) $|A^{-1}|$: Apply the fourth property:

$$|A^{-1}| = \frac{1}{|A|}$$

(D) matches (III) $\frac{1}{|A|}$

Conclusion: The correct match is (A)-(II), (B)-(IV), (C)-(I), (D)-(III).

[The correct option is 1.]

Question28

Consider a closed cylinder of radius r with a fixed surface area.

The volume of the cylinder is maximum when its height is:

1. $3r$

2. $\frac{1}{3}r$

3. $\frac{r}{2}$

4. $2r$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

Maximum Cylinder Volume

Let S be the fixed surface area and V be the volume.

$$S = 2\pi r^2 + 2\pi rh \implies h = \frac{S}{2\pi r} - r$$

Substitute h into the volume formula $V = \pi r^2 h$:

$$V = \pi r^2 \left(\frac{S}{2\pi r} - r \right) = \frac{Sr}{2} - \pi r^3$$

For maximum volume, set $\frac{dV}{dr} = 0$:

$$\begin{aligned} \frac{dV}{dr} &= \frac{S}{2} - 3\pi r^2 = 0 \\ \implies S &= 6\pi r^2 \end{aligned}$$

Substitute this expression for S back into the surface area equation $S = 2\pi r^2 + 2\pi rh$:

$$6\pi r^2 = 2\pi r^2 + 2\pi rh$$

$$4\pi r^2 = 2\pi rh$$

Divide by $2\pi r$ to solve for h :

$$h = \frac{4\pi r^2}{2\pi r} = 2r$$

[The correct option is 4.]

Question 29

The function $f(x) = 2 \log_e(x - 2) - x^2 + 4x + 1$, ($x > 2$) is increasing on the interval:

1. $(1, 2)$
2. $(2, 6)$
3. $(2, 3)$
4. $(3, \infty)$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Interval where $f(x)$ is Increasing

Given $f(x) = 2 \log_e(x - 2) - x^2 + 4x + 1$, with domain $x > 2$.

The function is increasing where the first derivative $f'(x) > 0$.

1. Calculate the first derivative $f'(x)$

$$f'(x) = \frac{d}{dx}(2 \log_e(x - 2) - x^2 + 4x + 1)$$

$$f'(x) = 2 \cdot \frac{1}{x - 2} - 2x + 4$$

Factor the polynomial terms:

$$f'(x) = \frac{2}{x - 2} - 2(x - 2)$$

2. Set $f'(x) > 0$ for Increasing Interval

$$\frac{2}{x - 2} - 2(x - 2) > 0$$

Since $x > 2$, $x - 2$ is positive. Multiply by $(x - 2)$:

$$2 - 2(x - 2)^2 > 0$$

Divide by 2 :

$$1 - (x - 2)^2 > 0$$

Rearrange and solve:

$$(x - 2)^2 < 1$$

$$\sqrt{(x - 2)^2} < \sqrt{1}$$

$$\implies |x - 2| < 1$$

$$-1 < x - 2 < 1$$

Add 2 to all parts:

$$-1 + 2 < x < 1 + 2$$

$$\implies 1 < x < 3$$

3. Apply Domain Restriction

The increasing interval $(1, 3)$ must be intersected with the domain $x > 2$:

$$\text{Increasing Interval} = (1, 3) \cap (2, \infty) = (2, 3)$$

[The correct option is 3.]

Question30

The value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angle, is:

1. $\frac{70}{11}$

2. $\frac{10}{11}$

3. $\frac{11}{70}$

4. $\frac{11}{10}$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Perpendicular Lines: Value of λ

Two lines are perpendicular if the dot product of their direction ratios $\vec{b}_1 \cdot \vec{b}_2 = 0$.

1. Extract Direction Ratios

The lines must be in standard form $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$, where a, b, c are the direction ratios.

Line 1

$$L_1: \frac{1-x}{3} = \frac{x-1}{-3} \Rightarrow a_1 = -3$$

$$\frac{7y-14}{2\lambda} = \frac{7(y-2)}{2\lambda} = \frac{y-2}{2\lambda/7} \Rightarrow b_1 = \frac{2\lambda}{7}$$

$$\frac{z-3}{2} \Rightarrow c_1 = 2$$

$$\vec{b}_1 = \left(-3, \frac{2\lambda}{7}, 2\right)$$

Line 2

$$L_2: \frac{7-7x}{3\lambda} = \frac{7(1-x)}{3\lambda} = \frac{7(x-1)}{-3\lambda} = \frac{x-1}{-3\lambda/7} \Rightarrow a_2 = -\frac{3\lambda}{7}$$

$$\frac{y-5}{1} \Rightarrow b_2 = 1$$

$$\frac{6-z}{5} = \frac{z-6}{-5} \Rightarrow c_2 = -5$$

$$\vec{b}_2 = \left(-\frac{3\lambda}{7}, 1, -5\right)$$

2. Apply Perpendicularity Condition $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(-3)\left(-\frac{3\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right)(1) + (2)(-5) = 0$$

$$\frac{9\lambda}{7} + \frac{2\lambda}{7} - 10 = 0$$

$$\frac{11\lambda}{7} = 10$$

$$\lambda = \frac{70}{11}$$

[The correct option is 1.]

Question 31

The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x-y}$ is

1. $y = \frac{x}{2} \log_e(x^2 + y^2) + C$: C is an arbitrary constant

2. $y = \sqrt{x^2 + y^2} + C$: C is an arbitrary constant

3. $y = \frac{1}{2} \log_e(x^2 + y^2) + C$: C is an arbitrary constant

4. $\tan^{-1} \frac{y}{x} = \frac{1}{2} \log_e(x^2 + y^2) + C : C \text{ is an arbitrary constant}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

Solution of $\frac{dy}{dx} = \frac{x+y}{x-y}$

Given the homogeneous equation:

$$\frac{dy}{dx} = \frac{1+y/x}{1-y/x}$$

Substitute $y = vx$ so $\frac{dy}{dx} = v + x \frac{dv}{dx} :$

$$v + x \frac{dv}{dx} = \frac{1+v}{1-v}$$

Separate variables:

$$x \frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v^2}{1-v} \implies \int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x}$$

Integrate:

$$\int \frac{1}{1+v^2} dv - \frac{1}{2} \int \frac{2v}{1+v^2} dv = \int \frac{1}{x} dx$$

$$\tan^{-1} v - \frac{1}{2} \log_e(1+v^2) = \log_e |x| + C'$$

Substitute $v = \frac{y}{x} :$

$$\tan^{-1} \frac{y}{x} = \log_e |x| + \frac{1}{2} \log_e \left(1 + \frac{y^2}{x^2} \right) + C'$$

$$\tan^{-1} \frac{y}{x} = \log_e |x| + \frac{1}{2} \log_e \left(\frac{x^2 + y^2}{x^2} \right) + C'$$

Simplify using logarithm properties ($\log_e(A/B) = \log_e A - \log_e B$) :

$$\tan^{-1} \frac{y}{x} = \log_e |x| + \frac{1}{2} \log_e(x^2 + y^2) - \frac{1}{2} \log_e(x^2) + C'$$

Use $\log_e(x^2) = 2 \log_e |x|$ and simplify constants into $C :$

$$\tan^{-1} \frac{y}{x} = \log_e |x| + \frac{1}{2} \log_e(x^2 + y^2) - \log_e |x| + C$$

Final solution:

$$\tan^{-1} \frac{y}{x} = \frac{1}{2} \log_e(x^2 + y^2) + C$$

[The correct option is 4.]

Question32

The corner points of the bounded feasible region for an LPP are: $(5, 5)$, $(15, 15)$, $(0, 20)$ and $(0, 10)$.

Let $z = 3x + 9y$ be the objective function.

Then the value of $\text{maximum}(z) - \text{minimum}(z)$ is:

1. 60
2. 90
3. 120
4. 180

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Maximum Minus Minimum Value for LPP

The objective function is $z = 3x + 9y$.

The corner points of the bounded feasible region are:

$$P_1 = (5, 5)$$

$$P_2 = (15, 15)$$

$$P_3 = (0, 20)$$

$$P_4 = (0, 10)$$

Evaluate z at each corner point:

$$z(5, 5) = 3(5) + 9(5) = 15 + 45 = \mathbf{60}$$

$$z(15, 15) = 3(15) + 9(15) = 45 + 135 = \mathbf{180}$$

$$z(0, 20) = 3(0) + 9(20) = 0 + 180 = \mathbf{180}$$

$$z(0, 10) = 3(0) + 9(10) = 0 + 90 = \mathbf{90}$$

$$\text{Maximum}(z) = 180$$

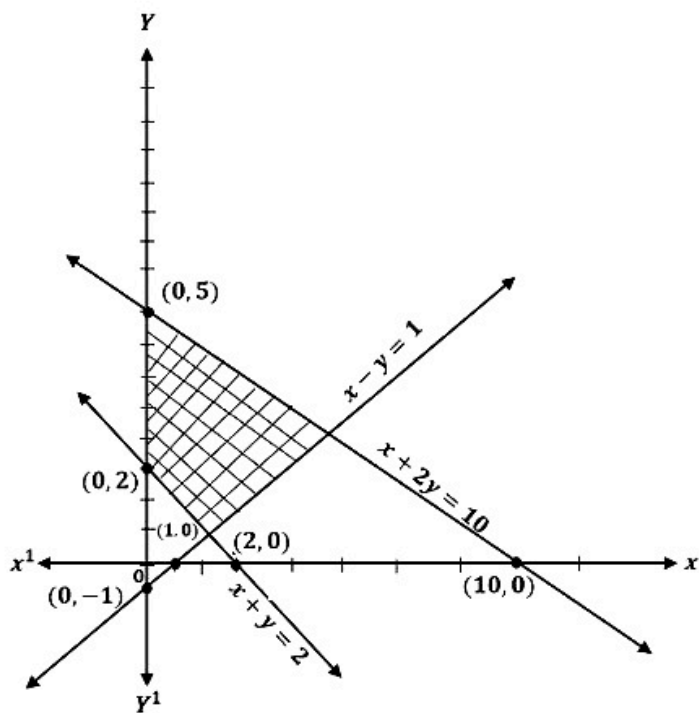
$$\text{Minimum}(z) = 60$$

$$\text{Maximum}(z) - \text{Minimum}(z) = 180 - 60 = \mathbf{120}$$

[The correct option is 3.]

Question33

The feasible region corresponding to the linear constraints of a Linear Programming Problem (LPP) is represented by the shaded region in the given figure. Which of the following is not a constraint to the given LPP?



1. $x + 2y \leq 10$
2. $x + y \geq 2$
3. $x - y \geq 1$
4. $x - y \leq 1$

Options:

- A.
1
B.
2
C.
3
D.
4

Answer: C

Solution:

LPP: Identifying the Non-Constraint

The feasible region is the shaded area bounded by four lines and restricted to $x \geq 0, y \geq 0$ (first quadrant).

Lines forming the boundary are $x + 2y = 10$, $x + y = 2$, and $x - y = 1$.

Line 1: $x + 2y = 10$

The shaded region is below this line.

Test origin $(0, 0)$: $0 + 2(0) = 0 \leq 10$.

Constraint: $x + 2y \leq 10$ ($\text{\textbf{Option 1 is a constraint}}$)

Line 2: $x + y = 2$

The shaded region is above this line.

Test origin $(0, 0)$: $0 + 0 = 0 \not\geq 2$.

Constraint: $x + y \geq 2$ (\textbf{Option 2 is a constraint})

Line 3: $x - y = 1$

The shaded region is above this line.

Test origin $(0, 0)$: $0 - 0 = 0 \not\geq 1$.

Constraint: $x - y \geq 1$

The three main constraints derived are $x + 2y \leq 10$, $x + y \geq 2$, and $x - y \geq 1$ (plus $x \geq 0, y \geq 0$).

Comparing this with the provided options:

Option 1: $x + 2y \leq 10$ (A constraint)

Option 2: $x + y \geq 2$ (A constraint)

Option 3: $x + 2y \geq 10$ (NOT a constraint)

The boundary is $x + 2y = 10$, and the feasible region satisfies $x + 2y \leq 10$. Therefore, $x + 2y \geq 10$ is NOT a constraint defining the shaded region.

[The correct option is 3.]

Question 34

If $x = -1$ and $x = -2$ are the extreme points of $f(x) = \alpha \log |x| + \beta x^2 + x$ then

1. $\alpha = \frac{-2}{3}, \beta = \frac{1}{6}$

2. $\alpha = \frac{-2}{3}, \beta = -\frac{1}{6}$

3. $\alpha = \frac{2}{3}, \beta = \frac{1}{6}$

4. $\alpha = \frac{2}{3}, \beta = -\frac{1}{6}$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Extreme Points: Find α and β

Given $f(x) = \alpha \log |x| + \beta x^2 + x$. Extreme points $x = -1, x = -2$ satisfy $f'(x) = 0$.

For $x \neq 0$, the derivative is:

$$f'(x) = \frac{\alpha}{x} + 2\beta x + 1$$

Set $f'(x) = 0$ and multiply by x :

$$\frac{\alpha}{x} + 2\beta x + 1 = 0 \implies \alpha + 2\beta x^2 + x = 0$$

Substitute the extreme points $x = -1$ and $x = -2$:

$$x = -1: \quad \alpha + 2\beta(-1)^2 + (-1) = 0 \implies \alpha + 2\beta = 1 \quad \textbf{(1)}$$

$$x = -2: \quad \alpha + 2\beta(-2)^2 + (-2) = 0 \implies \alpha + 8\beta = 2 \quad \textbf{(2)}$$

Subtract (1) from (2):

$$(\alpha + 8\beta) - (\alpha + 2\beta) = 2 - 1 \implies 6\beta = 1 \implies \beta = \frac{1}{6}$$

Substitute $\beta = \frac{1}{6}$ into (1):

$$\alpha + 2\left(\frac{1}{6}\right) = 1 \implies \alpha + \frac{1}{3} = 1 \implies \alpha = 1 - \frac{1}{3} = \frac{2}{3}$$

Final values: $\alpha = \frac{2}{3}, \beta = \frac{1}{6}$.

[The correct option is 3.]

Question 35

The value of λ for which the two vectors $(2\hat{i} - \hat{j} + 2\hat{k})$ and $(3\hat{i} + \lambda\hat{j} + \hat{k})$ are perpendicular, is:

1. 4

2. 8

3. 6

4. 1

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

Perpendicular Vectors: Find λ

Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular if their dot product is zero, i.e. $\vec{a} \cdot \vec{b} = 0$.

Let $\vec{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\vec{b} = 3\mathbf{i} + \lambda\mathbf{j} + \mathbf{k}$.

Applying the dot product condition:

$$\vec{a} \cdot \vec{b} = (2)(3) + (-1)(\lambda) + (2)(1) = 0$$

$$6 - \lambda + 2 = 0$$

$$8 - \lambda = 0$$

$$\text{Hence, } \lambda = 8$$

[The correct option is 2.]
