

CUET Mathematics Solved Paper-2022

Held on 17 August 2022

- If $x = 2at$, $y = at^2$, then $\frac{d^2y}{dx^2}$ is
 - 1
 - $\frac{1}{2a}$
 - t
 - 0
- A die is thrown once. If E is the event that 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even', then the incorrect option is
 - $P(E) = \frac{1}{3}$
 - $P(F) = \frac{1}{2}$
 - $P(E \cap F) = \frac{1}{6}$
 - E and F are dependent events
- Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. The probability that there is at least one defective egg is
 - $\frac{10^{10} - 9^{10}}{10^{10}}$
 - $\frac{9^{10} - 10^{10}}{10^{10}}$
 - $\frac{10^9 - 9^{10}}{10^{10}}$
 - $\frac{10^{10} + 9^{10}}{10^{10}}$
- If m is the degree and n is the order of the given differential equation

$$\frac{x^3 \left(\frac{d^3y}{dx^3} \right)^2 + 2x^2 \left(\frac{d^2y}{dx^2} \right)^3}{(x+1)^5} = \left(3x - \frac{d^2y}{dx^2} \right)^4$$
 - $m - n = 2$
 - $m + n = 5$
 - $m = 4, n = 3$
 - Order (n) is 3 but degree (m) is not defined
- The differential equation representing the family of curves $y = m(x - d)$ where m and d are arbitrary constants, is
 - $\frac{dy}{dx} = 0$
 - $\frac{d^2y}{dx^2} = 0$
 - $x \frac{d^2y}{dx^2} + y = 0$
 - $x \frac{d^2y}{dx^2} - y = 0$
- $\int_1^2 \frac{dx}{x(x^4+1)} = ?$
 - $\log\left(\frac{32}{17}\right)$
 - $\log\left(\frac{16}{17}\right)$
 - $\frac{1}{4} \log\left(\frac{16}{17}\right)$
 - $\frac{1}{4} \log\left(\frac{32}{17}\right)$
- If the area of the region in first quadrant, bounded by the curve $y^2 = 9x$, $x = 2$, $x = 4$ and the x -axis is $a + b\sqrt{2}$, then the value of $a + b$ is
 - 16
 - 12
 - 20
 - 8
- If $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \\ 8 \end{bmatrix}$, then the value of $x + 2y - 3z$ is
 - 5
 - 4
 - 3
 - 7
- If $x = 3t^2 + 5t + 6$ and $y = -4t^3 - 2t^2 + 5t + 7$, $t \neq \frac{-5}{6}$ then the value of $\frac{dy}{dx}$ is
 - $-2t + 1$
 - $\frac{-12t^2 - 4t - 5}{6t + 5}$
 - $\frac{-4t^3 - 2t^2 + 5t + 7}{3t^2 + 5t + 6}$
 - $\frac{-4t^3 - 2t^2 + 5t + 7}{6t + 5}$
- The interval in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing is
 - $(-\infty, 2)$
 - $(-\infty, -2)$
 - $(2, \infty)$
 - $(-2, \infty)$
- If $y = \log_e \left(\frac{2x}{1-x} \right)$, then $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}$ is
 - $\frac{1}{2}$
 - $\frac{1}{4}$
 - 0
 - $\frac{3}{5}$

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12. If a, b, c are mutually unequal real numbers, then the value

$$\text{of } \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} =$$

- (a) $-(a+b+c)$
(b) $a+b+c$
(c) $a^2+b^2+c^2$
(d) $a^3+b^3+c^3$

13. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $n \in \mathbb{N}$ (where $\mathbb{N}$ is the set of natural numbers), then A^n is equal to

- (a) nA (b) $2nA$
(c) $2^{n-1}A$ (d) 2^nA

14. If $y = \frac{1}{\sqrt{1+x^2-x}}$, then the value of $(1+x^2)^{\frac{3}{2}} \cdot \frac{d^2y}{dx^2}$ is

- (a) x (b) x^2-1
(c) $\sqrt{1+x^2}-1$ (d) 1

15. The equation of the tangent to the curve $y = x^2 - 2x + 7$, which is parallel to the line $2x - y + 9 = 0$, is

- (a) $2x - y + 3 = 0$
(b) $2x - y + 6 = 0$
(c) $2x - y + 1 = 0$
(d) $2x - y + 4 = 0$

16. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b . Then R is

- (a) Symmetric but not transitive
(b) Transitive but not symmetric
(c) Neither symmetric nor transitive
(d) Both symmetric and transitive

17. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is

- (a) Reflexive only
(b) Reflexive and symmetry relation
(c) Transitive only
(d) Equivalence relation

18. If $f: \mathbb{R} - \{-1\} \rightarrow \mathbb{R} - \{1\}$ be a function defined by

$$f(x) = \frac{x-1}{x+1}, \text{ then}$$

- A. f is one-one but not onto
B. f is onto but not one-one
C. f is one-one and onto

D. $f^{-1}(x) = \frac{x+1}{x-1}$

E. $(f \circ f)(x) = -\frac{1}{x}; x \neq 0, -1$

Choose the correct answer from the options given below

- (a) A, D, E only (b) C, D only
(c) B, E only (d) C, E only

19. The domain of the function $\cos^{-1}(2x-1)$ is

- (a) $[0, 1]$ (b) $[-1, 1]$
(c) $(-1, 1)$ (d) $[0, \pi]$

20. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) =$

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

21. If the matrix $A = \begin{bmatrix} 3 & 2a & -5 \\ -4 & 0 & b \\ -5 & 3 & 7 \end{bmatrix}$ is symmetric then the

value of $(a+b)$ is

- (a) 1 (b) 5 (c) 3 (d) 4

22. If A is square matrix of size 4 and $|A| = 6$. If $|\text{Adj}(\text{Adj}(3A))| = 2^a \cdot 3^b$, then value of $a+b$ is

- (a) 24 (b) 54 (c) 72 (d) 216

23. The value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$, is

- (a) 2 (b) $\pm 2\sqrt{2}$ (c) 4 (d) $\pm 2\sqrt{3}$

24. If $y = \left(\frac{1}{x}\right)^x$, then $\frac{d^2y}{dx^2} =$

- (a) $x^{-x}(1+\log x)^2 - x^{-(x+1)}$
(b) $x^{-x}(1+\log x)^2 - x^{-(x-1)}$
(c) $x^{-x}(1+\log x)^2 - x^{-(x+1)}$
(d) $x^{-x}(1+\log x)^{-1} + x^{-(x-1)}$

25. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$ then $|\vec{x}|$ is

- (a) 2 (b) 3 (c) ± 3 (d) 5

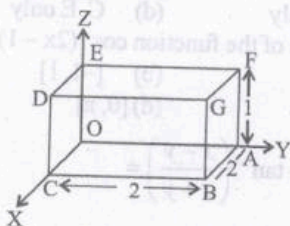
26. $\begin{vmatrix} 0 & \sin 2\alpha & -\cos^2 \alpha \\ -\sin^2 \alpha & 0 & \sin \alpha \sin \beta \\ -\cos \alpha \sin \beta & 2\sin^2 \beta & 0 \end{vmatrix} =$

- (a) 0 (b) -1
(c) Independent of α (d) Independent of β

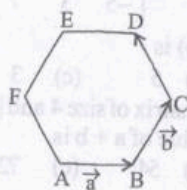
27. $\int \frac{2}{x^4-1} dx =$

- (a) $\log \left| \frac{x^2-1}{x^2+1} \right| + C$
(b) $2\tan^{-1}(x^2) + C$
(c) $\frac{1}{2} \log \left| \frac{x-1}{x+1} \right| - \tan^{-1} x + C$
(d) $\tan^{-1} x + \frac{1}{2} \log \left| \frac{x+1}{x-1} \right| + C$

28. The acute angle between the diagonals OG and AD of the cuboid (shown in the figure) is



- (a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\cos^{-1}\left(-\frac{1}{3}\right)$
 (c) $\cos^{-1}\left(\frac{7}{9}\right)$ (d) $\cos^{-1}\left(\frac{1}{9}\right)$
29. In the following figure ABCDEF is a regular hexagon. If $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$ then $\overrightarrow{CD}$ in terms of $\vec{a}$ and $\vec{b}$ is:



- (a) $\vec{a} + \vec{b}$ (b) $\vec{a} - \vec{b}$
 (c) $\vec{b} - \vec{a}$ (d) $3\vec{b} - \vec{a}$
30. The integrating factor of the differential equation $x \frac{dy}{dx} + y - x + xy \cot x = 0$, ($x \neq 0$) is:

- (a) $x \sin x$ (b) $x \cos x$
 (c) x (d) $\sin x$

31. $\int_{\frac{1}{4}}^1 \frac{dx}{\sqrt{-x^2 - 2x + 3}} =$

- (a) $\sin^{-1}\left(\frac{1}{4}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$
 (c) $\sin^{-1}\left(\frac{5}{8}\right)$ (d) $\cos^{-1}\left(\frac{5}{8}\right)$

32. A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k ²	2k ²	7k ² +k

The value of $P(0 < X < 5)$ is

- (a) $\frac{1}{5}$ (b) $\frac{2}{5}$
 (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

33. Two independent events A and B are such that

$P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, the $P(A|B) =$

- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

34. Probability that A speaks truth is $\frac{4}{5}$. He tosses a coin and reports that a head appears. The probability that actually there was a head, is

- (a) $\frac{4}{5}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

35. The expectation of a number obtained when throwing a die having 1 written on three faces, 2 on two faces and 5 on one face is given by

- (a) 2 (b) $\frac{3}{2}$ (c) $\frac{71}{30}$ (d) 6

36. If $\int \frac{dx}{\sqrt{x+2}-\sqrt{x+1}} = \frac{2}{3} \left[(\lambda+1)^{\frac{3}{2}} - \lambda^{\frac{3}{2}} \right] + C$, then the value of λ is

- (a) $x-1$ (b) x (c) $x+1$ (d) $\frac{1}{x}$

37. $\int_0^{\pi} \sin^3 x \cdot \cos^2 x \cdot dx =$

- (a) $-\frac{4}{15}$ (b) $\frac{2}{15}$
 (c) 0 (d) $\frac{4}{15}$

38. The distance of the point (3, -2, 1) from the plane $2x - y + 2z + 3 = 0$ is:

- (a) $\frac{3}{13}$ (b) $\frac{13}{3}$
 (c) $\frac{14}{3}$ (d) $\frac{3}{14}$

39. The maximum value of the function $z = 3x + 3y$, subject to the constraints $x + 2y \leq 30$, $2x + y \leq 50$, $x \geq 0$, $y \geq 0$ is:

- (a) 75 (b) 90
 (c) 80 (d) 45

40. Let R be the relation in the set $A = \{a, b, c, d\}$ given by $R = \{(a, a), (b, b), (c, c), (a, b), (b, a), (c, d), (d, d), (d, c)\}$

- (a) R is reflexive and symmetric but not transitive
 (b) R is reflexive and transitive but not symmetric
 (c) R is symmetric and transitive but not reflexive
 (d) R is an equivalence relation

Passage:

Three pizza outlets A, B and C sell three types of pizza namely cheese pizza, veg pizza and paneer pizza. In a day, A can sell 40 cheese pizza, 30 veg pizza and 20 paneer pizza; B can sell 20 cheese pizza, 40 veg pizza and 60 paneer pizza; C can sell 60 cheese pizza, 20 veg pizza and 30 paneer pizza. If the revenue generated in a day by A is ₹6000, by B is ₹9000 and by C is ₹7000. If x denotes selling price of cheese pizza, y is selling price of veg pizza and z be the selling price of Paneer pizza then based on this information, answer the following question:

41. The revenue generated by three outlets A, B and C are:

- (a) 6000 (b) 22000
 (c) 16000 (d) 15000

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42. The matrix representation of the above problem is :

(a)
$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 4 & 2 \\ 2 & 6 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 600 \\ 900 \\ 700 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 600 \\ 900 \\ 700 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 600 \\ 450 \\ 700 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 4 & 2 \\ 2 & 6 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 9000 \\ 7000 \end{bmatrix}$$

43. The price of a cheese pizza is:

- (a) ₹50 (b) ₹80 (c) ₹500 (d) ₹800

44. The price of a paneer pizza is:

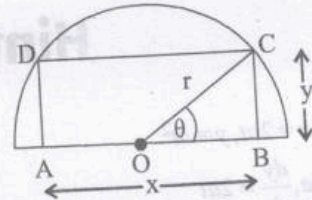
- (a) ₹50 (b) ₹60 (c) ₹65 (d) ₹80

45. If the cost price of a cheese pizza is ₹30, a veg pizza is ₹50 and a paneer pizza is ₹50, what is the profit of outlet A in a day?

- (a) ₹6300 (b) ₹3300
(c) ₹2300 (d) ₹18300

Passage

A rectangle of length 'x' and breadth 'y' is inscribed in a semi-circle of fixed radius 'r' as shown in the figure given below.



Based on the above information answer the following question:

46. Area $A(\theta)$, $0 < \theta < \frac{\pi}{2}$ of the rectangle ABCD, is

- (a) $r^2 \sin \theta$ (b) $r^2 \sin 2\theta$
(c) $r^2 \cos 2\theta$ (d) $r^2 \cos \theta$

47. The value of θ , for which $A'(\theta) = 0$ is

- (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

48. Dimensions x, y of the rectangle ABCD, when area is maximum are:

- (a) $r \frac{\sqrt{3}}{2}, \frac{2r}{\sqrt{3}}$ (b) $r\sqrt{2}, \frac{r}{\sqrt{2}}$

- (c) $\frac{r}{\sqrt{2}}, \sqrt{2}r$ (d) $r, \frac{r}{\sqrt{2}}$

49. Maximum area of the rectangle is:

- (a) $2r^2$ (b) $3r^2$ (c) r^2 (d) $4r^2$

50. Perimeter of rectangle when its area is maximum is:

- (a) $\frac{8\sqrt{3}r}{3}$ (b) $4r$

- (c) $\frac{7\sqrt{3}r}{3}$ (d) $3\sqrt{2}r$

Hints & Explanations

1. (b) Given, $x = 2at, y = at^2$

$$\text{So, } \frac{dx}{dt} = 2a, \frac{dy}{dx} = 2at$$

$$\frac{dy}{dx} = \frac{dt}{dx} \therefore \frac{dy}{dx} = t$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx} = 1 \cdot \frac{1}{2a} = \frac{1}{2a}$$

2. (d) Since, $E = \{3, 6\}$ and $F = \{2, 4, 6\}$, $E \cap F = \{6\}$

$$\text{Now, } P(E) \cdot P(F) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$\therefore P(E \cap F) = P(E) \cdot P(F)$$

$\therefore E$ and F are independent events

3. (a) Since, $P(\text{No defective eggs}) = \left(\frac{9}{10}\right)^{10}$

$$P(E) = 1 - \left(\frac{9}{10}\right)^{10} = \frac{10^{10} - 9^{10}}{10^{10}}$$

4. (b) The given differential equation is

$$\frac{x^3 \left(\frac{d^3y}{dx^3} \right)^2 + 2x^2 \left(\frac{d^2y}{dx^2} \right)^3}{(x+1)^5} = \left(3x - \frac{d^2y}{dx^2} \right)^4$$

$$\Rightarrow x^3 \left(\frac{d^3y}{dx^3} \right)^2 + 2x^2 \left(\frac{d^2y}{dx^2} \right)^3 - (x+1)^5 \left(3x - \frac{d^2y}{dx^2} \right)^4 = 0$$

$\therefore$ Order (n) = 3, Degree (m) = 2

5. (b) Given, $y = m(x-d)$

$$\frac{dy}{dx} = m, \frac{d^2y}{dx^2} = 0$$

6. (d) Let $I = \int_1^2 \frac{dx}{x(x^4+1)} = \int_1^2 \frac{dx}{x^5(1+x^{-4})}$

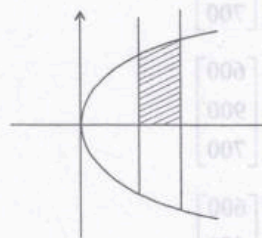
$$\text{Let } 1+x^{-4} = t, -4x^{-5} dx = dt$$

$$\text{If } x=1 \Rightarrow t=2, x=2 \Rightarrow t=\frac{17}{16}$$

$$\therefore I = \int_2^{17/16} \frac{-1}{4t} dt = \frac{-1}{4} [\ln t]_2^{17/16}$$

$$= \frac{-1}{4} \left[\ln \frac{17}{16} - \ln 2 \right] = \frac{1}{4} \ln \left[\frac{32}{17} \right]$$

7. (b) Given curves: $y^2 = 9x, x = 2, x = 4$ and $y = 0$
Now, graph of the curves



$$\therefore \text{Required area} = \int_2^4 \sqrt{9x} dx = 3 \left[\frac{x^{3/2}}{3/2} \right]_2^4$$

$$= 2[8 - 2\sqrt{2}] = 16 - 4\sqrt{2}$$

$$\text{Now, } a = 16 \text{ and } b = -4 \therefore a + b = 12$$

8. (b) Given $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \\ 8 \end{bmatrix}$

$$\therefore x + y + z = 11$$

$$x + z = 6$$

$$y + z = 8$$

$$\text{By (i) \& (ii), } y = 5$$

$$\text{By (ii) \& (i), } x = 3$$

$$\therefore z = 3$$

$$\text{Now, } x + 2y - 3z = 4$$

9. (b) Given, $x = 3t^2 + 5t + 6, y = -4t^3 - 2t^2 + 5t + 7$

$$\text{So, } \frac{dy}{dt} = 6t + 5, \frac{dy}{dx} = \frac{-12t^2 - 4t + 5}{6t + 5}$$

$$\frac{dy}{dx} = \frac{dt}{dx} \therefore \frac{dy}{dx} = \frac{-12t^2 - 4t + 5}{6t + 5}$$

10. (c) Given, $f(x) = x^2 - 4x + 6$

For increasing of the function, $f'(x) > 0$

$$\Rightarrow 2x - 4 > 0 \Rightarrow x > 2$$

11. (c) Given, $y = \log_e \left(\frac{2x}{1-x} \right)$

$$\frac{d[\ln f(x)]}{dx} = \frac{1}{f(x)} \cdot \frac{d(f(x))}{dx}$$

$$\text{So, } \frac{dy}{dx} = \frac{1}{\frac{2x}{1-x}} \cdot \frac{d \left(\frac{2x}{1-x} \right)}{dx}$$

$$= \frac{1-x}{2x} \left[\frac{(1-x)2 + 2x}{(1-x)^2} \right] = \frac{2}{2x(1-x)} = \frac{1}{1-x} + \frac{1}{x}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{(1-x)^2} - \frac{1}{x^2}$$

$$\text{At } x = \frac{1}{2}, \frac{d^2y}{dx^2} = 4 - 4 = 0$$

12. (b) Given,

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & b^3-a^3 \\ 0 & c-a & c^3-a^3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

Using, $R_2 \rightarrow R_2 + R_1, R_3 \rightarrow R_3 - R_1$

$$\begin{vmatrix} 1 & a & a^3 \\ (b-a)(c-a) & 0 & 1 \\ 0 & 1 & b^2+a^2+ab \\ 0 & 1 & c^2+a^2+ac \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ (b-a)(c-a) & 0 & 1 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$$= \frac{c^2+ac-b^2-ab}{(c-b)} = \frac{(c-b)(a+b+c)}{(c-b)}$$

$$= a+b+c$$

13. (c) Given, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2A$$

$$A^3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = 2^2 A$$

$$A^4 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = 2^3 A$$

As, so on we get; $A^n = 2^{n-1} A$

14. (d) Given, $y = \frac{1}{\sqrt{1+x^2} - x}$

$$y = \frac{1}{\sqrt{1+x^2} - x} \times \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2} + x} = \frac{\sqrt{1+x^2} + x}{1}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}} \times 2x + 1 = \frac{y}{\sqrt{1+x^2}}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{\sqrt{1+x^2} \frac{dy}{dx} - y \frac{1}{2\sqrt{1+x^2}} \cdot 2x}{(1+x^2)}$$

$$(1+x^2)^{3/2} \frac{d^2y}{dx^2} = \frac{(1+x^2)y}{\sqrt{1+x^2}} - xy$$

$$\Rightarrow (1+x^2)^{3/2} \frac{d^2y}{dx^2} = y(\sqrt{1+x^2} - x) = 1$$

15. (a) Given the curve,

$$y = x^2 - 2x + 7; \frac{dy}{dx} = 2x - 2$$

Given, $y = 2x + 9$

$\Rightarrow$ Slope = 2

If point of tangency be (x_1, y_1) .

$$\text{then } 2x_1 - 2 = 2$$

$$x_1 = 2, y_1 = 7$$

Now, equation of tangent,

$$y - 7 = 2(x - 2) \Rightarrow 2x - y + 3 = 0$$

16. (b) aRb if a is brother of b .

$\nexists bRa$, so it can't be symmetric.

If $aRb, bRc \Rightarrow aRc$

So, it is transitive.

17. (a) Given the relation,

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$$

$$\text{and } A = \{1, 2, 3\}$$

Now, $\because (a, a) \in R \forall a \in A$

$\Rightarrow R$ is reflexive.

Since, $(1, 2) \in R$ but $(2, 1) \notin R$

So, R is not symmetric.

Now, $(1, 2)(2, 3) \in R$ but $(1, 3) \notin R$

So, R is not transitive.

18. (d) Given,

$$f(x) = \frac{x-1}{x+1} = \frac{(x+1)}{x+1} - \frac{2}{x+1} = 1 - \frac{2}{x+1}$$

$$f'(x) = \frac{2}{(x+1)^2} > 0$$

So, $f(x)$ is one-one.

$$y = \frac{x-1}{x+1} \Rightarrow y+1 = x - yx$$

$$x = \frac{1+y}{1-y}, y \in R - \{1\}$$

So, f is onto.

$$\text{And } y = \frac{x-1}{x+1} \Rightarrow \frac{y+1}{y-1} = \frac{x}{-1}$$

$$\therefore f^{-1}(x) = \frac{x+1}{1-x}$$

$$\text{And } f \circ f(x) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = -\frac{2}{2x} = -\frac{1}{x}$$

19. (a) Given, $\cos^{-1}(2x-1)$ Now, $-1 \leq 2x-1 \leq 1$
 $0 \leq 2x \leq 2; 0 \leq x \leq 1$
 Domain = $[0, 1]$.

20. (c) Let $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = t$

$$\tan^{-1}\left(\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \frac{x}{y} \cdot \frac{x-y}{x+y}}\right) = t$$

$$\therefore \tan t = \frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \frac{x}{y} \cdot \frac{x-y}{x+y}} = \frac{x^2 + xy - xy + y^2}{xy + y^2 + x^2 - xy} = 1$$

$$t = \tan^{-1}(1); \therefore t = \frac{\pi}{4}$$

21. (a) Given, $A = \begin{bmatrix} 3 & 2a & -5 \\ -4 & 0 & b \\ -5 & 3 & 7 \end{bmatrix}$

Since, A is symmetric.

So, $2a = -4 \Rightarrow a = -2$ and $b = 3$

So, $a + b = 1$

22. (b) We know that

$$\therefore |\text{Adj.}(\text{Adj } A)| = |A|^{(n-1)^2}$$

$$\text{Now, } |\text{Adj.}(\text{Adj } (3A))| = |3A|^{3^2}$$

$$= (3^4 |A|)^9 = 3^{36} 6^9 = 3^{45} 6^9$$

$$\text{So, } a = 45 \text{ and } b = 9 \therefore a + b = 54$$

23. (b) Given, $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

$$\Rightarrow 3 - x^2 = -5$$

$$\Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}$$

24. (a) Given, $y = \left(\frac{1}{x}\right)^x$

$$y = x^{-x}$$

Taking log on both sides,

$$\therefore \log y = -x \log x$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{-x}{x} + \log x(-1) = -1 - \log x$$

$$\Rightarrow \frac{dy}{dx} = -y(1 + \log x)$$

$$\therefore \frac{d^2 y}{dx^2} = -y \left[\frac{1}{x} \right] - (1 + \log x) \frac{dy}{dx}$$

$$= -x^{-x-1} + x^{-x}(1 + \log x)^2$$

$$= x^{-x}(1 + \log x)^2 - x^{-(x+1)}$$

25. (b) Given, $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$

$$|\vec{x}|^2 - |\vec{a}|^2 = 8 \Rightarrow |\vec{x}|^2 - 1 = 8$$

$$\therefore |\vec{x}| = 3$$

26. (a, c, d) Given,

$$\begin{vmatrix} 0 & \sin 2\alpha & -\cos^2 \alpha \\ -\sin^2 \alpha & 0 & \sin \alpha \sin \beta \\ -\cos \alpha \sin \beta & 2 \sin^2 \beta & 0 \end{vmatrix}$$

Expanding by R_1

$$= -\sin 2\alpha(\sin \alpha \cos \alpha \sin^2 \beta) - \cos^2 \alpha(-2 \sin^2 \alpha \sin^2 \beta)$$

$$= -2 \sin^2 \alpha \cos^2 \alpha \sin^2 \beta + 2 \cos^2 \alpha \sin^2 \alpha \sin^2 \beta$$

$$= 0$$

27. (c) Since, $\int \frac{2}{x^4 - 1} dx = \int \frac{2}{(x^2 - 1)(x^2 + 1)} dx$

$$= \int \frac{(x^2 + 1) - (x^2 - 1)}{(x^2 - 1)(x^2 + 1)} dx$$

$$= \int \left(\frac{1}{x^2 - 1} - \frac{1}{x^2 + 1} \right) dx$$

$$= \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| - \tan^{-1} x + C$$

28. (a) The vector

$$\vec{OG} = (2-0)\hat{i} + (2-0)\hat{j} + (1-0)\hat{k}$$

$$\vec{OG} = (2\hat{i} + 2\hat{j} + \hat{k})$$

$$\vec{AD} = \vec{OD} - \vec{OA} = (2\hat{i} + \hat{k}) - (2\hat{j})$$

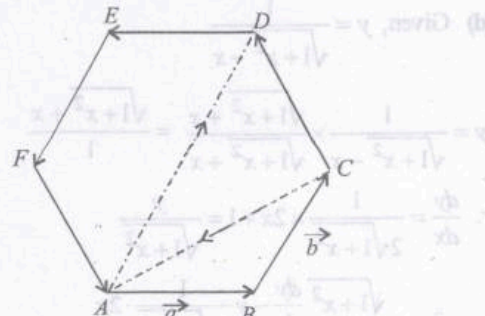
$$= 2\hat{i} - 2\hat{j} + \hat{k}$$

Let θ be the angle between $\vec{OG}$ and $\vec{AD}$,

$$\text{then } \cos \theta = \frac{|\vec{OG} \cdot \vec{AD}|}{|\vec{OG}| |\vec{AD}|}$$

$$\cos \theta = \frac{4 - 4 + 1}{\sqrt{3} \cdot \sqrt{3}} = \frac{1}{3} \therefore \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

29. (c) Given, regular hexagon



$$\therefore \vec{CD} = \vec{CA} + \vec{AD}$$

$$\overline{CD} = -(\overline{AB} + \overline{BC}) + 2\vec{b}$$

$$= -(\vec{a} + \vec{b}) + 2\vec{b} = \vec{b} - \vec{a}$$

30. (a) Given, the differential equation

$$x \frac{dy}{dx} + y - x + xy \cot x = 0 (x \neq 0)$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$$

$$\therefore \text{I.F.} = e^{\int \left(\frac{1}{x} + \cot x\right) dx} = e^{\ln x + \ln \sin x}$$

$$= e^{\ln(x \sin x)} = x \sin x$$

31. (d) Let $I = \int \frac{dx}{\sqrt{-x^2 - 2x + 3}} = \int \frac{dx}{\sqrt{4 - (x+1)^2}}$

$$= \left(\sin^{-1} \frac{x+1}{2} \right) \Big|_1^4$$

$$= \sin^{-1} \left(\frac{1+1}{2} \right) - \sin^{-1} \left(\frac{5}{8} \right)$$

$$= \frac{\pi}{2} - \sin^{-1} \left(\frac{5}{8} \right) = \cos^{-1} \left(\frac{5}{8} \right)$$

32. (c) We know, $P(0 \leq x \leq 7) = 1$

$$\Rightarrow k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k = 1$$

$$\Rightarrow (k+1)(10k-1) = 0$$

$$k = \frac{1}{10}, -1$$

$$\text{Since, } k \neq -1$$

$$\text{Now, } P(0 < x < 5) = k + 2k + 2k + 3k$$

$$= 8k = \frac{8}{10} = \frac{4}{5}$$

33. (b) Given, $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

Since, A and B are independent events.

$$\text{So, } P(A \cap B) = P(A)P(B)$$

Now,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{3}} = \frac{1}{2}$$

34. (a) Given, the probability, $P(T) = \frac{4}{5}$

Now, by conditional probability, we get

$$P\left(\frac{T}{H}\right) = \frac{P(H \cap T)}{P(H)}$$

$$\frac{P(T)P\left(\frac{H}{T}\right)}{P(T)P\left(\frac{H}{T}\right) + P(F)P\left(\frac{H}{F}\right)}$$

$$= \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{1}{2} + \frac{1}{5} \times \frac{1}{2}} = \frac{4}{5}$$

35. (a) Sample space = $\{1, 1, 1, 2, 2, 5\}$

$$\text{Given: } P(1) = \frac{1}{2}, P(2) = \frac{1}{3}, P(5) = \frac{1}{6}$$

So, the expectation

$$\therefore E = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 5 \cdot \frac{1}{6} = \frac{3-4+5}{6} = \frac{12}{6} = 2$$

36. (c) $I = \int \frac{dx}{\sqrt{x+2} - \sqrt{x+1}} = \int \frac{\sqrt{x+2} + \sqrt{x+1}}{x+2 - (x+1)} dx$

$$= \frac{2}{3} (x+2)^{3/2} + \frac{2}{3} (x+1)^{3/2} + C$$

$$\text{So, } \int \frac{dx}{\sqrt{x+2} - \sqrt{x+1}} = \frac{2}{3} [(x+2)^{3/2} + (x+1)^{3/2}] + C$$

$$\text{So, } \lambda = x+1$$

37. (d) Let $I = \int_0^\pi \sin^3 x \cos^2 x dx$

$$I = 2 \int_0^{\pi/2} \sin^3 x \cos^2 x dx$$

$$= 2 \int_0^{\pi/2} \sin x (1 - \cos^2 x) \cos^2 x dx$$

Let $\cos x = t$, then $\sin x dx = -dt$

If $x = 0$, then $t = 1$,

$x = \pi/2$, then $t = 0$

$$\Rightarrow -2 \int_1^0 (t^2 - t^4) dt = 2 \int_0^1 (t^2 - t^4) dt$$

After solving integral and apply the limit

$$I = \frac{4}{15}$$

38. (b) Given, the plane

$$2x - y + 2z + 3 = 0; P_1 = (3, -2, 1)$$

So, distance

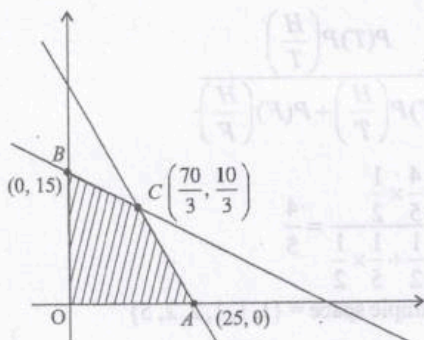
$$\therefore d = \frac{6+2+2+3}{\sqrt{4+1+4}} = \frac{13}{3}$$

39. (c) The maximise function

$$z = 3x + 3y$$

So, corner points of feasible region are

$$O(0,0), A(25,0), B(0,15) \text{ and } C\left(\frac{70}{3}, \frac{10}{3}\right)$$



$$Z_A = 75, \quad Z_B = 45$$

$$Z_C = 80, \quad Z_O = 0$$

$\therefore$ Maximum value of $z = 80$

40. (d) Given, set $A = \{a, b, c, d\}$ and
 $R = \{(a, a) (b, b) (c, c), (a, b) (b, a) (c, d) (d, d) (d, c)\}$

(i) $\because (a, a) \in R \forall a \in A$

So, R is reflexive.

(ii) If $(x, y) \in R \Rightarrow (y, x) \in R$

$\therefore R$ is symmetric.

(iii) If $xRy, yRz \Rightarrow xRz \forall x, y, z \in A$

R is transitive also.

$\therefore R$ is an equivalence relation.

41. (b) Since outlet by A

$$40x + 30y + 20z = 6000$$

Outlet by B

$$20x + 40y + 60z = 9000$$

and outlet by C

$$60x + 20y + 30z = 7000$$

$$\text{Total revenue} = 6000 + 9000 + 7000 = ₹22000$$

42. (c) Now, the obtained equations are

$$4x + 3y + 2z = 600$$

$$x + 2y + 3z = 450$$

$$6x + 2y + 3z = 700$$

$$\text{So, } \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 600 \\ 450 \\ 700 \end{bmatrix}$$

43. (a) Now, price of cheese pizza be x .

$$\text{So, } x = ₹50$$

44. (d) Now, price of paneer pizza be z .

$$\text{So, } z = ₹80$$

45. (c) Now, CP of 40 cheese + 30 veg + 20 paneer

$$= 40 \times 30 + 50 \times 30 + 20 \times 50$$

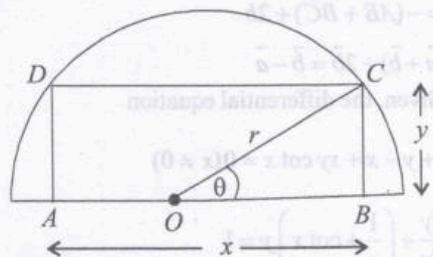
$$= 1200 + 1500 + 1000 = ₹3700$$

$$\text{and SP of 40 cheese + 30 veg + 20 paneer} \\ = ₹6000$$

$$\text{Now, profit of outlet A in a day}$$

$$= 6000 - 3700 = ₹2300$$

46. (b)



From given figure,

$$x = 2r \cos \theta$$

$$y = r \sin \theta$$

$$\text{Area } (ABCD) = x \times y$$

$$\begin{aligned} \text{Area } (ABCD) &= 2r^2 \cos \theta \sin \theta \\ &= r^2 \sin 2\theta \end{aligned}$$

47. (c) Since, $A = r^2 \sin 2\theta$

$$\text{Now, } A' = 2r^2 \cos 2\theta = 0$$

$$\therefore 2\theta = (2n+1)\frac{\pi}{2} \quad (n \in \mathbb{Z})$$

$$\theta = (2n+1)\frac{\pi}{4}$$

$$\therefore \theta = \frac{\pi}{4}$$

48. (b) For maximum,

Area of rectangular $ABCD$.

$$A' = 0 \Rightarrow \theta = \frac{\pi}{4}$$

$$\therefore x = 2r \cos \theta \quad y = r \sin \theta$$

$$x = \frac{2r}{\sqrt{2}} \quad y = \frac{r}{\sqrt{2}}$$

$$x = \sqrt{2}r \quad y = \frac{r}{\sqrt{2}}$$

49. (c) For maximum,

$$\text{Area} = \max (r^2 \sin 2\theta) = r^2$$

$$\Rightarrow \text{Maximum area} = r^2$$

50. (d) For maximum area,

$$x = \sqrt{2}r; \quad y = \frac{r}{\sqrt{2}}$$

$$\text{So, perimeter} = 2(x+y)$$

$$= 2\left(r\sqrt{2} + \frac{r}{\sqrt{2}}\right)$$

$$= 2r\left[\frac{2+1}{\sqrt{2}}\right] = 3\sqrt{2}r$$