

JEE (Main)-2026 Session-1
Question Paper with Solutions
(Mathematics, Physics, & Chemistry)
2nd April 2026 Shift -1

Time: 3 hrs.

M.M: 300

IMPORTANT INSTRUCTIONS:

- (1) The test is of 3 hours duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
- (4) **Section - A:** Attempt all questions.
- (5) **Section - B:** Attempt all questions.
- (6) **Section - A** (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.
- (7) **Section - B** (21 - 25) contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS

SECTION-A

1. Let $\alpha, \alpha + 2, \alpha \in \mathbf{Z}$, be the roots of the quadratic equation $x(x + 2) + (x + 1)(x + 3) + (x + 2)(x + 4) + \dots + (x + n - 1)(x + n + 1) = 4n$ for some $n \in \mathbf{N}$.

Then $(n + \alpha)$ is equal to :

- (1) 0 (2) 1
(3) 2 (4) 3

Ans. (3)

Sol. $nx^2 + x(2 + 4 + 6 + \dots + 2n) + (1 \cdot 3 + \dots + (n - 1)(n + 1)) = 4n$

$$nx^2 + n(n + 1)x + \frac{n(n - 1)(2n + 5)}{6} = 4n$$

$$x^2 + (n + 1)x + \frac{(n - 1)(2n + 5)}{6} = 4$$

D must be a perfect square

$$D = \frac{122 - 2n^2}{6} = 20 - \left(\frac{n^2 - 1}{3}\right)$$

$$\text{If make perfect square} \Rightarrow \frac{n^2 - 1}{3} = 16 \Rightarrow n = 7$$

So Equation is

$$\Rightarrow x^2 + 8x + \frac{8 \times 15}{6} - 5 = 0$$

$$x^2 + 8x + 15 = 0$$

$$x = -3, -5$$

$$\alpha = -5, \alpha + 2 = -3$$

$$\alpha + n = 7 - 5 = 2$$

TEST PAPER WITH SOLUTION

2. Let x and y be real number such that

$$50 \left(\frac{2x}{1 + 3i} - \frac{y}{1 - 2i} \right) = 31 + 17i, i = \sqrt{-1}. \text{ Then the}$$

value of $10(x - 3y)$ is :

- (1) 20 (2) 31
(3) 35 (4) 75

Ans. (4)

$$\text{Sol. } 50 \left(\frac{2x(1 - 3i)}{10} - \frac{y(1 + 2i)}{5} \right) = 31 + 17i$$

$$10x - (30x)i - 10y - 20yi = 31 + 17i$$

$$\begin{cases} 10(x - y) = 31 \\ -30x - 20y = 17 \end{cases}$$

$$30x - 30y = 93$$

$$-10(3x + 2y) = 17$$

$$x = 0.9, y = -2.2$$

$$10(x - 3y) = 75$$

3. Let $\alpha, \beta \in \mathbf{R}$ be such that the system of linear equations

$$x + 2y + z = 5$$

$$2x + y + \alpha z = 5$$

$$8x + 4y + \beta z = 18$$

has no solution. Then $\frac{\beta}{\alpha}$ is equal to :

- (1) -4 (2) 4
(3) 8 (4) -8

Ans. (2)

Sol. $x + 2y + z = 5$

$$2x + y + \alpha z = 5$$

$$8x + 4y + \beta z = 18$$

for no solution

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & \alpha \\ 8 & 4 & \beta \end{vmatrix} = 0$$

$$\Delta = 1(\beta - 4\alpha) - 2(2\beta - 8\alpha) + 1(0) = 0$$

$$= \beta - 4\alpha - 4\beta + 16\alpha = 0$$

$$\Rightarrow 4\alpha = \beta$$

4. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & \alpha \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 3 \\ \beta & 2 \end{bmatrix}$.

If $A^2 - 4A + I = O$ and $B^2 - 5B - 6I = O$, then among the two statements :

$$(S1) : [(B - A)(B + A)]^T = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}$$

and

$$(S2) : \det(\text{adj}(A + B)) = -5,$$

(1) only (S1) is correct

(2) only (S2) is correct

(3) both (S1) and (S2) are correct

(4) both (S1) and (S2) are wrong

Ans. (2)

Sol. $A = \begin{bmatrix} 1 & 2 \\ 1 & \alpha \end{bmatrix}$

$$A^2 - 4A + I = 0$$

trace of matrix

$$\alpha + 1 = 4$$

$$\alpha = 3$$

$$B = \begin{bmatrix} 3 & 3 \\ \beta & 2 \end{bmatrix}$$

$$B^2 - 5B - 6I = 0$$

$$\det.(B) = 6 - 3\beta = -6$$

$$3\beta = 12$$

$$\beta = 4$$

$$B - A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}; B + A = \begin{bmatrix} 4 & 5 \\ 5 & 5 \end{bmatrix}$$

$$(B - A)(B + A) = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}, \text{adj}B = \begin{bmatrix} 2 & -3 \\ -4 & 3 \end{bmatrix}$$

$$\text{adj}(A + B) = \begin{bmatrix} 5 & -5 \\ -5 & 4 \end{bmatrix}$$

5. Let A be the set of first 101 terms of an A.P., whose first term is 1 and the common difference is 5 and let B be the set of first 71 terms of an A.P., whose first term is 9 and the common difference is 7. Then the number of elements in $A \cap B$, which are divisible by 3, is :

(1) 4 (2) 5

(3) 6 (4) 7

Ans. (2)

Sol. AP_1 Set $A = \{1, 6, 11, 16, \dots, 101 \text{ terms}\}$

AP_2 Set $B = \{9, 16, \dots, 71 \text{ terms}\}$

$$D = \text{L.C.M. } \{d_1, d_2\} = 35$$

1st Common term is 16

$$16 + (n - 1) 35 \leq 499$$

$$n \leq 14.8$$

$$\Rightarrow n = 14$$

$$A \cap B = \{16, 51, 86, 121, 156, 191, 226, 261, 296, 331, 366, 401, 436, 471\}$$

terms divisible by '3'

$$= \{51, 156, 261, 366, 471\}$$

Ans. 5 terms

6. The number of seven-digit numbers, that can be formed by using the digits 1, 2, 3, 5 and 7 such that each digit is used at least once, is ;

- (1) 15400 (2) 17800
(3) 16800 (4) 29400

Ans. (3)

Sol. Exactly one digit repeated $\rightarrow {}^5C_1 \times \frac{7!}{3!} = 4200$

Exactly two digits repeated $\rightarrow {}^5C_2 \times \frac{7!}{2!2!} = 12600$

Total numbers = 16800

7. The number of elements in the set

$$S = \left\{ (r, k) : k \in \mathbf{Z} \text{ and } {}^{36}C_{r+1} = \frac{6({}^{35}C_r)}{(k^2 - 3)} \right\}, \text{ is}$$

- (1) 2 (2) 4 (3) 8 (4) 16

Ans. (2)

Sol. $\frac{36}{r+1} \cdot {}^{35}C_r = 6 \cdot \frac{{}^{35}C_r}{k^2 - 3}$

$$k^2 - 3 = \frac{r+1}{6}$$

$$k^2 = \frac{r+19}{6}$$

$$0 \leq r \leq 35$$

$$19 \leq r+19 \leq 54$$

$$3.1 \leq \frac{r+19}{6} \leq 9$$

$$\therefore k^2 = 4, 9$$

$$k = \pm 2, \pm 3$$

$$\therefore (r, k) \Rightarrow (5, \pm 2) \Rightarrow (35, \pm 3)$$

$$\therefore 4$$

8. If the mean of the data

Class	5-10	10-15	15-20	20-25	25-30	30-35
Frequency	2	k	28	54	k+1	5

is 21, then k is one of the roots of the equation :

- (1) $2x^2 - 23x - 10 = 0$
(2) $4x^2 - 35x + 24 = 0$
(3) $2x^2 - 19x - 10 = 0$
(4) $2x^2 - 35x + 98 = 0$

Ans. (3)

Sol.

Class	5-10	10-15	15-20	20-25	25-30	30-35
Freq.	2	k	28	54	k+1	5

mean

$$21 = \frac{15 + 12.5k + 490 + 1215 + 27.5 + 27.5k + 162.5}{90 + 2k}$$

$$1890 + 42k = 1910 + 40k$$

$$k = 10$$

9. Let the mid points of the sides of a triangle ABC

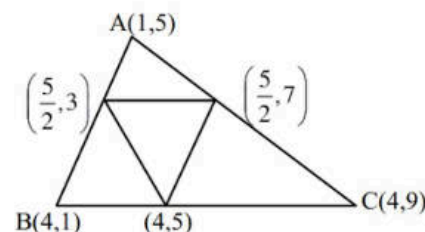
be $\left(\frac{5}{2}, 7\right)$, $\left(\frac{5}{2}, 3\right)$ and $(4, 5)$. If its incentre is

(h, k) , then $3h + k$ is equal to :

- (1) 11 (2) 12
(3) 13 (4) 14

Ans. (3)

Sol.



$$I = \left(\frac{24}{9}, 5 \right)$$

$$\therefore 3h + k = 8 + 5 = 13$$

10. Let an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a < b$, pass through the

point $(4, 3)$ and have eccentricity $\frac{\sqrt{5}}{3}$. Then the

length of its latus rectum is :

- (1) $\frac{4\sqrt{5}}{3}$ (2) $2\sqrt{5}$
(3) $\frac{7\sqrt{5}}{3}$ (4) $\frac{8\sqrt{5}}{3}$

Ans. (4)

Sol. $e^2 = 1 - \frac{a^2}{b^2} \Rightarrow \frac{a^2}{b^2} = 1 - e^2 = 1 - \frac{5}{9} = \frac{4}{9}$
 $\Rightarrow \frac{a^2}{b^2} = \frac{4}{9} \quad \dots(1)$

Passes through (4,3) $\Rightarrow \frac{16}{a^2} + \frac{9}{b^2} = 1 \quad \dots(2)$

from (1) and (2)

$a^2 = 20$ and $b^2 = 45$

$LR = \frac{2a^2}{b} = \frac{2(20)}{3\sqrt{5}} = \frac{8\sqrt{5}}{3}$

11. If $\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right) = K$, then the value of $\sin\left(\frac{10K\pi}{3}\right)$ is :

- (1) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (2) $\frac{\sqrt{3}-1}{\sqrt{2}}$
 (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{2}$

Ans. (1)

Sol. $K = \sin 10^\circ \sin 50^\circ \sin 70^\circ$

$= \frac{1}{4} \sin 30^\circ = \frac{1}{8}$

$\sin 10K \frac{\pi}{3} = \sin\left(10 \times \frac{1}{8} \cdot \frac{\pi}{3}\right) = \sin \frac{5\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

12. Let $S = \{x \in [-\pi, \pi] : \sin x (\sin x + \cos x) = a, a \in \mathbb{Z}\}$. Then $n(S)$ is equal to :

- (1) 3 (2) 6
 (3) 7 (4) 9

Ans. (4)

Sol. $\sin x (\sin x + \cos x) \in \left[\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right]$

2 integer will be there $\Rightarrow a = 0, 1$

If $a = 0$ $\sin x (\sin x + \cos x) = 0$

$\Rightarrow \sin x = 0$ or $\sin x + \cos x = 0$

$\boxed{x = -\pi, 0, \pi}$ $= \tan x = -1$

3 solutions

$\boxed{x = -\frac{\pi}{4}, \frac{3\pi}{4}}$ 2 solutions

If $a = 1$ $\sin x (\sin x + \cos x) = 1$

$\sin^2 x + \sin x \cos x = 1$

$2\sin^2 x + 2\sin x \cos x = 2$

$1 - \cos 2x + \sin 2x = 2$

$\sin 2x - \cos 2x = 1 \rightarrow \text{square}$

$\sin 4x = 0$

$4x = -4\pi, -3\pi, \dots, 3\pi, 4\pi$

$x = -\pi, -\frac{3\pi}{4}, -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

$\Rightarrow x = -\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{2}$ 4 solutions

Total 9 solution

13. If the point of intersection of the lines $\frac{x+1}{3} =$

$\frac{y+a}{5} = \frac{z+b+1}{7}$ and $\frac{x-2}{1} = \frac{y-b}{4} = \frac{z-2a}{7}$ lies

on xy-plane, then the value of $a+b$ is :

- (1) 2 (2) 5
 (3) 7 (4) 9

Ans. (3)

Sol. Line (L_1)

$\frac{x+1}{3} = \frac{y+a}{5} = \frac{z+b+1}{7} = r_1$

$\Rightarrow$ General point P on L_1 is

$(3r_1 - 1, 5r_1 - a, 7r_1 - b - 1)$

Line (L_2)

$\frac{x-2}{1} = \frac{y-b}{4} = \frac{z-2a}{7} = r_2$

$\Rightarrow$ General point Q on L_2 is

$(r_2 + 2, 4r_2 + b, 7r_2 + 2a)$

For point of intersection

$3r_1 - 1 = r_2 + 2 \Rightarrow r_2 = 3r_1 - 3 \quad \dots(1)$

$5r_1 - a = 4r_2 + b \quad \dots(2)$

$7r_1 - b - 1 = 7r_2 + 2a \quad \dots(3)$

Since point lies on XY plane

$\Rightarrow z - \text{coordinate} = 0$

From L_1 : $7r_1 - b - 1 = 0 \Rightarrow 7r_1 = b + 1$

From L_2 : $7r_2 + 2a = 0 \Rightarrow 2a = -7r_2$

Substitute $r_2 = 3r_1 - 3$

$$2a = -7(3r_1 - 3)$$

$$\Rightarrow a = \frac{-21r_1 + 21}{2}$$

Put in equation (2)

$$5r_1 - a = 4r_2 + b$$

$$5r_1 - \left(\frac{-21r_1 + 21}{2} \right) = 4(3r_1 - 3) + (7r_1 - 1)$$

$$\Rightarrow r_1 = \frac{5}{7}$$

$$\Rightarrow b = 7r_1 - 1 = 4$$

$$\Rightarrow a = 3 \therefore a + b = 7$$

14. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$, then the maximum value of $3|(3\vec{a} + 2\vec{b})| + 4|(3\vec{a} - 2\vec{b})|$ is :

- (1) 30 (2) 36
(3) 60 (4) 72

Ans. (3)

Sol. $E = 3\sqrt{9a^2 + 4b^2 + 12\vec{a} \cdot \vec{b}} + 4\sqrt{9a^2 + 4b^2 - 12\vec{a} \cdot \vec{b}}$
 $= 3\sqrt{36 + 36 + 12 \times 6 \cos \theta} + 4\sqrt{36 + 36 - 72 \cos \theta}$
 $= 3\sqrt{72 + 72 \cos \theta} + 4\sqrt{72 - 72 \cos \theta}$
 $= 18\sqrt{2}\sqrt{1 + \cos \theta} + 24\sqrt{2}\sqrt{1 - \cos \theta}$
 $= 18\sqrt{2}\sqrt{2} \cos \frac{\theta}{2} + 24\sqrt{2}\left(\sqrt{2} \sin \frac{\theta}{2}\right)$
 $= 36\left(\cos \frac{\theta}{2}\right) + 48\sin\left(\frac{\theta}{2}\right)$
 $E_{\max} = \sqrt{(36)^2 + (48)^2} = \sqrt{6^4 + 6^2 \times 8^2} = 60$

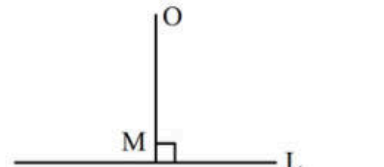
15. Let a line L passing through the point (1, 1, 1) be perpendicular to both the vectors $2\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. If P(a, b, c) is the foot of perpendicular from the origin on the line L, then the value of $34(a + b + c)$ is :

- (1) 50 (2) 80
(3) 100 (4) 120

Ans. (3)

Sol. Direction vector of line is

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$$



$$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-1}{2}$$

So equation of line is

$$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-1}{2} = \lambda \text{ let}$$

let $(2\lambda + 1, -3\lambda + 1, 2\lambda + 1)$ be the foot of perpendicular from (0, 0, 0) upon given line

$$\Rightarrow \vec{OM} \cdot (2\hat{i} - 3\hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow 2(2\lambda + 1) - 3(-3\lambda + 1) + 2(2\lambda + 1) = 0$$

$$\Rightarrow \lambda = -\frac{1}{17}$$

$$\text{So } (a, b, c) \equiv \left(\frac{-2}{17} + 1, \frac{3}{17} + 1, \frac{-2}{17} + 1 \right)$$

$$\Rightarrow 34(a + b + c) = 100$$

16. If $\lim_{x \rightarrow 2} \frac{\sin(x^3 - 5x^2 + ax + b)}{(\sqrt{x} - 1) \log_e(x - 1)} = m$, then $a + b + m$

is equal to:

- (1) 5 (2) 6
(3) 8 (4) 10

Ans. (2)

Sol. $\therefore$ Denominator = 0 at $x = 2$

$\Rightarrow$ Numerator = 0 at $x = 2$

$$\Rightarrow 2^3 - 5(2)^2 + a(2) + b = 0$$

$$\Rightarrow 2a + b = 12 \quad \dots (1)$$

$$\therefore m = \lim_{x \rightarrow 2} \frac{\sin(x^3 - 5x^2 + ax + b) \cdot (x^3 - 5x^2 + ax + b)}{\left(\frac{x-1-1}{\sqrt{x-1}+1}\right) \cdot \log_e(1+(x-2)) \cdot (x-2)}$$

$$\Rightarrow m = \lim_{x \rightarrow 2} 2 \left(\frac{x^3 - 5x^2 + ax + b}{(x-2)^2} \right)$$

$$\Rightarrow m = \lim_{x \rightarrow 2} 2 \left(\frac{3x^2 - 10x + a}{2(x-2)} \right)$$

$\therefore$ Denominator = 0 at $x = 2$

$\Rightarrow$ Numerator = 0 at $x = 2$

$$\Rightarrow 3(2)^2 - 10(2) + a = 0 \Rightarrow a = 8$$

put $a = 8$ in (1), we get $b = -4$

$$\therefore m = \lim_{x \rightarrow 2} 2 \left(\frac{6x - 10}{2} \right) \Rightarrow m = 2$$

$$\therefore a + b + m = 6$$

- 17.** If the curve $y = f(x)$ passes through the point $(1, e)$ and satisfies the differential equation $dy = y(2 + \log x) dx$, $x > 0$, then $f(e)$ is equal to :

- (1) e^e (2) e^{e^2}
(3) e^{2e} (4) e^{2e^e}

Ans. (3)

Sol. $\frac{dy}{dx} = y(2 + \ln x)$

$$\int \frac{dy}{y} = \int (2 + \ln x) dx$$

$$\ln y = 2x + x \ln x - x + C$$

$$\ln y = x + x \ln x + C$$

Since it passes through $(1, e)$

$$1 = 1 + 0 + C \Rightarrow C = 0$$

$$\ln y = x + x \ln x$$

$$f(x) = y = e^{x + x \ln x}$$

$$\Rightarrow f(e) = e^{e+e} = e^{2e}$$

- 18.** The number of critical points of the function

$$f(x) = \begin{cases} \left| \frac{\sin x}{x} \right|, & x \neq 0 \\ 1, & x = 0 \end{cases} \text{ in the interval } (-2\pi, 2\pi) \text{ is}$$

equal to:

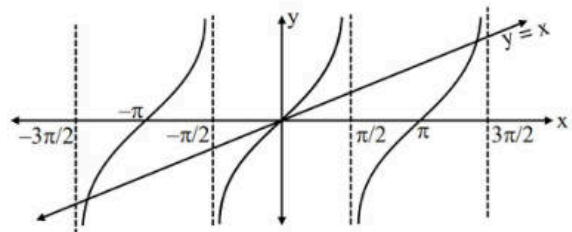
- (1) 1 (2) 3
(3) 5 (4) 7

Ans. (3)

Sol. $\lim_{x \rightarrow 0} \left| \frac{\sin x}{x} \right| = 1 = f(0) \rightarrow f(x) \text{ is continuous}$

$$\text{Now } \frac{d}{dx} \left(\frac{\sin x}{x} \right) = \frac{x \cos x - \sin x}{x^2}$$

$$\Rightarrow f'(x) = 0 \Rightarrow \tan x = x$$



3 solution in $(-2\pi, 2\pi)$

also $f'(0)$ does not exist at $x = -\pi, \pi$

total 5 points

- 19.** Let $[\cdot]$ denote the greatest integer function. Then

the value of $\int_0^3 \left(\frac{e^x + e^{-x}}{[x]!} \right) dx$ is:

- (1) $e^2 + e^3 - \frac{1}{e^2} - \frac{1}{e^3}$
(2) $\frac{1}{2} \left(e^2 + e^3 - \frac{1}{e^2} - \frac{1}{e^3} \right)$
(3) $e^2 + e^3 - \frac{1}{2e^2} - \frac{1}{2e^3}$
(4) $\frac{1}{2} (e^2 + e^3) - \frac{1}{e^2} - \frac{1}{e^3}$

Ans. (2)

Sol. $\int_0^1 \frac{e^x + e^{-x}}{1} dx + \int_1^2 \frac{e^x + e^{-x}}{1} dx + \int_2^3 \frac{e^x + e^{-x}}{2} dx$

$$\int_0^2 e^x dx + \int_0^2 e^{-x} dx + \frac{1}{2} \int_2^3 (e^x + e^{-x}) dx$$

$$(e^2 - 1) + \frac{(e^{-2} - 1)}{-1} + \frac{1}{2} ((e^3 - e^2) - (e^{-3} - e^{-2}))$$

$$\frac{e^3 + e^2}{2} + 1 - 1 - \frac{e^{-2}}{2} - \frac{e^{-3}}{2}$$

$$\frac{e^3 + e^2 - e^{-2} - e^{-3}}{2}$$

- 20.** Let $y = y(x)$ be the solution curve of the differential equation $(1 + \sin x) \frac{dy}{dx} + (y + 1) \cos x = 0$, $y(0) = 0$. If the curve $y = y(x)$ passes through the point $\left(\alpha, \frac{-1}{2}\right)$, then a value of α is :

- (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$
 (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Ans. (4)

Sol. $\int \frac{dy}{y+1} = - \int \frac{\cos x}{1 + \sin x} dx$

$$\Rightarrow \ln(y+1) = -\ln(1 + \sin x) + C$$

$$\because y(0) = 0 \Rightarrow C = 0$$

$$\Rightarrow y+1 = \frac{1}{1 + \sin x}$$

passes through $\left(\alpha, -\frac{1}{2}\right)$

$$\Rightarrow \sin x = 1$$

$$\Rightarrow x = \frac{\pi}{2} \text{ (as per options)}$$

SECTION-B

- 21.** If the domain of the function $f(x) = \sqrt{\log_{(0.6)} \left(\frac{2x-5}{x^2-4} \right)}$ is $(-\infty, a] \cup \{b\} \cup [c, d) \cup (e, \infty)$, then the value of $a + b + c + d + e$ is _____.

Ans. (4)

Sol. For domain $\log_{0.6} \left| \frac{2x-5}{x^2-4} \right| \geq 0$

$$\left| \frac{2x-5}{x^2-4} \right| \leq 1 \quad \& \quad x \neq \frac{5}{2} \quad \dots (1)$$

$$-1 \leq \frac{2x-5}{x^2-4} \leq 1$$

$$\frac{2x-5}{x^2-4} + 1 \geq 0$$

$$\frac{x^2 + 2x - 9}{x^2 - 4} \geq 0$$

$$\frac{(x+1)^2 - 10}{(x-2)(x+2)} \geq 0$$

$$\begin{array}{ccccccc} + & - & + & - & + \\ \leftarrow & | & | & | & | & | & \rightarrow \\ -1-\sqrt{10} & -2 & 2 & -1+\sqrt{10} & \end{array}$$

$$x \in (-\infty, -1-\sqrt{10}] \cup (-2, 2) \cup [-1+\sqrt{10}, \infty) \dots (2)$$

$$\frac{2x-5}{x^2-4} - 1 \leq 0$$

$$\frac{2x-5-x^2+4}{x^2-4} \leq 0$$

$$\frac{x^2 - 2x + 1}{x^2 - 4} \geq 0$$

$$\frac{(x-1)^2}{(x-2)(x+2)} \geq 0$$

$$x \in (-\infty, -2) \cup (2, \infty) \cup \{1\} \dots (3)$$

$$(1) \cap (2) \cap (3)$$

$$x \in (-\infty, -1-\sqrt{10}] \cup \{1\} \cup [-1+\sqrt{10}, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$$

$$a + b + c + d + e = -2 + 1 + 5 = 4$$

22. If $\sum_{k=1}^n a_k = 6n^3$, then $\sum_{k=1}^6 \left(\frac{a_{k+1} - a_k}{36} \right)^2$ is equal to ____.

Ans. (91)

Sol. $a_1 + a_2 + \dots + a_n = 6n^3$
 $a_1 + a_2 + \dots + a_n + a_{n+1} = 6(n+1)^3$
 $a_{n+1} = 6(n+1)^3 - 6n^3$
 $= 6((n+1) - n)((n+1)^2 + n^2 + n(n+1))$
 $a_{n+1} = 6(1)(3n^2 + 3n + 1)$
 $a_n = 6(3(n-1)^2 + 3(n-1) + 1)$
 $= 6(3(n^2 - 2n + 1) + 3(n-1) + 1)$
 $= 6(3(n^2 - n) + 1)$
 $a_n = 6(3n^2 - 3n + 1)$
 $\sum_{k=1}^6 \left(\frac{6(3k^2 + 3k + 1) - 6(3k^2 - 3k + 1)}{36} \right)^2$
 $\sum_{k=1}^6 \left(\frac{36k}{36} \right)^2 = \sum_{k=1}^6 k^2 = \frac{6 \times 7 \times 13}{6} = 91$

23. Let $a, b, c \in \{1, 2, 3, 4\}$. If the probability, that $ax^2 + 2\sqrt{2}bx + c > 0$ for all $x \in \mathbf{R}$, is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to ____.

Ans. (81)

Sol. $8b^2 - 4ac < 0 \Rightarrow 2b^2 < ac$

b	a	c	
1	1	3, 4	→ 2
1	2	2, 3, 4	→ 3
1	3	1, 2, 3, 4	→ 4
1	4	1, 2, 3, 4	→ 4
2	3	3, 4	→ 2
2	4	3, 4	→ 2
17			

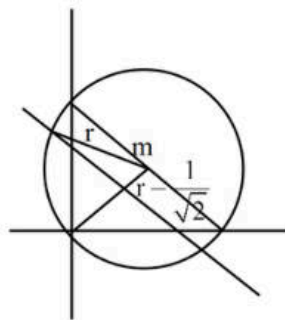
Required probability = $\frac{17}{64} \equiv \frac{m}{n}$

$\therefore m + n = 17 + 64 = 81$

24. Let a circle C have its centre in the first quadrant, intersect the coordinate axes at exactly three points and cut off equal intercepts from the coordinate axes. If the length of the chord of C on the line $x + y = 1$ is $\sqrt{14}$, then the square of the radius of C is ____.

Ans. (8)

Sol.



$$r^2 = \left(\frac{\sqrt{14}}{2} \right)^2 + \left(r - \frac{1}{\sqrt{2}} \right)^2$$

$$r^2 = \frac{7}{2} + r^2 - \sqrt{2}r + \frac{1}{2}$$

$$\sqrt{2}r = 4$$

$$r = 2\sqrt{2}$$

$$r^2 = 8$$

25. If $\alpha = \int_0^{2\sqrt{3}} \log_2(x^2 + 4) dx + \int_2^4 \sqrt{2^x - 4} dx$, then α^2 is equal to ____.

Ans. (192)

Sol. As we know that

$$\int_a^b f(x) dx + \int_{f(a)}^{f(b)} f^{-1}(x) dx = bf(b) - af(a)$$

$$\therefore \alpha = (2\sqrt{3})(4) - (0)(2)$$

$$= 8\sqrt{3}$$

$$\Rightarrow \alpha^2 = 192$$

PHYSICS

SECTION-A

26. The dimensional formula of $\frac{1}{2} \epsilon_0 E^2$ (ϵ_0 = permittivity of vacuum and E = electric field) is $M^a L^b T^c$. The value of $2a - b + c =$ _____.

- (1) 0 (2) 1
(3) -1 (4) 2

Ans. (2)

Sol. $\frac{1}{2} \epsilon_0 E^2$ = Energy density

$$\therefore \left[\frac{1}{2} \epsilon_0 E^2 \right] = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

$$\therefore a = 1, b = -1, c = -2$$

$$\therefore 2a - b + c = 2 + 1 - 2 = 1$$

27. The diameter of a wire measured by a screw gauge of least count 0.001 cm is 0.08 cm. The length measured by a scale of least count 0.1 cm is 150 cm. When a weight of 100 N is applied to the wire, the extension in length is 0.5 cm, measured by a micrometer of least count 0.001 cm. The error in the measured Young's modulus is $\alpha \times 10^9 \text{ N/m}^2$. The value of α is _____.

(Ignore the contribution of the load to Young's modulus error calculation)

- (1) 1.3 (2) 1.65
(3) 0.13 (4) 0.25

Ans. (2)

TEST PAPER WITH SOLUTION

Sol. $Y = \frac{F\ell}{A\Delta\ell}$

$$\therefore \frac{\Delta Y}{Y} = \frac{\Delta F}{F} + \frac{\Delta \ell}{\ell} + 2 \frac{\Delta d}{d} + \frac{\Delta(\Delta \ell)}{\Delta \ell}$$

$$\left[A = \pi \frac{d^2}{4} \Rightarrow \frac{\Delta A}{A} = \frac{2\Delta d}{d} \right]$$

Since F is known exactly $\Rightarrow \Delta F = 0$

$$\frac{\Delta Y}{Y} = \frac{0.1}{150} + \frac{2 \times 0.001}{0.08} + \frac{0.001}{0.5} = 0.0277$$

$$\therefore \Delta Y = 0.0277 Y$$

$$Y = \frac{100 \times 1.5}{\frac{\pi}{4} \left(\frac{0.08}{100} \right)^2 \times 0.5 \times 10^{-2}} = 5.97 \times 10^{10} \text{ N/m}^2$$

$$\therefore \Delta Y = 0.0277 \times 5.97 \times 10^{10} \text{ N/m}^2 = 1.65 \times 10^9 \text{ N/m}^2$$

28. The velocity of a particle is given as

$\vec{v} = -x\hat{i} + 2y\hat{j} - z\hat{k} \text{ m/s}$. The magnitude of acceleration at point (1, 2, 4) is _____ m/s^2 .

- (1) $\sqrt{6}$ (2) 9
(3) $\sqrt{33}$ (4) 0

Ans. (2)

Sol. $\vec{v} = -x\hat{i} + 2y\hat{j} - z\hat{k}$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{dx}{dt}\hat{i} + 2\frac{dy}{dt}\hat{j} - \frac{dz}{dt}\hat{k}$$

$$= -(-x)\hat{i} + 2(2y)\hat{j} - (-z)\hat{k} = x\hat{i} + 4y\hat{j} + z\hat{k}$$

$$\therefore \vec{a} \text{ at } (1, 2, 4) = \hat{i} + 8\hat{j} + 4\hat{k}$$

$$\therefore |\vec{a}| = 9 \text{ m/s}^2$$

29. The position of an object having mass 0.1 kg as a function of time t is given as $\vec{r} = (10t^2\hat{i} + 5t^3\hat{j})\text{m}$.

At $t = 1$ s, which of the following statements are **correct**?

A. The linear momentum $\vec{p} = (2\hat{i} + 1.5\hat{j})\text{kg.m/s}$.

B. The force acting on the object $\vec{F} = (2\hat{i} + 3\hat{j})\text{N}$.

C. The angular momentum of the object about its origin $\vec{L} = 15\hat{k}\text{Js}$.

D. The torque acting on the object about its origin $\vec{\tau} = 20\hat{k}\text{Nm}$.

Choose the **correct** answer from the option given below.

(1) A, B and C only

(2) B, C and D only

(3) A, C and D only

(4) A, B and D only

Ans. (4)

Sol. Given $\vec{r} = 10t^2\hat{i} + 5t^3\hat{j} \Rightarrow \vec{r}_{t=1} = 10\hat{i} + 5\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 20t\hat{i} + 15t^2\hat{j} \Rightarrow \vec{v}_{t=1\text{sec}} = 20\hat{i} + 15\hat{j}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 20\hat{i} + 30t\hat{j} \Rightarrow \vec{a}_{t=1\text{sec}} = 20\hat{i} + 30\hat{j}$$

$$\therefore \vec{P} = m\vec{v} = 2\hat{i} + 1.5\hat{j}$$

$$\vec{F} = m\vec{a} = 2\hat{i} + 3\hat{j}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = 20\hat{k}$$

$$\vec{L} = \vec{r} \times \vec{P} = 5\hat{k}$$

30. A planet (P_1) is moving around the star of mass $2M$ in the orbit of radius R . Another planet (P_2) is moving around another star of mass $4M$ in a orbit of radius $2R$. Ratio of time periods of revolution of P_2 and P_1 is _____.

(1) $\frac{1}{2}$ (2) 2

(3) 4 (4) $\frac{1}{4}$

Ans. (2)

Sol. From Kepler's law,

$$T^2 = \frac{4\pi^2}{GM_s} r^3$$

$$\therefore T_{P_1} = \frac{2\pi}{\sqrt{G(2M)}} R^{3/2}$$

$$T_{P_2} = \frac{2\pi}{\sqrt{G(4M)}} (2R)^{3/2}$$

$$\therefore \frac{T_{P_2}}{T_{P_1}} = \frac{(2)^{3/2}}{\sqrt{4}} \times \frac{\sqrt{2}}{1} = \frac{2\sqrt{2} \times \sqrt{2}}{2} = 2$$

31. A particle is rotating in a circular path and at any instant its motion can be described as $\theta = \frac{5t^4}{40} - \frac{t^3}{3}$.

The angular acceleration of the particle after 10 seconds is _____ rad/s^2 .

(1) 150 (2) 120

(3) 130 (4) 170

Ans. (3)

Sol. $\theta = \frac{5t^4}{40} - \frac{t^3}{3}$

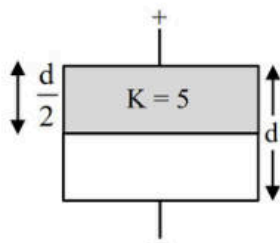
$$\frac{d\theta}{dt} = \frac{5}{40}(4t^3) - \frac{3t^2}{3} = \frac{t^3}{2} - t^2$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{3t^2}{2} - 2t$$

at $t = 10$

$$\alpha = \frac{3}{2}(10)^2 - 2 \times 10 = 130 \text{ rad/sec}^2$$

32. A parallel plate air capacitor has a capacitance C . When it is half filled as show in figure with a dielectric constant $K = 5$, the percentage increase in the capacitance is _____.



- (1) 33.34 (2) 66.67
(3) 200 (4) 400

Ans. (2)

Sol. Initial capacitance

$$C_i = \frac{A \epsilon_0}{d} = C$$

Final capacitance

$$C_1 = \frac{A \epsilon_0 \times 5}{\frac{d}{2}} = 10C$$

$$C_2 = \frac{A \epsilon_0}{\frac{d}{2}} = 2C$$

$$C_f = \frac{C_1 C_2}{C_1 + C_2} = \frac{10 \times 2}{10 + 2} C = \frac{5}{3} C$$

$$\% \text{ increase} = \frac{\frac{5}{3}C - C}{C} \times 100 = 66.67$$

33. Heat is supplied to a diatomic gas at constant pressure. Then the ratio of $\Delta Q : \Delta U : \Delta W$ is _____.
(1) 2 : 3 : 5
(2) 5 : 3 : 2
(3) 2 : 5 : 7
(4) 7 : 5 : 2

Ans. (4)

Sol. $\Delta Q_p = nC_p \Delta T = \frac{7}{2} nR \Delta T$

$$\Delta U = nC_v \Delta T = \frac{5}{2} nR \Delta T$$

$$W = nR \Delta T$$

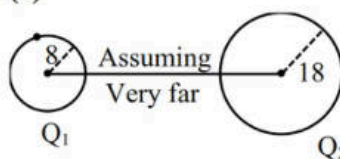
$$\Delta Q : \Delta U : W :: 7 : 5 : 2$$

34. Two charged conducting spheres S_1 and S_2 of radii 8 cm and 18 cm are connected to each other by a wire. After equilibrium is established, the ratio of electric fields on S_1 and S_2 spheres are E_{S_1} and E_{S_2} respectively. The value of $\frac{E_{S_1}}{E_{S_2}}$ is _____.

- (1) $\frac{3}{2}$ (2) $\frac{2}{3}$
(3) $\frac{4}{9}$ (4) $\frac{9}{4}$

Ans. (4)

Sol.



$$V_1 = V_2$$

$$\frac{kQ_1}{8} = \frac{kQ_2}{18} \quad \left\{ v_{\text{surface}} = \frac{kQ}{R} \right\}$$

$$\frac{Q_1}{Q_2} = \frac{4}{9}$$

$$\frac{E_{S_1}}{E_{S_2}} = \frac{kQ_1}{r_1^2} \frac{r_2^2}{kQ_2} \quad \left\{ E_{\text{surface}} = \frac{kQ}{R^2} \right\}$$

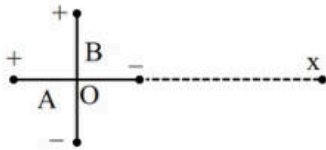
$$= \frac{Q_1}{Q_2} \left(\frac{r_2}{r_1} \right)^2 = \frac{4}{9} \times \left(\frac{9}{4} \right)^2 = \frac{9}{4}$$

35. The equation of a plane progressive wave is given by $y = 5 \cos \pi \left(200t - \frac{x}{150} \right)$ where x and y are in cm and t is in second. The velocity of the wave is _____ m/s
(1) 120 (2) 150
(3) 200 (4) 300

Ans. (4)

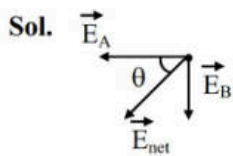
Sol. $v = \frac{\text{coeff of } t}{\text{coeff of } x} = \frac{200}{\frac{1}{150} \times 100} = 300 \text{ m/s}$

36. Two short electric dipoles A and B having dipole moment p_1 and p_2 respectively are placed with their axis mutually perpendicular as shown in the figure. The resultant electric field at a point x is making an angle of 60° with the line joining points O and x. The ratio of the dipole moments p_2/p_1 is _____.



- (1) $\frac{\sqrt{3}}{2}$
 (2) $2\sqrt{3}$
 (3) $\frac{1}{\sqrt{3}}$
 (4) $\sqrt{3}$

Ans. (2)



$$\vec{E}_A = \frac{2kP_1}{x^3}(-\hat{i}) \text{ \{Axial point\}}$$

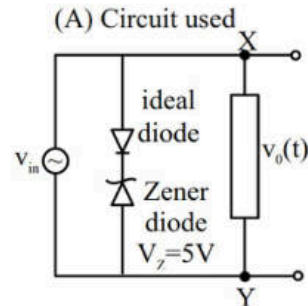
$$\vec{E}_B = \frac{kP_2}{x^3}(-\hat{j}) \text{ \{Equatorial point\}}$$

$$\text{Now : } \tan \theta = \frac{|\vec{E}_B|}{|\vec{E}_A|}$$

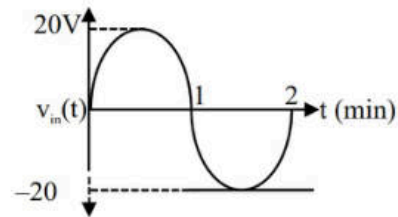
$$\Rightarrow \tan 60^\circ = \frac{P_2}{2P_1}$$

$$\Rightarrow \frac{P_2}{P_1} = 2\sqrt{3}$$

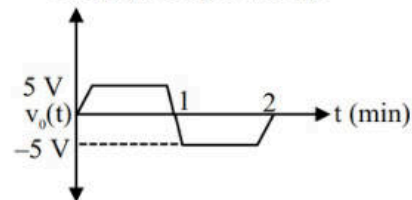
37. For the given circuit (shown in part (A)) the time dependent input voltage $v_{in}(t)$ and corresponding output $v_o(t)$ are shown in part (B) and part (C), respectively. Identify the components that are used in the circuit between points X and Y.



Part(B) : Input voltage



Part(C) : Output voltage



- (1) X Y
 (2) X Y
 (3) X Y
 (4) X Y

Ans. (2)

Sol. For the +ve half cycle of input

Middle branch has ideal diode in forward bias and Zener diode in reverse bias. Thus, we get constant 5V output due to Zener breakdown. For this situation component between x & y should act as open circuit.

For -ve half cycle of input

middle branch has ideal diode in reverse bias, which makes it open branch without any current. for this situation component between x & y should give a constant -5V output due to Zener breakdown.

(Note : The input side must contain some resistance else V_{in} is equal to V_{out} for every cases)

38. When a coil is placed in a time dependent magnetic field the power dissipated in it is P. The number of turns, area of the coil and radius of the coil wire are N, A and r respectively. For a second coils number of turns, area of the coil and radius of the coil wire are 2N, 2A and 3r respectively. When the first coil is replaced with second coil the power dissipated in it is $\sqrt{2} \alpha P$. The value of α is _____.

- (1) 36 (2) $128\sqrt{2}$
(3) 16 (4) 64

Ans. (1)

Sol. Power dissipated : $P = \frac{\varepsilon^2}{R}$

$$\text{Induced emf } \varepsilon = -NA \frac{dB}{dt}$$

$$\text{Resistance of coil : } R = \rho \frac{\ell}{\pi r^2}$$

$$\text{Now : } \ell = N(2\pi R_{\text{coil}})$$

$$\text{Also : } A = \pi R_{\text{coil}}^2 \Rightarrow R_{\text{coil}} = \sqrt{\frac{A}{\pi}}$$

$$\text{Thus ; } R = \frac{\rho N(2\pi)}{\pi r^2} \sqrt{\frac{A}{\pi}}$$

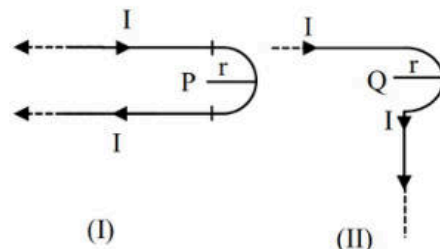
$$P = \frac{N^2 A^2 \left(\frac{dB}{dt}\right)^2}{\frac{2\rho N}{r^2} \sqrt{\frac{A}{\pi}}} \Rightarrow P \propto NA^{3/2} r^2 \dots\dots(1)$$

$$P' \propto (2N)(2A)^{3/2}(3r)^2 \dots\dots(2)$$

$$\frac{(2)}{(1)} : \frac{P'}{P} = 36\sqrt{2} \Rightarrow P' = (\sqrt{2})(36)P$$

$$\text{Thus : } \alpha = 36$$

39. Two identical long current carrying wires are bent into the shapes shown in the following figures. If the magnitude of magnetic fields at the centres P and Q of a semicircular arc are B_1 and B_2 respectively, then the ratio $\frac{B_1}{B_2}$ is _____.



- (1) $\frac{2+\pi}{1+\pi}$ (2) $\frac{1+\pi}{1-\pi}$
(3) $\frac{2+\pi}{1-\pi}$ (4) $\frac{1+\pi}{2-\pi}$

Ans. (1)

$$\text{Sol. } B_1 = \frac{\mu_0 I}{4\pi r} \times 2 + \frac{\mu_0 I}{4r} = \frac{\mu_0 I}{4r} \left(\frac{2}{\pi} + 1 \right)$$

$$B_2 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r} = \frac{\mu_0 I}{4r} \left(\frac{1}{\pi} + 1 \right)$$

$$\frac{B_1}{B_2} = \frac{2+\pi}{1+\pi}$$

40. For a thin symmetric prism made of glass (refractive index 1.5), the ratio of incident angle and minimum deviation will be ____.

- (1) 3 : 4 (2) 3 : 2
(3) 2 : 1 (4) 1 : 2

Ans. (2)

Sol. We have for thin prism minimum deviation :

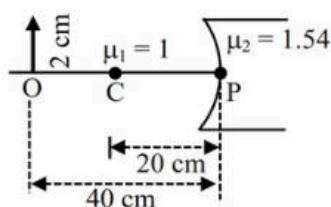
$$\delta_m = (\mu - 1)A$$

$$\Rightarrow \delta_m = (1.5 - 1)A \quad \Rightarrow \delta_m = \frac{A}{2}$$

$$\text{Also, } i = \frac{A + \delta_m}{2} \Rightarrow i = \frac{A + \frac{A}{2}}{2} \Rightarrow i = \frac{3A}{4}$$

$$\text{Thus } \frac{i}{\delta_m} = \frac{3}{2}$$

41. Refer the figure given below. μ_1 and μ_2 are refractive indices of air and lens material. The height of image will be ____ cm.



- (1) 1 (2) 0.5
(3) 1.2 (4) 0.25

Ans. (1)

$$\text{Sol. } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.54}{v} - \frac{1}{(-40)} = \frac{1.54 - 1}{-20}$$

$$\frac{1.54}{v} = -\frac{1}{40} - \frac{0.54}{20}$$

$$\frac{1.54}{v} = \frac{-2.08}{40}$$

$$v = -29.61 \text{ cm}$$

$$\frac{h_i}{h_o} = \frac{\mu_1 v}{\mu_2 u} = \frac{1(-29.61)}{1.54(-40)} = 0.48$$

$$h_i = 0.48 \times 2 = 0.96 \approx 1 \text{ cm}$$

42. For a certain metal, when monochromatic light of wavelength λ is incident, the stopping potential for photoelectrons is $3V_0$. When the same metal is illuminated by light of wavelength 2λ , then the stopping potential becomes V_0 . The threshold wavelength for photoelectric emission for the given metal is $\alpha\lambda$. The value of α is ____.

- (1) 1 (2) 4
(3) 2 (4) 3

Ans. (2)

$$\text{Sol. } \frac{hc}{\lambda} - \phi = 3eV_0 \quad \dots\dots(i)$$

$$\frac{hc}{2\lambda} - \phi = eV_0 \quad \dots\dots(ii)$$

$$\frac{hc}{\lambda} - \phi = 3 \left[\frac{hc}{2\lambda} - \phi \right]$$

$$\phi = \frac{hc}{4\lambda}$$

$$\frac{hc}{\lambda_0} = \frac{hc}{4\lambda}$$

$$\lambda_0 = 4\lambda$$

$$\alpha = 4$$

43. An electromagnetic wave travelling in x-direction is described by field equation $E_y = 300 \sin \omega \left(t - \frac{x}{c} \right)$.

If the electron is restricted to move in y-direction only with speed of 1.5×10^6 m/s then ratio of maximum electric and magnetic forces acting on the electron is ____.

- (1) 200 (2) 150
(3) 400 (4) 300

Ans. (1)

$$\text{Sol. } E_y = 300 \sin \omega \left(t - \frac{x}{c} \right)$$

$$F_M = qvB \quad \dots\dots(i)$$

$$F_e = qE \quad \dots\dots(ii)$$

$$\frac{F_e}{F_m} = \frac{qE}{qvB} = \frac{E}{vB} = \frac{c}{v} = \frac{3 \times 10^8}{1.5 \times 10^6} = 200$$

44. Angular momentum of an electron in a hydrogen atom is $\frac{3h}{\pi}$, then the energy of the electron is ____ eV.

- (1) -1.51 (2) -0.85
(3) -0.38 (4) -0.28

Ans. (3)

Sol. $L = \frac{nh}{2\pi}$

$$\frac{3h}{\pi} = \frac{nh}{2\pi}$$

$$n = 6$$

$$E = \frac{-13.6}{n^2} \text{ eV}$$

$$= \frac{-13.6}{6^2} \approx -0.38 \text{ eV}$$

45. A liquid drop of diameter 2 mm breaks into 512 droplets. The change in surface energy is $\alpha \times 10^{-6}$ J. The value of α is _____. (Take surface tension of liquid = 0.08 N/m)

- (1) 10 (2) 7
(3) 8 (4) 11

Ans. (2)

Sol. $V_1 = V_2$

$$\frac{4}{3}\pi R^3 = 512 \left(\frac{4}{3}\pi r^3 \right)$$

$$R = (512)^{1/3} r$$

$$1 = 8r$$

$$r = \frac{1}{8} \text{ mm}$$

$$\Delta SE = SE_2 - SE_1$$

$$= 512 [T \times 4\pi r^2] - [T \times 4\pi R^2]$$

$$= 4\pi T [512r^2 - R^2]$$

$$= 4\pi(0.08) \left[512 \left(\frac{1}{8} \right)^2 - 1^2 \right] \times 10^{-6} \approx 7 \times 10^{-6}$$

$$\alpha = 7$$

SECTION-B

46. In single slit diffraction pattern, the wavelength of light used is 628 nm and slit width is 0.2 mm, the angular width of central maximum is $\alpha \times 10^{-2}$ degrees. The value of α is _____.

Ans. (36)

Sol. Angular width of central maxima $\theta_{\text{cm}} = \frac{2\lambda}{d}$

$$= \frac{2 \times 628 \times 10^{-9}}{0.2 \times 10^{-3}} \text{ rad.}$$

$$= (628 \times 10^{-5}) \times \frac{180}{\pi} \text{ degrees}$$

$$\approx 36 \times 10^{-2} \text{ degrees}$$

$$\therefore \text{Answer : } \alpha = 36$$

47. A vessel contains 0.15 m³ of a gas at pressure 8 bar and temperature 140°C with $c_p = 3R$ and $c_v = 2R$. It is expanded adiabatically till pressure falls to 1 bar. The work done during this process is ____ k J. (R is gas constant)

Ans. (120)

Sol. $r = \frac{C_p}{C_v} = \frac{3}{2}$

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\Rightarrow V_2 = \left(\frac{P_1}{P_2} \right)^{1/\gamma} V_1$$

$$\Rightarrow V_2 = \left(\frac{8}{1} \right)^{2/3} \times 0.15$$

$$\Rightarrow V_2 = 0.6 \text{ m}^3$$

$$\therefore W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$= \left(\frac{8 \times 0.15 - 1 \times 0.6}{1.5 - 1} \right) \times 10^5 = 1.2 \times 10^5$$

$$\Rightarrow W = 120 \text{ kJ}$$

48. A $1 \mu\text{C}$ charge moving with velocity $\vec{v} = (\hat{i} - 2\hat{j} + 3\hat{k}) \text{ m/s}$ in the region of magnetic field $\vec{B} = (2\hat{i} + 3\hat{j} - 5\hat{k}) \text{ T}$. The magnitude of force acting on it is $\sqrt{\alpha} \times 10^{-6} \text{ N}$. The value of α is ____.

Ans. (171)

Sol. $\vec{F} = q(\vec{v} \times \vec{B})$

$$= 1 \times 10^{-6} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix}$$

$$= 10^{-6} [\hat{i} + 11\hat{j} + 7\hat{k}]$$

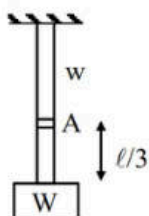
$$\Rightarrow \vec{F} = \sqrt{171} \times 10^{-6} \text{ N}$$

$$\therefore \alpha = 171$$

49. A uniform wire of length l of weight w is suspended from the roof with a weight of W at the other end. The stress in the wire at $\frac{l}{3}$ distance from the top is $\left(\frac{W}{A} + \frac{2}{\gamma} \frac{w}{A} \right)$, where, A is the cross sectional area of the wire. The value of γ is ____.

Ans. (6)

Sol.



$$\sigma = \frac{T}{A} = \frac{W + \frac{w}{3}}{A}$$

$$\Rightarrow \sigma = \frac{W}{A} + \frac{2}{6} \frac{w}{A}$$

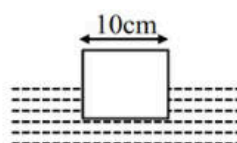
$$\therefore \gamma = 6$$

50. A tube is filled with water and a wooden cube $10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm}$ is placed in the water. The wooden cube is found to float on the water with a part of it submerged in water. When a metal coin is placed on the wooden cube, the submerged part is increased by 3.87 cm . The mass of the metal coin is ____ gram.

(Take water density as 1 g/cm^3 and density of wood as 0.4 g/cm^3)

Ans. (387)

Sol.



Weight of coin = Additional upthrust

$$\Rightarrow m_c g = \rho_{\ell} V_{\text{add}} g$$

$$\Rightarrow m_c = 1 \times 3.87 \times (10)^2$$

$$\Rightarrow m_c = 387 \text{ gram}$$

CHEMISTRY

SECTION-A

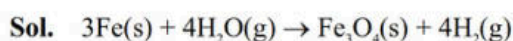
51. The mass of iron converted into Fe_3O_4 by the action of 18 g of steam is :

(Given : Molar mass of H, O and Fe are 1, 16 and 56 g mol^{-1} respectively)

Assume iron is present in excess :

- (1) 2.1 g (2) 4.2 g
(3) 21 g (4) 42 g

Ans. (4)



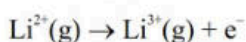
$$n_{\text{steam}} = \frac{18}{18} = 1$$

$$n_{\text{Fe}} \text{ required} = \frac{3}{4}$$

$$\text{mass of Fe required} = \frac{3}{4} \times 56$$

$$= 42 \text{ gm}$$

52. What is the energy (in J atom^{-1}) required for the following process ?



(Take the ionization energy for the H atom in the ground state as $2.18 \times 10^{-18} \text{ J atom}^{-1}$)

- (1) 8.72×10^{-18}
(2) 1.962×10^{-18}
(3) 1.962×10^{-17}
(4) 6.54×10^{-17}

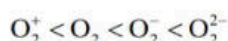
Ans. (3)

Sol. $E_{\text{Li}^{2+}} = E_{\text{H}} \times Z^2$
 $= 2.18 \times 10^{-18} \times (3)^2$
 $= 1.962 \times 10^{-17} \text{ J atom}^{-1}$

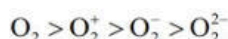
TEST PAPER WITH SOLUTION

53. Given below are two statements :

Statement (I) : The correct sequence of bond lengths in the following species is :



Statement (II) : The correct sequence of number of unpaired electrons in the following species is :



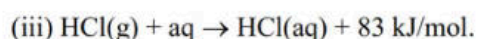
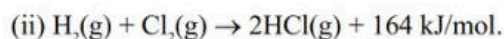
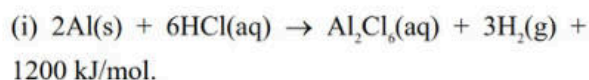
In the light of the above statements, choose the **correct answer** from the options given below :

- (1) Both **Statement I** and **Statement II** are true
(2) Both **Statement I** and **Statement II** are false
(3) **Statement I** is true but **Statement II** is false
(4) **Statement I** is false but **Statement II** is true

Ans. (3)

Sol.	Species	Bond order	unpaired electrons
	$\text{O}_2^{\oplus}$	2.5	1
	O_2	2	2
	O_2^-	1.5	1
	O_2^{2-}	1	0

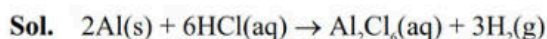
54. Consider the following data



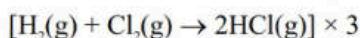
The enthalpy of formation of anhydrous solid Al_2Cl_6 is :

- (1) -648 kJ mol^{-1} (2) $-1350 \text{ kJ mol}^{-1}$
(3) $-2002 \text{ kJ mol}^{-1}$ (4) $-1527 \text{ kJ mol}^{-1}$

Ans. (4)



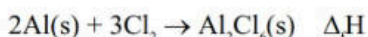
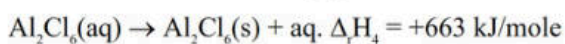
$$\Delta_r H_1 = -1200 \text{ kJ/mol}$$



$$\Delta_r H_2 = -163 \times 3 \text{ kJ/mol}$$



$$\Delta_r H_3 = -83 \times 6 \text{ kJ/mol}$$



$$\Delta_r H = \Delta_r H_1 + \Delta_r H_2 + \Delta_r H_3 + \Delta_r H_4$$

$$= -1200 - 163 \times 3 - 83 \times 6 + 663$$

$$= -1527 \text{ kJ/mole}$$

- 55.** 19.5 g of fluoro acetic acid (molar mass = 78 g mol^{-1}) is dissolved in 500g of water at 298 K. The depression in the freezing point of water was 1°C . What is K_a of fluoro acetic acid ?

(For water, $K_f = 1.86 \text{ K kg mol}^{-1}$). Assume molarity and molality to have same values.

$$(1) 10^{-6}$$

$$(2) 4 \times 10^{-4}$$

$$(3) 3 \times 10^{-5}$$

$$(4) 3 \times 10^{-3}$$

Ans. (4)



$$\text{no. of moles} = \frac{19.5}{78} = \frac{1}{4} \text{ mole}$$

$$1 = i \times 1.86 \times \frac{1/4}{1/2}$$

$$i = \frac{2}{1.86}$$

$$i = 1 + (n-1)\alpha$$

$$i = 1 + \alpha$$

$$\alpha = \frac{2}{1.86} - 1 = 0.075$$

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

$$= \frac{1}{2} \times \frac{(0.075)^2}{(1-0.075)}$$

$$= 3 \times 10^{-3}$$

- 56.** The solubility product constants of Ag_2CrO_4 and AgBr are $32x$ and $4y$ respectively at 298 K.

The value of $\left(\frac{\text{molarity of } \text{Ag}_2\text{CrO}_4}{\text{molarity of } \text{AgBr}} \right)$ can be expressed as :

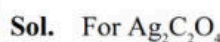
$$(1) \frac{2\sqrt[3]{x}}{y}$$

$$(2) 2\sqrt{\frac{x}{y}}$$

$$(3) \sqrt{\frac{x}{y}}$$

$$(4) \frac{\sqrt[3]{x}}{\sqrt{y}}$$

Ans. (4)



$$4S_1^3 = 32x$$

$$\Rightarrow S_1 = 2x^{1/3}$$

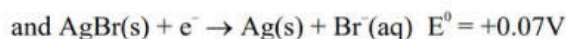
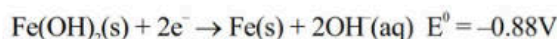


$$S_2^2 = 4y$$

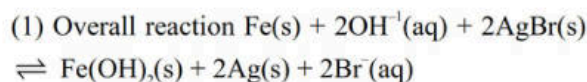
$$\Rightarrow S_2 = 2y^{1/2}$$

$$\text{Ratio} \left(\frac{S_1}{S_2} \right) = \frac{\sqrt[3]{x}}{\sqrt{y}}$$

- 57.** An electrochemical cell is constructed using half cells in the direction of spontaneous change



Which of the following option is **correct** ?

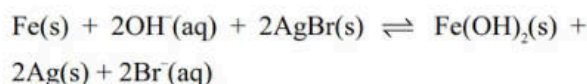
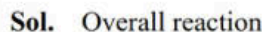


$$(2) E_{\text{cell}}^\circ = -0.95\text{V}$$

(3) Fe is reduced in the electrochemical cell

(4) E_{cell}° is an extensive property

Ans. (1)



$$E_{\text{cell}}^\circ = E_{\text{R(cathode)}}^\circ - E_{\text{R(anode)}}^\circ$$

$$= 0.07 - (-0.88) = 0.95 \text{ V}$$

58. $t_{100\%}$ is the time required for the 100% completion of the reaction while $t_{1/2}$ is the time required for 50% of the reaction to be completed. Which of the following option correctly represents the relation between $t_{100\%}$ and $t_{1/2}$ for zero and first order reactions respectively ?

- (1) $t_{100\%} = (t_{1/2})^2$ and $t_{100\%} = (t_{1/2})^{-\infty}$
 (2) $t_{100\%} = 2t_{1/2}$ and $t_{100\%} = (t_{1/2})^{\infty}$
 (3) $t_{100\%} = 2t_{1/2}$ and $t_{100\%} = (2t_{1/2})^2$
 (4) $t_{100\%} = (t_{1/2})^{\infty}$ and $t_{100\%} = 2t_{1/2}$

Ans. (2)

Sol. Zero –order

$$t_{100\%} = \frac{a}{k}; t_{1/2} = \frac{a}{2k}$$

$$t_{100\%} = 2t_{1/2}$$

1st order

$$t_{100\%} = \infty$$

$$t_{1/2} = \frac{\ell n 2}{k}$$

$$t_{100\%} = (t_{1/2})^{\infty}$$

59. Given below are two statements :

Statement (I) : The first ionisation enthalpy of the elements Na, Mg, Cl and Ar follows the order
 $\text{Na} > \text{Mg} > \text{Cl} > \text{Ar}$

Statement (II) : Among Ca, Al, Fe and B, the third ionisation enthalpy is very high for Ca.

In the light of the above statements, choose the **correct answer** from the options given below :

- (1) Both **Statement I** and **Statement II** are true
 (2) Both **Statement I** and **Statement II** are false
 (3) **Statement I** is true but **Statement II** is false
 (4) **Statement I** is false but **Statement II** is true

Ans. (4)

Sol. (i) $\text{Na} < \text{Mg} < \text{Cl} < \text{Ar} \rightarrow I^{\text{st}}$ ionization energy
 (ii) Among B, Ca, Al, Fe

Ca has maximum 3rd ionization energy among given due to inert gas configuration.

60. Given below are two statements :

Statement (I) : Oxidising power of halogens decreases in the order $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$, which is the basis of “Layer test”.

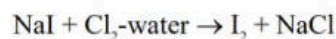
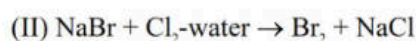
Statement (II) : “Layer test” to identify Br_2 and I_2 in aqueous solution involves the oxidation of bromide or iodide into Br_2 or I_2 respectively with Cl_2 , which is a type of displacement redox reaction.

In the light of the above statements, choose the **correct answer** from the options given below :

- (1) Both **Statement I** and **Statement II** are true
 (2) Both **Statement I** and **Statement II** are false
 (3) **Statement I** is true but **Statement II** is false
 (4) **Statement I** is false but **Statement II** is true

Ans. (1)

Sol. (I) $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$ (Oxidizing power order)



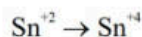
61. Which of the following sets includes all the species that will change the orange colour of $\text{K}_2\text{Cr}_2\text{O}_7$ in acidic medium ?

- (1) Fe^{2+} , Sn^{2+} , I^- , S^{2-}
 (2) S^{2-} , Fe^{3+} , I^- , $\text{C}_2\text{O}_4^{2-}$
 (3) Fe^{2+} , NO_2^- , SO_2 , Sn^{4+}
 (4) Fe^{3+} , SO_4^{2-} , S^{2-} , Sn^{4+}

Ans. (1)

Sol. Reducing agent + $\text{KmnO}_4 \xrightarrow[\text{medium}]{\text{acidic}} \text{Mn}^{+2} + \text{Product}$

Given Reducing Agent



62. Match List – I with List – II.

List – I	List – II
Chromium (III) Complexes	$\Delta_0(\text{cm}^{-1})$
(en = ethylene diamine)	

A. $[\text{Cr}(\text{CN})_6]^{3-}$	I. 15,060
B. $[\text{CrF}_6]^{3-}$	II. 17,400
C. $[\text{Cr}(\text{H}_2\text{O})_6]^{3+}$	III. 22,300
D. $[\text{Cr}(\text{en})_3]^{3+}$	IV. 26,600

Choose the **correct** answer from the options given below :

- (1) A-I, B-II, C-III, D-IV
- (2) A-II, B-III, C-IV, D-I
- (3) A-III, B-IV, C-I, D-II
- (4) A-IV, B-I, C-II, D-III

Ans. (4)

Sol. CFSE is proportional to strength of ligand followed by chelation.

CN^- is stronger ligand than en

$\text{CN}^- > \text{en} > \text{H}_2\text{O} > \text{F}^-$

63. Given below are two statements :

Statement (I) : 1,2,3-Trihydroxypropane can be separated from water by simple distillation.

Statement (II) : An azotropic mixture cannot be separated by fractional distillation.

In the light of the above statements, choose the **correct answer** from the options given below :

- (1) Both **Statement I** and **Statement II** are true
- (2) Both **Statement I** and **Statement II** are false
- (3) **Statement I** is true but **Statement II** is false
- (4) **Statement I** is false but **Statement II** is true

Ans. (4)

Sol. Glycerol is separated by distillation under reduced pressure.

64. Given below are two statements :

Statement (I) : Benzyl chloride reacts faster in $\text{S}_{\text{N}}1$ mechanism than ethyl chloride.

Statement (II) : Ethyl carbocation intermediate is less stabilized by hyperconjugation than benzyl carbocation by resonance.

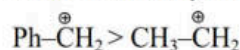
In the light of the above statements, choose the **correct answer** from the options given below :

- (1) Both **Statement I** and **Statement II** are true
- (2) Both **Statement I** and **Statement II** are false
- (3) **Statement I** is true but **Statement II** is false
- (4) **Statement I** is false but **Statement II** is true

Ans. (1)

Sol. Rate of $\text{S}_{\text{N}}1 \propto$ Stability of carbocation

Order of stability of carbocation :



Hyperconjugation in $\text{CH}_3-\overset{\oplus}{\text{C}}\text{H}_2$

Resonance in $\text{Ph}-\overset{\oplus}{\text{C}}\text{H}_2$

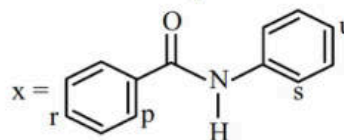
65. In IUPAC nomenclature, the **correct** order of decreasing priority of functional group is :

- (1) $-\text{CONH}_2$, $>\text{C}=\text{O}$, $-\text{CHO}$, $-\text{NH}_2$, $-\text{C}\equiv\text{C}-$
- (2) $-\text{CONH}_2$, $-\text{COOCH}_3$, $-\text{CHO}$, $-\text{NH}_2$, $-\text{OH}$
- (3) $-\text{CONH}_2$, $-\text{CHO}$, $>\text{C}=\text{O}$, $-\text{NH}_2$, $-\text{C}\equiv\text{C}-$
- (4) $-\text{CONH}_2$, $-\text{CHO}$, $-\text{CN}$, $-\text{NH}_2$, $-\text{C}\equiv\text{C}-$

Ans. (3)

Sol. Theory based

66. For the given molecule, “x”, the preferred site for the attack of the electrophile is :



- (1) Predominantly at “r”
- (2) “r” and “u”
- (3) “p” and “s”
- (4) Predominantly at “u”

Ans. (4)

Sol. U is predominant site towards attack of electrophile because of +M nature of nitrogen.

67. Match List-I with List-II.

List-I Mixture of Compounds	List-II Reagent used to distinguish
A. Diethyl amine + Ethyl amine	I. Bromine water
B. Acetaldehyde + Acetone	II. $\text{CHCl}_3 + \text{KOH}, \Delta$
C. Ethanol + Phenol	III. Neutral FeCl_3
D. Benzoic acid + Cinnamic acid	IV. Ammonical silver nitrate

Choose the **correct** answer from the options given below :

- (1) A-IV, B-II, C-I, D-III
- (2) A-IV, B-II, C-III, D-I
- (3) A-II, B-IV, C-I, D-III
- (4) A-II, B-IV, C-III, D-I

Ans. (4)

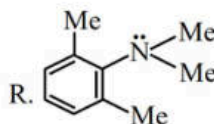
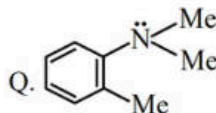
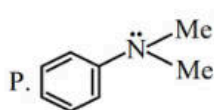
Sol. • Phenol gives violet colour with neutral FeCl_3 , not ethanol.

• 1° amine gives positive test with CHCl_3/KOH not 2° amine.

• Aldehyde gives positive test with ammonical silver nitrate not ketone.

• Cinnamic acid gives bromine water test.

68. Consider the three aromatic molecules (P, Q and R) whose structures have been given below :



The **correct** order regarding the reactivity of these compounds with $\text{Ph-N}\equiv\text{N Cl}^{(-)}$ under optimum but (+)

slightly acidic medium is :

- (1) $\text{P} > \text{Q} > \text{R}$
- (2) $\text{R} > \text{P} > \text{Q}$
- (3) $\text{R} > \text{Q} > \text{P}$
- (4) $\text{P} > \text{R} > \text{Q}$

Ans. (1)

Sol. Rate of ESR $\propto$ nucleophilicity of benzene ring.

69. Match List-I with List-II.

List-I Vitamin	List-II Name
A. Vitamin B_1	I. Pyridoxine
B. Vitamin B_2	II. Ascorbic acid
C. Vitamin B_6	III. Thiamine
D. Vitamin C	IV. Riboflavin

Choose the **correct** answer from the options given below :

- (1) A-II, B-I, C-III, D-IV
- (2) A-IV, B-III, C-II, D-I
- (3) A-III, B-IV, C-I, D-II
- (4) A-I, B-III, C-II, D-IV

Ans. (3)

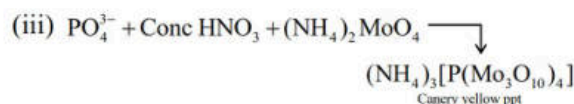
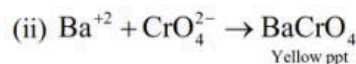
Sol. Theory based

70. A salt with few drops of conc. HCl gives apple green colour in flame test. The group precipitate of the salt is dissolved in acetic acid and treated with K_2CrO_4 to give yellow precipitate. When the sodium carbonate extract of the salt solution is heated with conc. HNO_3 and ammonium molybdate, it resulted a canary yellow precipitate. The cation and anion present in the salt are respectively.

- (1) Ca^{2+} and SO_4^{2-}
- (2) Ba^{2+} and PO_4^{3-}
- (3) Mn^{2+} and PO_4^{3-}
- (4) Ba^{2+} and SO_4^{2-}

Ans. (2)

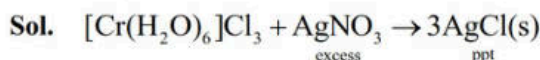
Sol. (i) Ba^{+2} gives apple green colour while Ca^{+2} and Sr^{+2} gives brick red and crimson red in flame test.



SECTION-B

71. 5.33 g of $\text{CrCl}_3 \cdot 6\text{H}_2\text{O}$, which is a 1 : 3 electrolyte, is dissolved in water and is passed through a cation exchanger. The chloride ions in the eluted solution, on treatment with AgNO_3 results in 8.61 g of AgCl . The ratio of moles of complex reacted and moles of AgCl formed is $\times 10^{-2}$. (Nearest integer)
[Molar mass in g mol^{-1} Cr : 52, Ag : 108, Cl : 35.5, H : 1, O : 16]

Ans. (33)

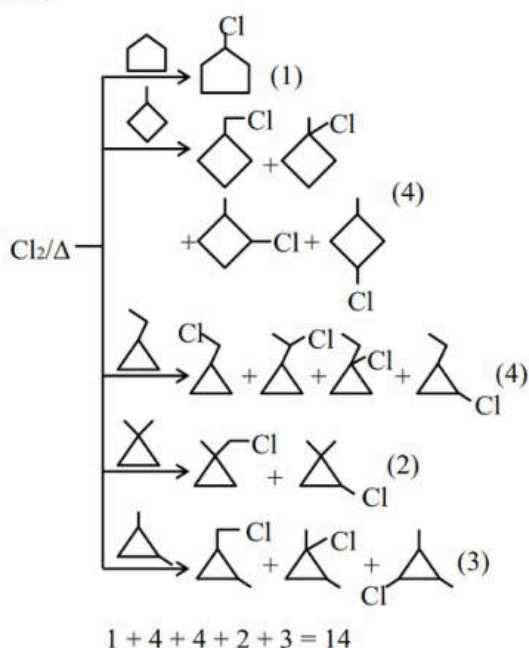


$$\frac{\text{no. of moles of complex reacted}}{\text{no. of moles of AgCl precipitated}} \times 100 = \frac{1}{3} \times 100 = 33.3$$

72. Consider the isomers of hydrocarbon with molecular formula C_5H_{10} . These isomers do not decolourise KMnO_4 solution. These isomers are subjected to chlorination with chlorine in presence of light to give monochloro compounds. The total number of monochloro compounds (structural isomers only) formed is ____.

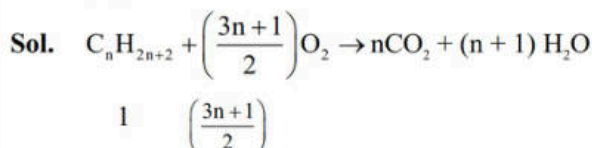
Ans. (14)

Sol.



73. One mole of an alkane (x) requires 8 mole oxygen for complete combustion. Sum of number of carbon and hydrogen atoms in the alkane (x) is ____.

Ans. (17)



$$\frac{3n+1}{2} = 8 \Rightarrow 3n+1 = 16 \Rightarrow 3n = 15 \Rightarrow n = 5$$

So alkane is $\text{C}_5\text{H}_{12} \Rightarrow 17$

74. For reaction $\text{A} \rightarrow \text{P}$, rate constant $k = 1.5 \times 10^3 \text{ s}^{-1}$ at 27°C . If activation energy for the above reaction is 60 kJ mol^{-1} , then the temperature (in $^\circ\text{C}$) at which rate constant, $k = 4.5 \times 10^3 \text{ s}^{-1}$ is _____. (Nearest integer)
Given : $\log 2 = 0.30$, $\log 3 = 0.48$, $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$, $\ln 10 = 2.3$

Ans. (41)

Sol.
$$\ln\left(\frac{K_2}{K_1}\right) = \frac{E_a}{R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$$

$$\ln\left(\frac{4.5 \times 10^3}{1.5 \times 10^3}\right) = \frac{60 \times 10^3}{8.3} \left[\frac{T_2 - 300}{300 \cdot T_2} \right]$$

$$\ln 3 = \frac{60 \times 10^3}{8.3} \left[\frac{T_2 - 300}{300 \cdot T_2} \right]$$

$$\Rightarrow T_2 = 314.4 \text{ K}$$

$$\Rightarrow t = 41.4^\circ\text{C}$$

75. At the transition temperature T , $\text{A} \rightleftharpoons \text{B}$ and $\Delta G^\circ = 105 - 35 \log T$ where A and B are two states of substance X. The transition temperature in $^\circ\text{C}$ when pressure is 1 atm is _____. (Nearest integer)

Ans. (727)

Sol. $\Delta G^\circ = 0$ (eqm. at standard pressure)
 $105 - 35 \log T = 0$
 $\log T = 3$
 $T = 1000 \text{ K}$
 $T = 727^\circ\text{C}$