JEE (Main)-2026 Session-1

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

24 January 2026 Shift – 1

Time: 3 hrs. **M.M: 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A: Attempt all questions.

(5) Section - B: Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 - 25) contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS

SECTION-A

1. If the function f(x)

$$= \frac{e^x\left(e^{\tan x - x}-1\right)+\log_e(\sec x+\tan x)-x}{\tan x - x}$$ is

Continuous at x = 0, then the value of f(0) is equal to

(1) 2 (2) $\frac{2}{3}$

(3) $\frac{1}{2}$ (4) $\frac{3}{2}$

Ans. (4)

Sol. $f(0) = \lim\limits_{x\to 0}\frac{e^{\tan x}-e^x+\ell n(\sec x+\tan x)-x}{\tan x - x}$

Applying L'hospital rule

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{e^{\tan x}.\sec^2 x - e^x + \sec x - 1}{\sec^2 x - 1}$$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{e^{\tan x}(\sec^2 x-1)+(e^{\tan x}-e^x)+\sec x-1}{\tan^2 x}$$

$$\Rightarrow f(0) = \lim_{x\to 0}\left(e^{\tan x}+\frac{e^x\left(e^{\tan x - x}-1\right)}{\tan^2 x}+\frac{1}{\sec x+1}\right)$$

$$\Rightarrow f(0) = 1 + 0 + \frac{1}{2} = \frac{3}{2}$$

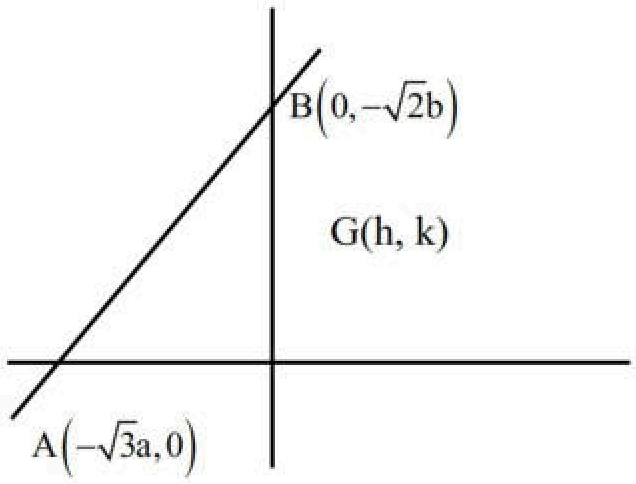
2. Let a circle of radius 4 pass through the origin O, the points A $\left(-\sqrt{3}a, 0\right)$ and B$\left(0, -\sqrt{2}\,b\right)$, where a and b are real parameters and ab ≠ 0. Then the locus of the centroid of ΔOAB is a circle of radius

(1) $\frac{5}{3}$ (2) $\frac{7}{3}$

(3) $\frac{8}{3}$ (4) $\frac{11}{3}$

Ans. (3)

Sol.



AB = 8

$3a^2 + 2b^2 = 64$

Centroid G(h, k)

$h = -\frac{\sqrt{3}a}{3}$, $k = -\frac{\sqrt{2}b}{3}$

$a = -\sqrt{3}h$, $b = \frac{-3}{\sqrt{2}}k$

$9h^2 + 9k^2 = 64$

$x^2 + y^2 = \frac{64}{9}$

$r = \frac{8}{3}$

3. Let the lines L_1: $\vec{r} = \hat{i}+2\hat{j}+3\hat{k}+\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$, $\lambda\in\mathbb{R}$ and L_2 : $\vec{r} = \left(4\hat{i}+\hat{j}\right)+\mu\left(5\hat{i}+2\hat{j}+\hat{k}\right)$, $\mu\in\mathbb{R}$, intersect at the point R. Let P and Q be the points lying on lines L_1 and L_2, respectively, such that $\left|\overrightarrow{PR}\right| = \sqrt{29}$ and $\left|\overrightarrow{PQ}\right| = \sqrt{\frac{47}{3}}$. If the point P lies in the first octant, then $27(QR)^2$ is equal to

(1) 340 (2) 360

(3) 320 (4) 348

Ans. (2)

Sol. For POI

$2\lambda + 1 = 5\mu + 4$; $3\lambda + 2 = 2\mu + 1$; $4\lambda + 3 = \mu$

$\Rightarrow \lambda = \mu = -1$

$R(-1, -1, -1) \quad P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

$PR^2 = 29 \Rightarrow (2\lambda + 2)^2 + (3\lambda + 3)^2 + (4\lambda + 4)^2 = 29$

$\Rightarrow \lambda = 0$ or $\lambda = -2$ (Reject)

$\Rightarrow P\ (1, 2, 3)$

$Q\ (5\mu + 4, 2\mu + 1, \mu)$

$$|PQ| = \sqrt{\frac{47}{3}} \Rightarrow PQ^2 = \frac{47}{3}$$

$$\Rightarrow (5\mu + 3)^2 + (2\mu - 1)^2 + (\mu - 3)^2 = \frac{47}{3}$$

$$\Rightarrow \mu = -\frac{1}{3}$$

$$Q = \left(\frac{7}{3}, \frac{1}{3}, -\frac{1}{3}\right)$$

$$(QR)^2 = \left(\frac{7}{3}+1\right)^2 + \left(\frac{1}{3}+1\right)^2 + \left(-\frac{1}{3}+1\right)^2$$

$$= \frac{100+16+4}{9} = \frac{120}{9}$$

$$\Rightarrow 27 \times (QR)^2 = 27 \times \frac{120}{9} = 360$$

4. Let 729, 81, 9, 1, …. be a sequence and P_n denote the product of the first n terms of this sequence.

If $2\sum_{n=1}^{40}\left(P_n\right)^{\frac{1}{n}} = \frac{3^{\alpha}-1}{3^{\beta}}$ and gcd $(\alpha, \beta) = 1$, then $\alpha + \beta$ is equal to

(1) 73 (2) 74

(3) 75 (4) 76

Ans. (1)

Sol. $P_n = 729.81.9$ …….(n terms)

$= 3^6.3^4.3^2 \ldots\ldots\ldots 3^{-2n+8}$

$P_n = 3^{6+4+2+\ldots\ldots+(-2n+8)} = 3^{n(7-n)}$

$P_n^{1/n} = 3^{7-n}$

$$\Rightarrow \sum_{n=1}^{40}(P_n)^{\frac{1}{n}} = 3^6 + 3^5 + \ldots\ldots + (40\ \text{terms})$$

$$= 3^6\left[\frac{1-\left(\frac{1}{3}\right)^{40}}{1-\frac{1}{3}}\right]$$

$$= \frac{3^6\left[3^{40}-1\right]\times 3^1}{3^{40}\times 2}$$

$$\sum(P_n)^{\frac{1}{n}} = \frac{(3^{40}-1)}{2\times 3^{33}}, \qquad \alpha = 40$$

$\beta = 33$

$\alpha + \beta = 73$

5. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \vec{a} \times \vec{b}$. Let $\vec{d}$ be a vector such that $\left|\vec{d} - \vec{a}\right| = \sqrt{11}, \left|\vec{c} \times \vec{d}\right| = 3$ and the angle between $\vec{c}$ and $\vec{d}$ is $\frac{\pi}{4}$. Then $\vec{a} \cdot \vec{d}$ is equal to

(1) 11

(2) 3

(3) 0

(4) 1

Ans. (3)

Sol. $$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$$

$\vec{c} = 2\hat{i} + 2\hat{k} + \hat{k}$, $|c| = 3$

$\left|\vec{c} \times \vec{d}\right| = 3$

$$\left|\vec{c}\right|\left|\vec{d}\right|\sin\frac{\pi}{4} = 3 \Rightarrow \left|\vec{d}\right| = \sqrt{2}$$

$$\left|\vec{d} - \vec{a}\right| = \sqrt{11}$$

$$\Rightarrow \left|\vec{a}\right|^2 + \left|\vec{d}\right|^2 - 2\vec{a}.\vec{d} = 11$$

$$9 + 2 - 2\vec{a}.\vec{d} = 11$$

$$\vec{a}.\vec{d} = 0$$

6. If the domain of the function

$$f(x) = \log_{(10x^2-17x+7)}(18x^2 - 11x + 1)$$

is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$, then

$90(a + b + c + d + e)$ equals:

(1) 170

(2) 177

(3) 307

(4) 316

Ans. **(4)**

Sol. $18x^2 - 11x + 1 > 0$

$(2x - 1)(9x - 1) > 0$

$x < \frac{1}{9}$ or $\frac{1}{2} < x$

Also $10x^2 - 17x + 7 > 0$

$(x - 1)(10x - 7) > 0$

$x < \frac{7}{10}$ or $1 < x$

& $10x^2 - 17x + 7 \neq 1$

$$x \in \left(-\infty, \frac{1}{9}\right) \cup \left(\frac{1}{2}, \frac{7}{10}\right) \cup (1, \infty) - \left\{\frac{6}{5}\right\}$$

$$90(a + b + c + d + e) = 90\left(\frac{1}{9} + \frac{1}{2} + \frac{7}{10} + 1 + \frac{6}{5}\right)$$

$= 10 + 45 + 63 + 90 + 108 = 316$

7. Let each of the two ellipses $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, (A < B) have eccentricity $\frac{4}{5}$. Let the lengths of the latus recta of E_1 and E_2 be ℓ_1 and ℓ_2, respectively, such that $2\ell_1^2 = 9\ell_2$. If the distance between the foci of E_1 is 8, then the distance between the foci of E_2 is

(1) $\frac{96}{5}$ (2) $\frac{32}{5}$

(3) $\frac{16}{5}$ (4) $\frac{8}{5}$

Ans. **(2)**

Sol. $2ae = 8 \Rightarrow a = 5$

$b^2 = a^2\ (1 - e^2)$

$b^2 = a^2 \times \frac{9}{25}\quad b^2 = 9$

$E_1 : \frac{x^2}{25} + \frac{y^2}{9} = 1$

$\ell_1 : \frac{2b^2}{a} = \frac{2 \times 9}{5} = \frac{18}{5}$

$A^2 = B^2\left(1 - e^2\right) \Rightarrow A^2 = \frac{9}{25}B^2 \Rightarrow A = \frac{3}{5}B$

$2\ell^2 = 9\ell_2 \Rightarrow 2\left(\frac{18}{5}\right)^2 = 9\ell_2 \Rightarrow \ell_2 = \frac{4 \times 18}{25}$

$\frac{2A^2}{B} = \frac{72}{25} \Rightarrow A^2 \frac{36}{25}B$

$\frac{9}{25}B^2 = \frac{36B}{25} \Rightarrow B = 4,$

Distance between focii $2Be = 2 \times \frac{4}{5} \times 4 = \frac{32}{5}$

8. The value of $\dfrac{\sqrt{3}\operatorname{cosec}20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}$ is equal to

(1) 32 (2) 16

(3) 64 (4) 12

Ans. **(3)**

Sol. $E = \dfrac{\dfrac{\sqrt{3}}{\sin 20^\circ} - \dfrac{1}{\cos 20^\circ}}{\dfrac{1}{2}.\dfrac{1}{4}.\cos 60^\circ}$

$= \dfrac{\left(\sqrt{3}\cos 20^\circ - \sin 20^\circ\right)}{\cos 20^\circ.\sin 20^\circ}16$

$= \dfrac{\left(\dfrac{\sqrt{3}}{2}\cos 20^\circ - \dfrac{1}{2}\sin 20^\circ\right)32 \times 2}{2\cos 20^\circ.\sin 20^\circ}$

$= \dfrac{\sin 40^\circ}{\sin 40^\circ} \times 64 = 64$

9. Let $S=\left\{z\in\mathbb{C}:\left|\frac{z-6i}{z-2i}\right|=1 \text{ and } \left|\frac{z-8+2i}{z+2i}\right|=\frac{3}{5}\right\}$.

Then $\sum_{z\in s}|z|^2$ is equal to

(1) 398 (2) 413

(3) 423 (4) 385

Ans. (4)

Sol. Solving $\left|\frac{z-6i}{z-2i}\right|=1\Rightarrow y=4$ ..... (1)

(where $z = x + iy$)

Now solving $\left|\frac{z-8+2i}{z+2i}\right|=\frac{3}{5}$

$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0$ .... (2)

Solving (1) & (2) $\Rightarrow z = 17 + 4i$ & $8 + 4i$

$\Rightarrow \Sigma |z|^2 = (17)^2 + (4)^2 + (8)^2 + (4)^2 = 385$

10. If $\cot x=\frac{5}{12}$ for some $x\in\left(\pi,\frac{3\pi}{2}\right)$, then

$\sin 7x\left(\cos\frac{13x}{2}+\sin\frac{13x}{2}\right)+$

$\cos 7x\left(\cos\frac{13x}{2}-\sin\frac{13x}{2}\right)$ is equal to

(1) $\frac{4}{\sqrt{26}}$ (2) $\frac{6}{\sqrt{26}}$

(3) $\frac{1}{\sqrt{13}}$ (4) $\frac{5}{\sqrt{13}}$

Ans. (3)

Sol. $\cot x=\frac{5}{12}\Rightarrow\cos x=\frac{-5}{13}=2\cos^2\frac{x}{2}-1$

$\cos\left(\frac{x}{2}\right)=-\frac{2}{\sqrt{13}}$ or $\frac{2}{\sqrt{13}}$ (rejected)

$\left\{\because\frac{x}{2}\in\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\right\}$

$\left(\sin 7x\frac{\sin 13x}{2}+\cos 7x\frac{\cos 13x}{2}\right)+\left(\sin 7x\frac{\cos 13x}{2}-\cos 7x\frac{\sin 13x}{2}\right)$

$\cos\left(7x-\frac{13x}{2}\right)+\sin\left(7x-\frac{13x}{2}\right)$

$\cos\frac{x}{2}+\sin\left(\frac{x}{2}\right)$

$\frac{3}{\sqrt{13}}-\frac{2}{\sqrt{13}}=\frac{1}{\sqrt{13}}$

11. Let $f(t)=\int\left(\frac{1-\sin(\log_e t)}{1-\cos(\log_e t)}\right)dt, t>1$.

If $f(e^{\pi/2}) = -e^{\pi/2}$ and $f(e^{\pi/4}) = \alpha\, e^{\pi/4}$, then α equals

(1) $-1-\sqrt{2}$

(2) $-1-2\sqrt{2}$

(3) $1+\sqrt{2}$

(4) $-1+\sqrt{2}$

Ans. (1)

Sol. $f(t)=\int\frac{1-\sin(\ln t)}{1-\cos(\ln t)}dt$

Let $\ln t = x \Rightarrow t = e^x \Rightarrow dt = e^x\, dx$

$=\frac{1}{2}\int\left(\operatorname{cosec}^2\frac{x}{2}-2\cot\frac{x}{2}\right)e^x dx - t\cot\left(\frac{\ln t}{2}\right)+C$

$\left(\because\int(f(x)+f'(x))e^x dx=f(x).e^x+c\right)$

Now $f\left(e^{\pi/2}\right)=-e^{\pi/2}\cot\left(\frac{\pi}{4}\right)+C=-e^{\frac{\pi}{2}}$ (given)

$\Rightarrow C = 0$

Now $f\left(e^{\pi/4}\right)=-e^{\pi/4}\cot\left(\frac{\pi}{8}\right)+C=-e^{\frac{\pi}{4}}(\sqrt{2}+1)$

12. Let R be a relation defined on the set $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ by

$R = \{((a, b), (c, d)) : 2a + 3b = 3c + 4d\}$.

Then the number of elements in R is

(1) 6 (2) 18

(3) 12 (4) 15

Ans. (3)

Sol.

(a, b)	(c, d)
(1, 1)	x
(1, 2)	x
(1, 3)	(1, 2)
(1, 4)	(2, 2)
(2, 1)	(1, 1)
(2, 2)	(2, 1)
(2, 3)	(3, 1)
(2, 4)	(4, 1)
(3, 1)	x
(3, 2)	x
(3, 3)	(1, 3)
(3, 4)	(2, 3)
(4, 1)	(1, 2)
(4, 2)	(2, 2)
(4, 3)	(3, 2)
(4, 4)	(4, 2)

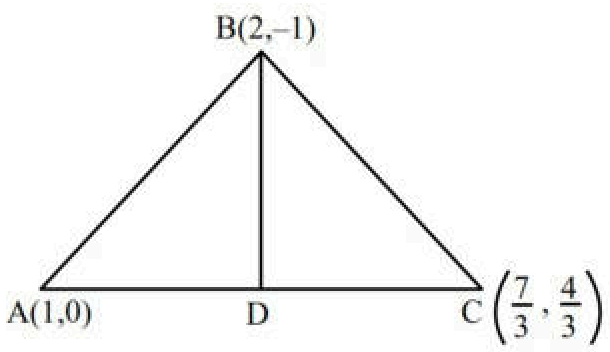
13. Let A(1, 0), B(2, –1) and $C\left(\frac{7}{3},\frac{4}{3}\right)$ be three points. If the equation of the bisector of the angle ABC is $\alpha x + \beta y = 5$, then the value of $\alpha^2 + \beta^2$ is

(1) 8 (2) 5

(3) 13 (4) 10

Ans. (4)

Sol.



$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{\sqrt{2}\times 3}{5\sqrt{2}} = \frac{3}{5}$$

$$D = \left(\frac{12}{8},\frac{4}{8}\right) = \left(\frac{3}{2},\frac{1}{2}\right)$$

$$\text{Slope of AD} = \frac{-3/2}{\frac{1}{2}} = -3$$

$3x + y = 5$

$\alpha = 3, \beta = 1;\ \alpha^2 + \beta^2 = 10$

14. Let $S = \frac{1}{25!} + \frac{1}{3!23!} + \frac{1}{5!21!} + \ldots$ up to 13 terms. If $13S = \frac{2^k}{n!}$, $k \in N$, then $n + k$ is equal to

(1) 51

(2) 52

(3) 49

(4) 50

Ans. (3)

Sol. $\frac{1}{26!}\left(\frac{26!}{25!1!} + \frac{26!}{3!23!} + \frac{26!}{5!21!} + \ldots.. + 13 \text{ terms}\right)$

$\frac{1}{26!}\left({}^{26}C_1 + {}^{26}C_3 + {}^{26}C_5 + \ldots. + 13\,\text{terms}\right)$

$\frac{1}{26!}\left({}^{26}C_1 + {}^{26}C_5 + \ldots. + {}^{26}C_{25}\right)$

$\Rightarrow S = \frac{1}{26!}\times 2^{25}$

$\Rightarrow 13S = \frac{2^{24}}{25!}$

so $n + k = 25 + 24 = 49$

15. Consider an A.P.: $a_1, a_2, \ldots, a_n$; $a_1 > 0$. If $a_2 - a_1 = \frac{-3}{4}$, $a_n = \frac{1}{4} a_1$, and $\sum_{i=1}^{n} a_i = \frac{525}{2}$, then $\sum_{i=1}^{17} a_i$ is equal to

(1) 476 (2) 952

(3) 238 (4) 136

Ans. (3)

Sol. $S_n = \frac{n}{2}\left[a_1 + a_n\right] = \frac{525}{2}$, $d = \frac{-3}{4}$

$\frac{n}{2}\left[a_1 + \frac{a_1}{4}\right] = \frac{525}{2}$

$\frac{5a_1 n}{4} = 525$

$a_1 n = 420$

$a_n = a_1 + (n-1)\left(\frac{-3}{4}\right)$

$\Rightarrow \frac{-3}{4}a_1 = \left(\frac{-3}{4}\right)(n-1) \Rightarrow a_1 = n-1$

$n(n-1) = 420$

$n^2 - n - 420 = 0$

$(n-21)(n+20) = 0$

$n = 21,\ a_1 = 20$

$\sum_{i=1}^{17} a_i = \frac{17}{2}\left[2a_1 + 16d\right]$

$= \frac{17}{2}\left[40 + 16\left(\frac{-3}{4}\right)\right]$

$= \frac{17}{2}[40 - 12]$

$= 17 \times 14 = 238$

16. Let $\alpha, \beta \in \mathbb{R}$ be such that the function

$$f(x) = \begin{cases} 2\alpha(x^2 - 2) + 2\beta x & ,x < 1 \\ (\alpha + 3)x + (\alpha - \beta) & ,x \geq 1 \end{cases}$$

be differentiable at all $x \in \mathbb{R}$. Then $34(\alpha + \beta)$ is equal to

(1) 84 (2) 48

(3) 36 (4) 24

Ans. (2)

Sol. $f(x) = \begin{cases} 2\alpha x^2 + 2\beta x - 4\alpha & ;\ x < 1 \\ (\alpha + 3)x + \alpha - \beta & ;\ x \geq 1 \end{cases}$

$f(1^+) = 2\alpha - \beta + 3$, $f(1^-) = -2\alpha + 2\beta$

$2\alpha - \beta + 3 = 2\beta - 2\alpha \Rightarrow 4\alpha - 3\beta + 3 = 0$ …(1)

$f'(1^+) = 4\alpha + 2\beta$, $f'(1^-) = \alpha + 3$

$4\alpha + 2\beta = \alpha + 3 \Rightarrow 3\alpha + 2\beta - 3 = 0$ …(2)

Solving (1) & (2)

We get $\alpha = \frac{3}{17}$, $\beta = \frac{21}{17}$

$\Rightarrow 34(\alpha + \beta) = 34 \times \frac{27}{17} = 48$

17. From a lot containing 10 defective and 90 non-defective bulbs, 8 bulbs are selected one by one with replacement. Then the probability of getting at least 7 defective bulbs is :-

(1) $\frac{7}{10^7}$ (2) $\frac{81}{10^8}$

(3) $\frac{67}{10^8}$ (4) $\frac{73}{10^8}$

Ans. (4)

Sol. 10 defective & 90 non-defective

Req. probability = (7 def 1 fair) or (8 defective)

Req. probability = $\frac{(10^7 \times 90) \times 8 + 10^8}{100^8}$

$= \frac{72 \times 10^8 + 10^8}{100^8} = \frac{73}{10^8}$

18. The mean and variance of a data of 10 observations are 10 and 2, respectively. If an observations α in this data is replaced by β, then the mean and variance become 10.1 and 1.99, respectively. Then $\alpha + \beta$ equals.

(1) 10 (2) 15

(3) 5 (4) 20

Ans. (4)

Sol. Let first 10 numbers are $x_1, x_2, \ldots\ldots x_9, \alpha$

$$\Rightarrow \alpha + \sum_{i=1}^{9} x_i = 100 \Rightarrow \sum_{i=1}^{9} x_i = 100 - \alpha$$

$$\text{Variance} = \left(\frac{\sum x_i^2}{n}\right) - \left(\frac{\sum x_i}{n}\right)^2$$

$$\Rightarrow \frac{\sum x_i^2}{n} = 98$$

$$\Rightarrow x_1^2 + x_2^2 + \ldots\ldots x_9^2 + \alpha^2 = 1020 \Rightarrow \sum x_i^2 = 1020 - \alpha^2$$

In second case, let number are

$x_1, x_2, \ldots\ldots x_9, \beta$

$100 - \alpha + \beta = 101$ $\alpha - \beta + 1 = 0$

$$\frac{\sum x_i^2 + \beta^2}{10} - (10.1)^2 = 1.99$$

$\beta^2 - \alpha^2 = 20$

$\alpha = \frac{19}{2}$

$\beta = \frac{21}{2}$

$$\alpha + \beta = \frac{19+21}{2} = 20$$

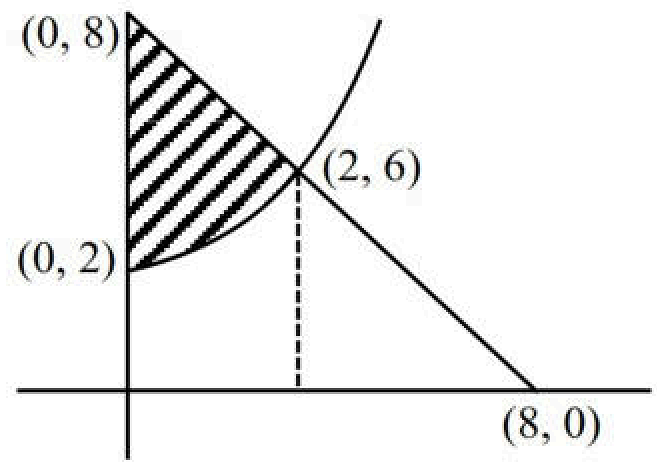
19. Let A_1 be the bounded area enclosed by the curves $y = x^2 + 2$, $x + y = 8$ and y-axis that lies in the first quadrant. Let A_2 be the bounded area enclosed by the curves $y = x^2 + 2$, $y^2 = x$, $x = 2$, and y-axis that lies in the first quadrant. Then $A_1 - A_2$ is equal to

(1) $\frac{2}{3}\left(2\sqrt{2}+1\right)$ (2) $\frac{2}{3}\left(4\sqrt{2}+1\right)$

(3) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (4) $\frac{2}{3}\left(3\sqrt{2}+1\right)$

Ans. **(1)**

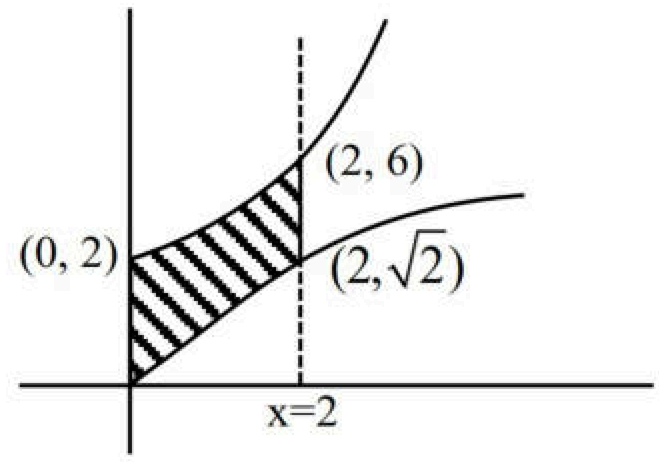
Sol.



$$A_1 = \int_0^2 ((8-x)-(x^2+2))\,dx$$

$$= A_1 = \int_0^2 (6 - x - x^2)\,dx$$

$$A_1\left(6x - \frac{x^2}{2} - \frac{x^3}{3}\right)_0^2 = 12 - 2 - \frac{8}{3} = 10 - \frac{8}{3} = \frac{22}{3}$$



$$A_2 = \int_0^2 (x^2+2)dx - \frac{2}{3}(2\sqrt{2})$$

$$A_2 = \left(\frac{x^3}{3} + 2x\right)_0^2 - \frac{4\sqrt{2}}{3}$$

$$A_2 = \frac{8}{3} + 4 - \frac{4\sqrt{2}}{3} = \frac{20}{3} - \frac{4\sqrt{2}}{3}$$

$$A_1 - A_2 = \frac{2}{3} + \frac{4\sqrt{2}}{3}$$

20. The number of the real solutions of the equation :

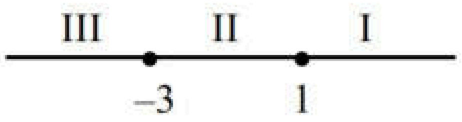
$x|x+3| + |x-1| - 2 = 0$ is

(1) 3 (2) 2

(3) 5 (4) 4

Ans. **(1)**

Sol.



(I)

$x^2 + 3x + x - 1 - 2 = 0$

$x^2 + 4x - 3 = 0$

$x = -2 + \sqrt{7}$ (rejected) , $-2 - \sqrt{7}$ (rejected)

(II)

$x^2 + 3x + 1 - x - 2 = 0$

$x^2 + 2x - 1 = 0$

$x = -1 + \sqrt{2}, -1 - \sqrt{2}$

(III)

$-x^2 - 3x + 1 - x - 2 = 0$

$x^2 + 4x + 1 = 0$

$x = -2 - \sqrt{3}, -2 + \sqrt{3}$ (rejected)

SECTION-B

21. Let a differentiable function f satisfy the equation

$$\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x).$$

If $y = f(x)$ is a standard parabola passing through the points (2, 1) and (–4, β),

Then β^{α} is equal to _____.

Ans. **(64)**

Sol. $\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x)$, Put $\frac{tx}{36} = y$

$$\frac{dy}{dt} = \frac{x}{36}$$

$$\int_0^x \frac{f(y)36dy}{x} = 4\alpha f(x)$$

$$\int_0^x f(y)dy = \frac{\alpha f(x)x}{9}$$

$$f(x) = \frac{\alpha}{9}\left(f(x) + xf'(x)\right)$$

$$\left(1 - \frac{\alpha}{9}\right)f(x) = \frac{\alpha x}{9}f'(x) \Rightarrow (9-\alpha)f(x) = \alpha x f'(x)$$

$$\frac{f'(x)}{f(x)} = \left(\frac{9}{\alpha} - 1\right)\frac{1}{x}$$

$$\log_e f(x) = \left(\frac{9}{\alpha} - 1\right)\log_e x + \log_e c$$

$f(x) = cx^{\left(\frac{9}{\alpha}-1\right)}$ for standard parabola

$$\frac{9}{\alpha} - 1 = 2$$

$\alpha = 3$

$f(x) = cx^2$

passing through (2,1)

$1 = 4c \Rightarrow c = 1/4$

$y = \frac{x^2}{4}$ passing through (–4, β)

$\beta = 4$

$\beta^{x} = 4^3 = 64$

22. Let a line L passing through the point P(1, 1, 1) be perpendicular to the lines $\frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0}$. Let the line L intersect the yz-plane at the point Q. Another line parallel to L and passing through the point S(1,0, –1) intersects the yz-plane at the point R. Then the square of the area of the parallelogram PQRS is equal to _____.

Ans. **(6)**

Sol. $d_1 = <4, 1, 1>$ and $d_2 = <1, 1, 0>$

$$d_L = d_1 \times d_2 = \begin{vmatrix} i & j & k \\ 4 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = <-1, 1, 3>$$

Line L passes through $P<1, 1, 1>$ with $d_2 = <-1, 1, 3>$

$r(t) = <1, 1, 1> + t<-1, 1, 3>$

$= <1 - t, 1 + t, 1 + 3t>$

$= <1 - t, 1 + t, 1 + 3t>$

For point Q; $x = 0$

$\Rightarrow t = 1$

$Q = <0, 2, 4>$

Another

Line parallel to L passes through $S <1, 0, -1>$

with $d_L = <-1, 1, 3>$

$r'(u) = <1, 0, -1 > + \mu<-1, 1, 3>$

$= <1 - \mu, \mu, -1 + 3\mu>$

for point R, $x = 0$

$\Rightarrow \mu = 1$

$R<0, 1, 2>$

Area of parallelogram with adjacent vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

$\overrightarrow{PQ} = <-1, 1, 3>$

$\overrightarrow{PS} = <0, -1, -2>$

Area of parallelogram

$$\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{vmatrix} i & j & k \\ -1 & 1 & 3 \\ 0 & -1 & -2 \end{vmatrix} = <1, -2, 1>$$

$$\text{Area} = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$$

23. The number of numbers greater than 5000, less than 9000 and divisible by 3, that can be formed using the digits 0, 1, 2, 5, 9, if the repetition of the digits is allowed, is _______

Ans. (42)

Sol. (1) all different

5, 0, 1, 9 $\Rightarrow \underline{|3} = 6$ ways

(2) 2 alike, 2 different

5, 0, 0, 1 $\Rightarrow$ 3 ways

5, 1, 1, 2 $\Rightarrow$ 3 ways

5, 2, 2, 0 $\Rightarrow$ 3 ways

5, 2, 2, 9 $\Rightarrow$ 3 ways

5, 5, 0, 2 $\Rightarrow$ 6 ways

5, 5, 2, 9 $\Rightarrow$ 6 ways

5, 1, 9, 9 $\Rightarrow$ 3 ways

(3) 3 alike, 1 different

5, 5, 5, 0 $\Rightarrow$ 3 ways

5, 5, 5, 9 $\Rightarrow$ 3 ways

(4) 2 alike, 2 other alike

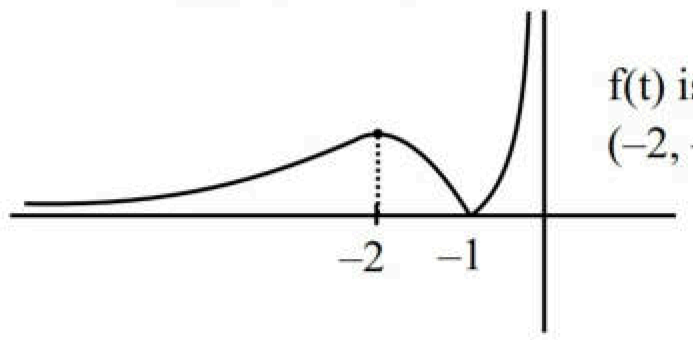
5, 5, 1, 1 $\Rightarrow$ 3 ways

Total ways = 42

24. Let $(2\alpha, \alpha)$ be the largest interval in which the function $f(t) = \dfrac{|t+1|}{t^2}$, $t < 0$, is strictly decreasing. Then the local maximum value of the function $g(x) = 2 \log_e(x - 2) + \alpha x^2 + 4x - \alpha$, $x > 2$, is ____

Ans. (4)

Sol. Drawing graph of f(t) for $t < 0$



f(t) is decreasing in $(-2, -1) \Rightarrow \alpha = -1$

$g(x) = \log_e(x-2) - x^2 + 4x + 1; \quad x > 2$

$$g'(x) = \frac{2}{x-2} - (2(x-2)\ ; \ x > 2$$

$$g'(x) = \frac{1-(x-2)^2}{(x-2)} = \frac{-(x-3)(x-1)}{(x-2)}$$

$\underset{2}{\vdash} \overset{+}{\quad} \underset{3}{+} \overset{-}{\quad}$ as $x > 2$

maxima occur at $x = 3$

$g(3) = 2\log_e 1 - 9 + 12 + 1 = 4$

25. The number of 3 × 2 matrices A, which can be formed using the elements of the set {–2, –1, 0, 1, 2} such that the sum of all the diagonal elements of A^TA is 5, is _____

Ans. (312)

Sol. $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3\times2}$

$$A^TA = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}_{2\times3} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3\times2}$$

$$= \begin{pmatrix} a_1^2 + a_2^2 + a_3^2 & \_\_\_\_\_ \\ \_\_\_\_\_ & b_1^2 + b_2^2 + b_3^2 \end{pmatrix}$$

$\text{Tr}\,(A^TA) = a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 = 5$

{2, 1, 0, 0, 0, 0}

{2, –1, 0, 0, 0, 0}

{–2, 1, 0, 0, 0, 0}

{–2, –1, 0, 0, 0, 0}

{1, 1, 1, 1, 1, 0}

$$\text{No. of ways} = \frac{6!}{4!}\times4 + 2\times\frac{6!}{5!} + 2\times\frac{6!}{4!} + 2\times\frac{6!}{3!2!}$$

$$= \frac{6!}{3!} + 2\times6 + 2\times15\times2\times\frac{6!}{3!}$$

$= 120 + 120 + 12 + 60 = 312$

PHYSICS

SECTION-A

26. Match the List-I with List-II

List-I		List-II	
A.	Magnetic induction	I.	$M L T^{-2} A^{-2}$
B.	Magnetic flux	II.	$M L^2 T^{-2} A^{-2}$
C.	Magnetic permeability	III.	$M L^0 T^{-2} A^{-1}$
D.	Self inductance	IV.	$M L^2 T^{-2} A^{-1}$

Choose the correct answer from the options given below:

(1) A-IV,B-III,C-I,D-II (2) A-III,B-IV,C-II,D-I

(3) A-I,B-III,C-IV,D-II (4) A-III,B-IV,C-I,D-II

Ans. **(4)**

Sol. Magnetic induction

F = qvB

$$[B] = \left[\frac{F}{qV}\right]$$

$[B] = [MT^{-2}A^{-1}]$

Magnetic Flux (ϕ)

ϕ = (B). (Area)

$[\phi] = [ML^2T^{-2}A^{-1}]$

Magnetic Permeability

$[\mu] = [MLT^{-2}A^{-2}]$

Self inductance

Using $U = \frac{1}{2}LI^2$

[Self inductance] = $[ML^2T^{-2}A^{-1}]$

A – III, B – IV, C – I, D – II

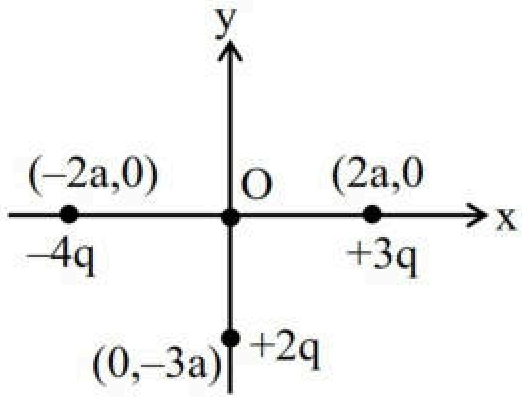
27. Three charges +2q. +3q and –4q are situated at (0,–3a), (2a, 0) and (–2a,0) respectively in the xy plane. The resultant dipole moment about origin is___

(1) $2qa\left(3\hat{j}-\hat{i}\right)$ (2) $2qa\left(3\hat{i}-7\hat{j}\right)$

(3) $2qa\left(7\hat{i}-3\hat{j}\right)$ (4) $2qa\left(3\hat{j}-7\hat{i}\right)$

Ans. **(3)**

Sol.



$$\vec{p} = q_1\vec{r}_1 + q_2\vec{r}_2 + q_3\vec{r}_3$$

$$\vec{p} = (2q)(-3a)\hat{j} + (3q)(2a)\hat{i} + (-4q)(-2a)\hat{i}$$

$$\vec{p} = 2qa\left(7\hat{i}-3\hat{j}\right)$$

28. A cylindrical block of mass M and area of cross section A is floating in a liquid of density ρ and with its axis vertical. When depressed a little and released the block starts oscillating. The period of oscillation is________.

(1) $2\pi\sqrt{\frac{M}{\rho Ag}}$

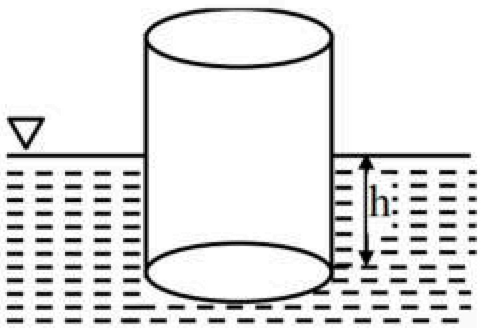
(2) $\pi\sqrt{\frac{2M}{\rho Ag}}$

(3) $\pi\sqrt{\frac{\rho A}{Mg}}$

(4) $2\pi\sqrt{\frac{\rho A}{Mg}}$

Ans. **(1)**

Sol.



At equilibrium

$\rho Ahg = Mg$

After displacing by x,

$Ma = -\rho A\,(h + x)\,g + Mg$

$Ma = -\rho Ahg - \rho Axg + Mg$

$Ma = -\rho Axg$

$a = \left(\dfrac{-\rho Ag}{M}\right)x$

on comparing with,

$a = -\omega^2 x$

$\omega = \sqrt{\dfrac{\rho Ag}{M}}$

$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{M}{\rho Ag}}$

29. Density of water at 4°C and 20°C are 1000 kg/m^3 and respectively. The increase in internal energy of 4 kg water when it is heated from 4 °C to 20°C is____J.

(Specific heat capacity of water = 4.2 J/kg. and 1 atmospheric pressure = 10^5 Pa)

(1) 315826.2

(2) 234699.2

(3) 258700.8

(4) 268799.2

Ans. (4)

Sol. $Q = mS\Delta T = 4 \times 4200 \times 16\text{ J} = 268800\text{ J}$

$W = P\Delta V$

$\Delta V = \left(\dfrac{m}{\rho_f} - \dfrac{m}{\rho_i}\right) = 4\left[\dfrac{1}{998} - \dfrac{1}{1000}\right]$

$P = 10^5$ Pa.

$\therefore\ W = 10^5 \times 4 \times \left[\dfrac{1}{998} - \dfrac{1}{1000}\right] = \dfrac{8\times 10^5}{10^3 \times 998} \approx 0.8\text{ J}$

$\Delta U = Q - W = 268799.2\text{ J}$

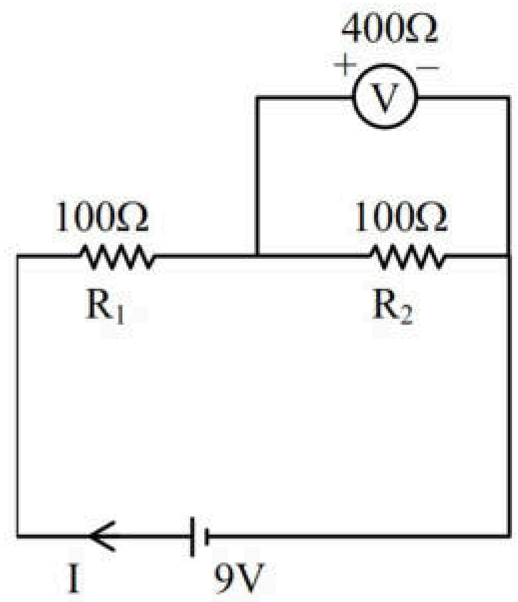
30. Two resistors of 100 Ω each are connected in series with a 9V battery. A voltmeter of 400Ω resistance is connected to measure the voltage drop across one of the resistors. The voltmeter reading is____V.

(1) 3 (2) 4.5

(3) 4 (4) 2

Ans. (3)

Sol.



Current in circuit.

$I = \dfrac{E}{Req}$

$Req = 100 + \dfrac{400\times 100}{400+100} = 180\Omega$

$\therefore\ I = \dfrac{9}{180} = \dfrac{1}{20}\text{A}$

Reading of voltmeter $= V = I \times 80 = \dfrac{1}{20}\times 80 = 4V$

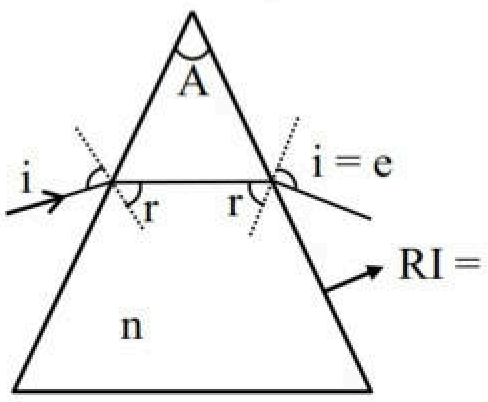
31. The exit surface of a prism with refractive index n is coated with a material having refractive index $\frac{n}{2}$. When this prism is set for minimum angle of deviation it exactly meets the condition of critical angle. The prism angle is____

(1) 60° (2) 15°

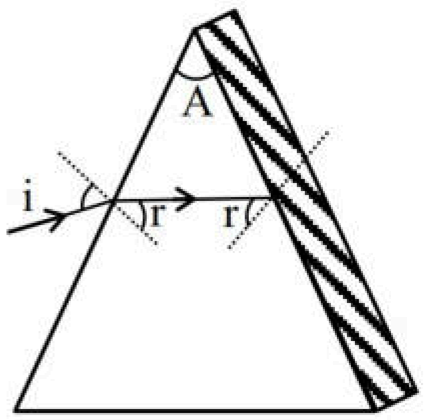
(3) 30° (4) 45°

Ans. **(1)**

Sol. i = e & r = A/2 for minimum deviation



For TIR ; $r = \theta_C$



$\text{Sin } r = \text{Sin } \theta_C$

$\text{Sin } r = \frac{n/2}{n}$

$\text{Sin } r = \frac{1}{2}$

$\text{Sin } \frac{A}{2} = \sin 30°$

$\frac{A}{2} = 30°$

$A = 60°$

32. Two electrons are moving in orbits of two hydrogen like atoms with speeds 3×10^5 m/s and 2.5×10^5 m/s respectively. If the radii of these orbits are nearly same then the possible order of energy states are_____ respectively.

(1) 6 and 5 (2) 9 and 8

(3) 8 and 10 (4) 10 and 12

Ans. **(1)**

Sol. $V \propto \frac{Z}{n}$

$r \propto \frac{n^2}{Z}$

Thus ; $r \propto \frac{n}{V}$

Radii are same then

$\frac{n_1}{V_1} = \frac{n_2}{V_2}$

$\frac{n_1}{n_2} = \frac{3 \times 10^5}{2.5 \times 10^5} = \frac{6}{5}$

Possible order is 6 and 5

33. In a microscope of tube length 10 cm two convex lenses are arranged with focal length of 2 cm and 5 cm. Total magnification obtained with this system for normal adjustment is $(5)^k$. The value of k is_____

(1) 2 (2) 5

(3) 3.5 (4) 4

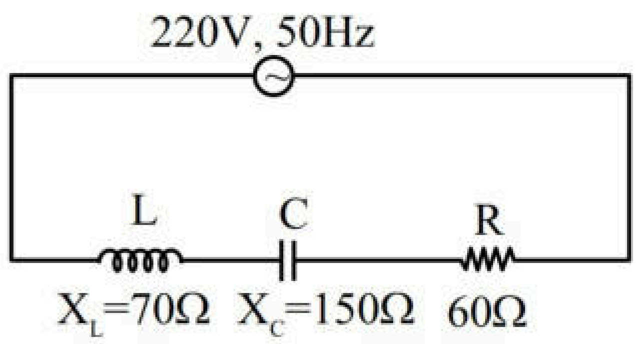
Ans. **(1)**

Sol. $f_0 = 2$ cm, fe = 5 cm

$\ell = 10$ cm, D = 25 cm

$M = \frac{\ell}{f_0} \cdot \frac{D}{f_e} = 25$

34. For the series LCR circuit connected with 220 V, 50 Hz a.c source as shown in the figure, the power factor is $\frac{\alpha}{10}$. The value of α is______



(1) 4 (2) 10

(3) 6 (4) 8

Ans. **(3)**

Sol. Power factor = $\frac{R}{Z}$

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$

$$=\sqrt{60^2+(150-70)^2}=100\Omega$$

$$\therefore \text{ Power factor} = \frac{60}{100}=\frac{6}{10}$$

Then a = 6

35. Match the **List-I** with **List-II**

List-I		List-II	
A	Radio-wave	I	is produced by Magnetron value
B	Micro-wave	II	Due to change in the vibrational modes of atoms
C	Infrared-wave	III	Due to inner shell electrons moving from higher energy level to lower energy level
D	X-ray	IV	Due to rapid acceleration of electrons

Choose the **correct** answer from the options given below:

(1) A-II,B-IV,C-III,D-I (2) A-IV,B-III,C-I,D-II

(3) A-IV,B-I,C-II,D-III (4) A-IV,B-II,C-I,D-III

Ans. **(3)**

Sol. Radio wave $\Rightarrow$ Produced by rapid acceleration of electrons

Micro wave $\Rightarrow$ By magnetron valve

Infrared wave $\Rightarrow$ Change in vibrational modes

X ray $\Rightarrow$ Transition of inner shell electrons from high energy level to low energy level.

A – IV, B – I, C – II, D – III

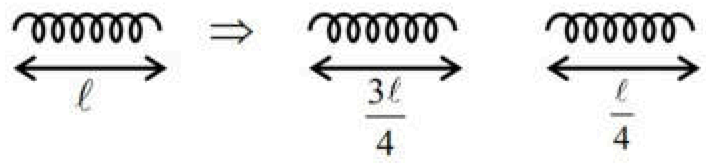
36. A spring of force constant 15 N/m is cut into two pieces. If the ratio of their length is 1:3, then the force constant of smaller piece is____N/m

(1) 15 (2) 20

(3) 60 (4) 45

Ans. **(3)**

Sol.



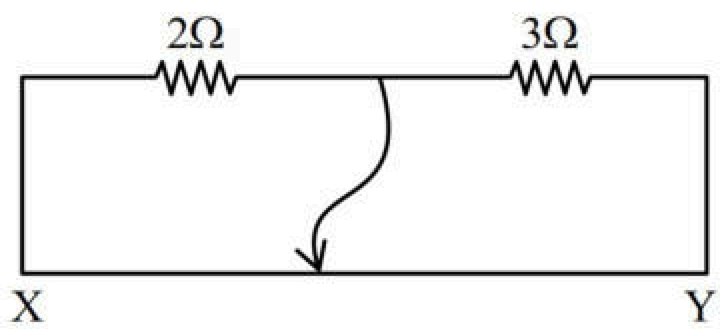
$K\ell$ = constant

$$K\ell = K'\left(\frac{\ell}{4}\right)$$

$K' = 4\ K$

$K' = 60$ N/m

37. Two resistors 2Ω and 3Ω are connected in the gaps of bridge as shown in figure. The null point is obtained with the contact of jockey at some point on wire XY. When an unknown resistor is connected in parallel with 3Ω resistor, the null point is shifted by 22.5 cm toward Y. The resistance of unknown resistor is____Ω.



(1) 3 (2) 2

(3) 4 (4) 1

Ans. **(2)**

Sol. Initially , $\frac{2}{3}=\frac{x}{100-x}$

$\Rightarrow x = 40$ cm

Now when 'R' connected in parallel

$$\frac{2}{\frac{3R}{3+R}}=\frac{40+22.5}{60-22.5}=\frac{62.5}{37.5}$$

$\therefore R = 2\Omega$

38. Given below are two statements:

Statement I : For all elements, greater the mass of the nucleus, greater is the binding energy per nucleon.

Statement II : For all elements, nuclei with less binding energy per nucleon transforms to nuclei with greater binding energy per nucleon.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is false but **Statement II** is true

Ans. **(4)**

Sol. Theoretical

39. A brass wire of length 2m and radius 1mm at 27°C is held taut between two rigid supports. Initially it was cooled to a temperature of –43°C creating a tension T in the wire. The temperature to which the wire has to be cooled in order to increase the tension in it to 1.4T, is ____°C

(1) –86 (2) –71

(3) –65 (4) –80

Ans. **(2)**

Sol. $T = \alpha YA(27 - (-43)$ .....(i)

$1.4\,T = \alpha YA(27 - \theta)$ ....(ii)

using (ii)/(i)

$$1.4 = \frac{27-\theta}{70}$$

$27 - \theta = 98 \quad \therefore \theta = -71°C$

40. The electrostatic potential in a charged spherical region of radius r varies as $V = ar^3 + b$, where a and b are constants. The total charge in the sphere of unit radius is $\alpha \times \pi a \in_0$. The value of α is _____.

(permittivity of vacuum is $\in_0$)

(1) –12 (2) –6

(3) –9 (4) –8

Ans. **(1)**

Sol. $v = ar^3 + b$

$$E = -\frac{dv}{dr} = -3ar^2$$

$$\phi_{closed} = \frac{q_{enc}}{\varepsilon_0}$$

$q_{enc} = \varepsilon_0 . E.A$

$= \varepsilon_0\left(-3a.(1)^2\right)4\pi(1)^2$

$= -12\pi a\varepsilon_0$

$\therefore x = -12$

41. Two masses 400 g and 350 g are suspended from the ends of a light string passing over a heavy pulley of radius 2 cm. When released from rest the heavier mass is observed to fall 81 cm in 9 s. The rotational inertia of the pulley is _______ kg.m^2.

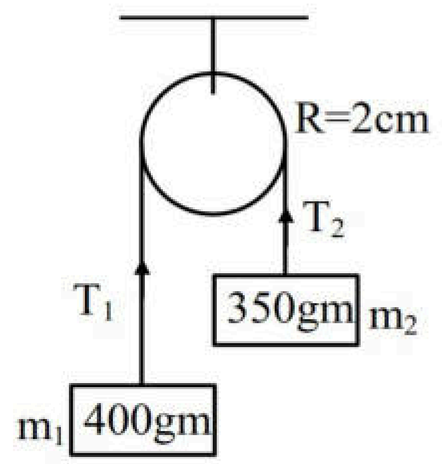
($g = 9.8$ m/s^2)

(1) 9.5×10^{-3} (2) 4.75×10^{-3}

(3) 1.86×10^{-2} (4) 8.3×10^{-3}

Ans. **(1)**

Sol.



$$S = ut + \frac{1}{2}at^2$$

$$a = \frac{2s}{t^2} = \frac{2\times 0.81}{81} = 0.02 \text{ m/s}^2$$

$m_1g - T_1 = m_1a$

$T_2 - m_2g = m_2a$

$$(T_1 - T_2)R = I.\frac{a}{R}$$

$$\therefore a = \frac{(m_1 - m_2)g}{m_1 + m_2 + \frac{I}{R^2}}$$

$$0.02 = \frac{(400-350)(10^{-3})g}{(400+350)(10^{-3})+\frac{I}{R^2}}$$

$$\frac{I}{R^2} = \frac{50\times10^{-3}g}{0.02} - 750\times10^{-3} = 23.75$$

$I = 23.75 \times 4 \times 10^{-4} = 9.5 \times 10^{-3}$ kg–m^2

42. An unpolarised light is incident at an interface of two dielectric media having refractive indices of 2 (incident medium) and $2\sqrt{3}$ (medium) respectively. To satisfy the condition that reflected and refracted rays are perpendicular to each other, the angle of incidence is ______.

(1) 60° (2) 10° (3) 30° (4) 45°

Ans. (1)

Sol. Brewester's law

$\tan\theta = \mu_{rel} = \sqrt{3}$

$\theta = 60°$

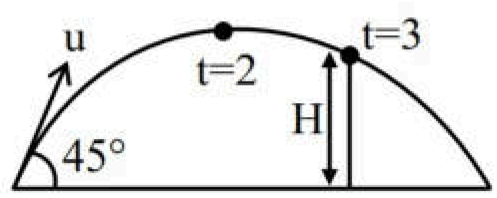
43. A boy thrown a ball into air at 45° from the horizontal to land it on a roof of a building of height H. If the ball attains maximum height in 2 s and lands on the building in 3 s after launch, then value of H is ________ m.

($g = 10$ m/s^2)

(1) 20 (2) 10 (3) 25 (4) 15

Ans. (4)

Sol. $T = \frac{2u_y}{g} = 4$



$\Rightarrow u_y = \frac{40}{2} = 20$ m/s

$y = u_y\Delta t - \frac{1}{2}g(\Delta t)^2$

$\Rightarrow H = 20 \times 3 - 5 \times 9$

$= 60 - 45$

$= 15$ m

44. There are three co-centric conducting spherical shells A, B and C of radii a, b and c respectively. The potential of the spheres A, B and C respectively, are :

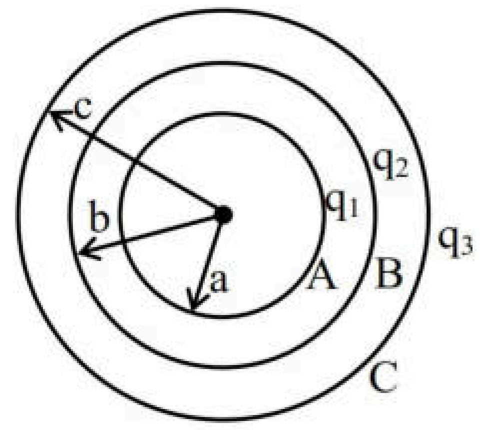
(1) $\frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{a}\right), \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{b}\right), \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right)$

(2) $\frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{a}\right), \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right)$

(3) $\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right)$

(4) $\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{b}\right), \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right)$

Ans. (3)



Sol.

$$V_A = \frac{Kq_1}{a}+\frac{Kq_2}{b}+\frac{Kq_3}{c} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right)$$

$$V_B = \frac{Kq_1}{b}+\frac{Kq_2}{b}+\frac{Kq_3}{c} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right)$$

$$V_C = \frac{Kq_1}{c}+\frac{Kq_2}{c}+\frac{Kq_3}{c} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right)$$

45. Three masses 200 kg, 300 kg and 400 kg are placed at the vertices of an equilateral triangle with sides 20 m. They are rearranged on the vertices of a bigger triangle of side 25 m and with the same centre. The work done in this process _________ J.

(Gravitational constant $G = 6.7 \times 10^{-11}$ N m^2 / kg^2)

(1) 9.86×10^{-6}

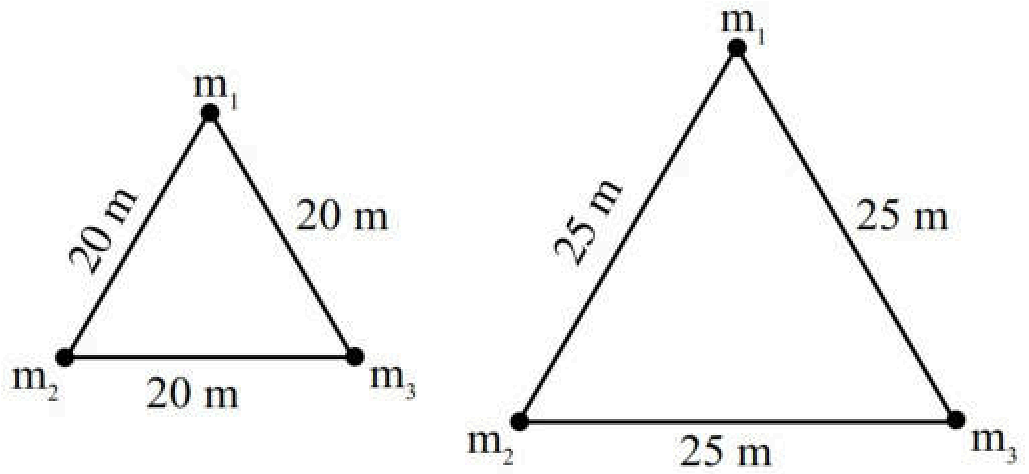
(2) 2.85×10^{-7}

(3) 1.74×10^{-7}

(4) 4.77×10^{-7}

Ans. (3)

Sol



Work done by external agent :

$w_{ext} = \Delta U$

$$U_i = -\frac{Gm_1m_2}{r_i} - \frac{Gm_2m_3}{r_i} - \frac{Gm_1m_3}{r_i} \;:\; r_i = 20\text{ m}$$

$$U_f = -\frac{Gm_1m_2}{r_f} - \frac{Gm_2m_3}{r_f} - \frac{Gm_1m_3}{r_f} \;:\; r_f = 25\text{ m}$$

$$U_i = \frac{-6.67\times10^{-11}}{20}\,[200\times300+300\times400+200\times400]$$

$$= \frac{-6.67\times10^{-11}}{20}\times26\times10^4 = -86.71\times10^{-8}\text{ J}$$

$$U_f = \frac{-6.67\times10^{-11}}{0.25}\,[200\times300+300\times400+200\times400]$$

$$= \frac{-6.67\times10^{-11}}{0.25}\times26\times10^4 = -693.68\times10^{-9}$$

$= -69.36 \times 10^{-8}$ J

$\Delta U = U_f - U_i = 1.74 \times 10^{-7}$ J

SECTION-B

46. A short bar magnet placed with its axis at 30° with an external field of 800 Gauss, experiences a torque of 0.016 N.m. The work done in moving it from most stable to most unstable position is $\alpha \times 10^{-3}$ J. The value of α is _______.

Ans. (64)

Sol. $\tau = \mu B \sin\theta \quad \Rightarrow 0.016 = \mu \times B \times \frac{1}{2}$

$$\Rightarrow \mu = \frac{0.032}{B}$$

$W_{ext} = U_f - U_i = \mu B - (\mu B) = 2\mu B$

$$= 2\times\frac{0.032}{B}\times B$$

$= 0.064$ J

47. A gas of certain mass filled in a closed cylinder at a pressure of 3.23 kPa has temperature 50°C. The gas is now heated to double its temperature. The modified pressure is _______ Pa.

Ans. (3730)

Sol. As per NTA

V = constant

so $P \propto T$

$T_i = 50°C = 323$ K

$T_f = 100°C = 373$ K

$$\Rightarrow \frac{P_f}{P_i} = \frac{T_f}{T_i}$$

$$\Rightarrow \frac{P_f}{3.23\text{kPa}} = \frac{373}{323}$$

$\Rightarrow P_f = 3730$ Pa

Other Solution :

As volume is constant

$\therefore P \propto T$

Since T is doubled (must be in Kelvin) so pressure must be doubled.

$\therefore P_f = 2P_i$

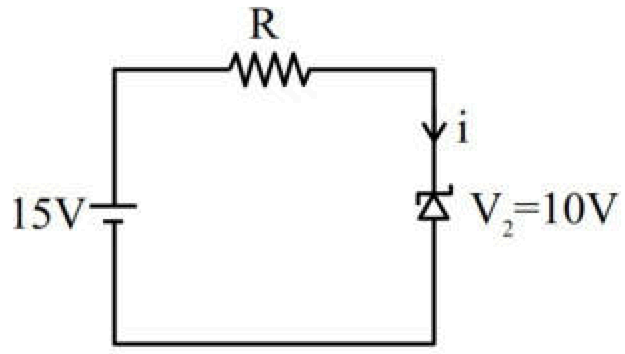
$P_f = 2 \times 3.23 = 6.46$ KPa

$P_f = 6460$ Pa

48. A voltage regulating circuit consisting of Zener diode, having break-down voltage of 10 V and maximum power dissipation of 0.4 W, is operated at 15 V. The approximate value of protective resistance in this circuit is ________ Ω.

Ans. (125)

Sol.

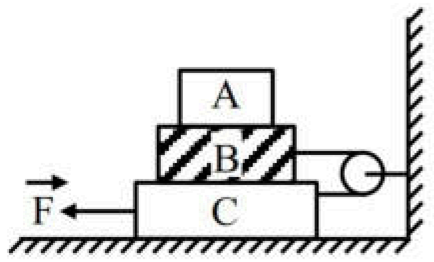


$P_D = 0.4W = 10i$

$i = 0.04A$

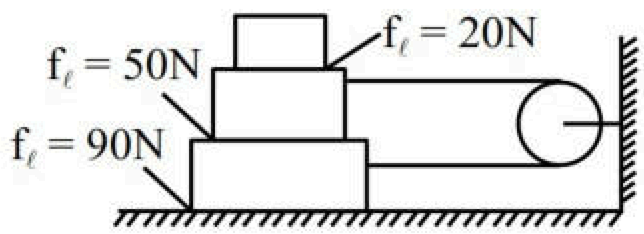
$R = \frac{15-10}{0.04} = \frac{5}{0.04} = 125\ \Omega$

49. In the given figure the blocks A, B and C weigh 4 kg, 6 kg and 8 kg respectively. The co-efficient of sliding friction between any two surfaces is 0.5. The force $\vec{F}$ required to slide the block C with constant speed is ________ N. (Used g = 10 m/s^2)



Ans. **(210)**

Sol. For 8kg to move with constant velocity $F_{net} = 0$.



$\therefore$ F = 90 + T + 50 (for 8kg block)

T = 20 + 50 (for 6kg block)

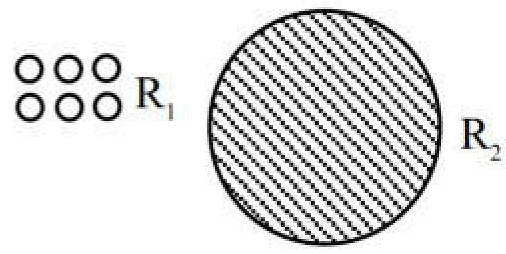
$\therefore$ F = 210 N.

50. Sixty four rain drops of radius 1 mm each falling down with a terminal velocity of 10 cm/s coalesce to form a bigger drop. The terminal velocity of bigger drop is ______ cm/s.

Ans. **(160)**

Sol. $V_T = \frac{2r^2g}{9n}[\sigma - \rho]$

$V_T \propto r^2$



64 drop

$64\left(\frac{4}{3}\pi R_1^3\right) = \frac{4}{3}\pi R_2^3$

$R_2 = 4R_1$

$\frac{(V_T)_1}{(V_T)_2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{1}{4}\right)^2$

$\frac{10}{(V_T)_2} = \frac{1}{16}$

$(V_T)_2 = 160$ cm/sec

CHEMISTRY

SECTION-A

51. Given below are two statements :

Statement-I Hybridisation, shape and spin only magnetic moment of $K_3[Co(CO_3)_3]$ is sp^3d^2, octahedral and 4.9 BM respectively.

Statement-II Geometry, hybridisation and spin only magnetic moment values (BM) of the ions $[Ni(CN)_4]^{2-}$, $[MnBr_4]^{2-}$ and $[CoF_6]^{3-}$ respectively are square planar, tetrahedral, octahedral : dsp^2, sp^3, sp^3d^2 and 0, 5.9, 4.9.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both statement-I and statement-II are false

(2) Statement I is false but statement-II is true

(3) Both statement-I and statement-II are true

(4) Statement-I is true but statement-II is false

Ans. **(3)**

Sol. In $K_3[Co(CO_3)_3] \Rightarrow sp^3d^2$ hybridized, octahedral

$\Rightarrow$ 4 unpaired electron

$\Rightarrow$ 4.9 B.M.

$[Ni(CN)_4]^{2-}$ $\Rightarrow dsp^2$ hybridized , square planar

$\Rightarrow$ 0 unpaired electron

$\Rightarrow$ 0 B.M.

$[MnBr_4]^{2-}$ $\Rightarrow sp^3$ hybridized , tetrahedral

$\Rightarrow$ 5 unpaired electron

$\Rightarrow$ 5.9 B.M.

$[CoF_6]^{3-}$ $\Rightarrow sp^3d^2$ hybridized, octahedral

$\Rightarrow$ 4 unpaired electron

$\Rightarrow$ 4.9 B.M.

52. $A \rightarrow D$ is an endothermic reaction occurring in three steps (elementary).

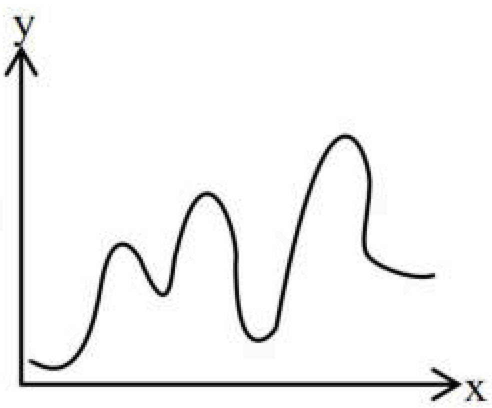
(i) $A \rightarrow B$ $\Delta H_i = +ve$

(ii) $B \rightarrow C$ $\Delta H_{ii} = -ve$

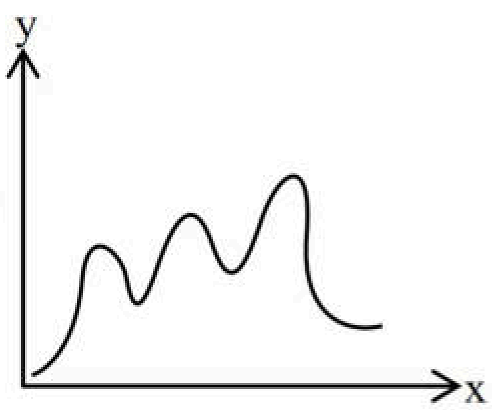
(iii) $C \rightarrow D$ $\Delta H_{iii} = -ve$

Which of the following graphs between potential energy (y-axis) vs reaction coordinate (x-axis) correctly represents the reaction profile of $A \rightarrow D$?

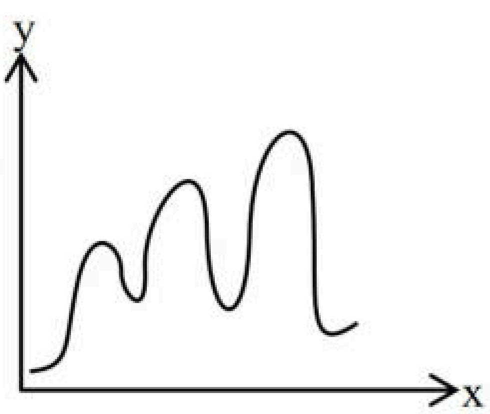
(1)



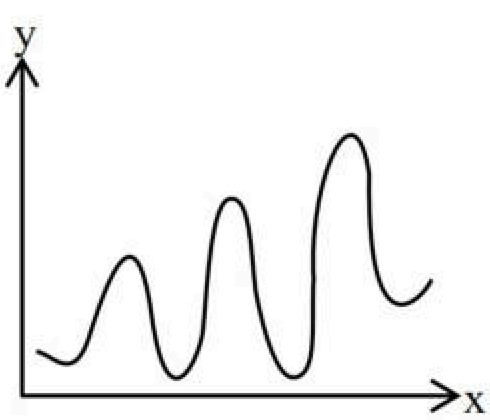
(2)



(3)



(4)



Ans. **(3)**

Sol. Given :

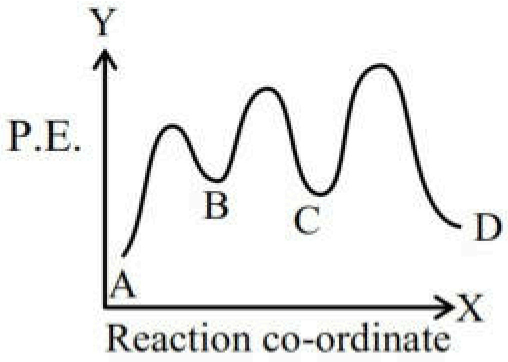
$A \rightarrow D$; $\Delta_r H = (+)ve = E_D - E_A \therefore E_D > E_A$

Mechanism

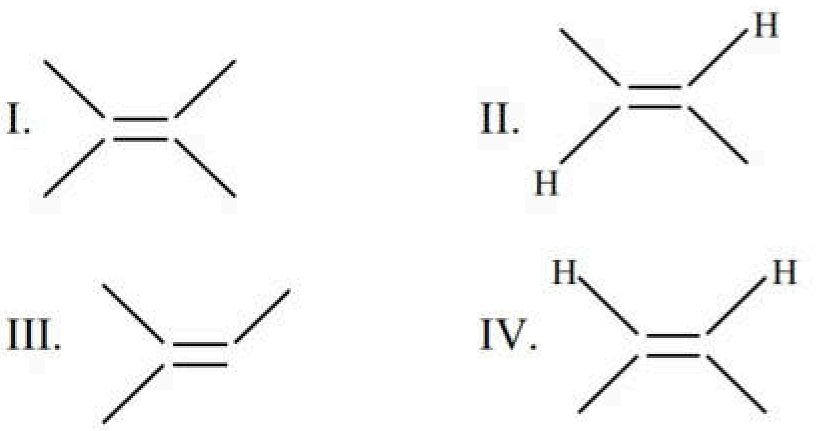
$A \rightarrow B$; $\Delta_r H = (+)ve \Rightarrow E_B > E_A$

$B \rightarrow C$; $\Delta_r H = (-)ve \Rightarrow E_C < E_B$

$C \rightarrow D$; $\Delta_r H = (-)ve \Rightarrow E_D < E_C$



53. Arrange the following alkenes in decreasing order of stability.



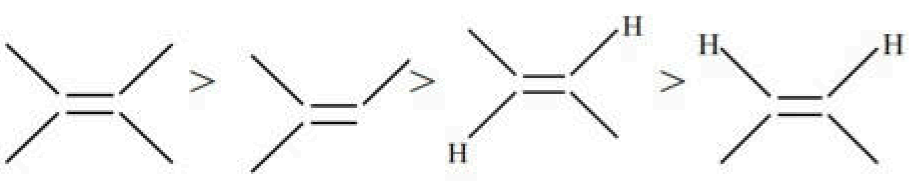
Choose the **correct** answer from the options given below :

(1) III > I > II > IV (2) III > II > I > IV

(3) I > III > II > IV (4) I > III > IV > II

Ans. **(3)**

Sol. Stability order :



$\alpha H = 12$ $\alpha H = 9$ $\alpha H = 6$ $\alpha H = 6$

trans cis

Trans is more stable than cis

54. Given below are statements about some molecules/ions.

Identify the **CORRECT** statements.

A. The dipole moment value of NF_3 is higher than that of NH_3.

B. The dipole moment value of BeH_2 is zero.

C. The bond order of O_2^{2-} and F_2 is same.

D. The formal charge on the central oxygen atom of ozone is –1.

E. In NO_2, all the three atoms satisfy the octet rule, hence it is very stable.

Choose the **correct** answer from the options given below :

(1) A, B, C, D & E (2) B & C only

(3) B, C & D only (4) A, C & D only

Ans. **(2)**

Sol. (A) Dipole moment : $NF_3 < NH_3$

(B) BeH_2 is 'sp' hybridized, linear molecule with zero dipole moment.

(C) $O_2^{-2} \Rightarrow$ bond order = 1

$F_2 \Rightarrow$ bond order = 1

(D) Formal charge on central oxygen atom in O_3 is +1.

(E) In NO_2; nitrogen does not follow octet rule.

55. A solution is prepared by dissolving 0.3 g of a non-volatile non-electrolyte solute 'A' of molar mass 60 g mol^{-1} and 0.9 g of a non-volatile non-electrolyte solute 'B' of molar mass 180 g mol^{-1} in 100 mL H_2O at 27°C. Osmotic pressure of the solution will be

[Given : R = 0.082 L atm K^{-1} mol^{-1}]

(1) 1.23 atm (2) 2.46 atm

(3) 0.82 atm (4) 1.47 atm

Ans. **(2)**

Sol. Mass of solute 'A' = 0.3 g

Moles of solute 'A' = $\dfrac{0.3g}{60\,g/mol} = \dfrac{1}{200}$ mol

Mass of solute 'B' = 0.9 g

Moles of solute 'B' $= \frac{0.9\,gm}{180\,g/mol} = \frac{1}{200}$ mol

Total molarity of all solutes

$= \frac{2/200}{100} \times 1000 = \frac{1}{10}$ M

$\therefore \pi = \frac{1}{10} \times 0.082 \times 300$

$\pi = 2.46$ atm.

56. Among the following, the CORRECT combinations are :

A. $IF_3 \rightarrow$ T-shaped (sp^3d)

B. $IF_5 \rightarrow$ Square pyramidal (sp^3d^2)

C. $IF_7 \rightarrow$ Pentagonal bipyramidal (sp^3d^3)

D. $ClO_4^- \rightarrow$ Square planar (sp^2d)

Choose the **correct** answer from the options given below :

(1) A, B and C only

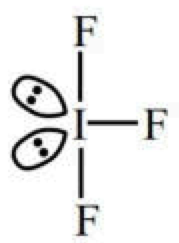
(2) A and B only

(3) A, B, C and D

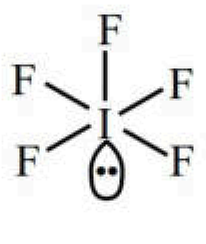
(4) B, C and D Only

Ans. **(1)**

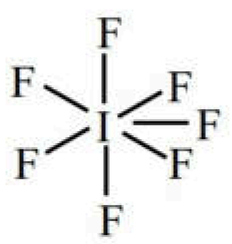
Sol.



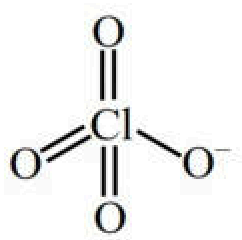
Hybridisation sp^3d
(T-shape)



Hybridisation sp^3d^2
(Squar pyramidal)

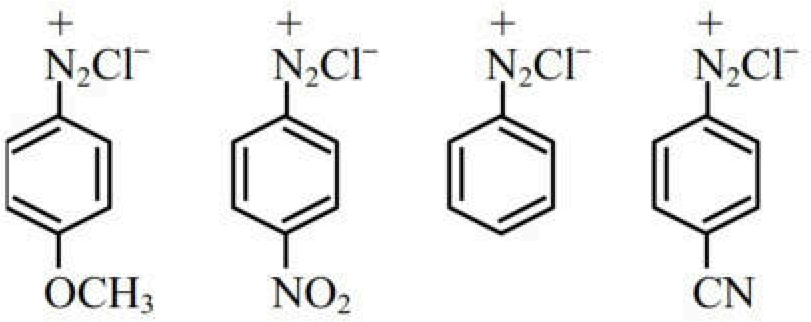


Hybridisation sp^3d^3
(Pentagonal bipyramidal)



Hybridisation sp^3
(Tetrahedral)

57. The correct stability order of the following diazonium salts is



(A) (B) (C) (D)

(1) A > B > C > D (2) C > D > B > A

(3) A > C > D > B (4) C > A > D > B

Ans. **(3)**

Sol. Correct order of stability

A > C > D > B

+M –M –M

EDG increases stability

EWG decreases stability

58. Consider a mixture 'X' which is made by dissolving 0.4 mol of $[Co(NH_3)_5SO_4]$ Br and 0.4 mol of $[Co(NH_3)_5Br]SO_4$ in water to make 4 L of solution. When 2 L of mixture 'X' is allowed to react with excess of $AgNO_3$, it forms precipitate 'Y'. The rest 2 L of mixture 'X' reacts with excess $BaCl_2$ to form precipitate 'Z'. Which of the following statements is **CORRECT**.

(1) 0.2 mol of 'Z' is formed

(2) 'Y' is $BaSO_4$ and 'Z' is AgBr

(3) 0.4 mol of 'Z' is formed

(4) 0.1 mol of 'Y' is formed

Ans. **(1)**

Sol. 0.4 mol $[Co(NH_3)_5SO_4]Br$ + 0.4 mol $[Co(NH_3)_5\,Br]SO_4$ are present in 4 lit. solution.

2 lit. of mixture will contain 0.2 mol of each complex.

2 lit. mixture on reaction with excess $AgNO_3$, 0.2 mole AgBr will be formed (Y).

2 lit. mixture on reaction with excess $BaCl_2$, 0.2 mole $BaSO_4$ will be formed (Z).

59. Given below are two statements

Statements-I : The number of paramagnetic species among $[CoF_6]^{3-}$, $[TiF_6]^{3-}$, V_2O_5 and $[Fe(CN)_6]^{3-}$ is 3.

Statement-II : $K_4[Fe(CN)_6]$ < $K_3[Fe(CN)_6]$ < $[Fe(H_2O)_6]\ SO_4.H_2O$ < $[Fe(H_2O)_6]Cl_3$ is the correct order in terms of number of unpaired electron(s) in the complexes.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both statement-I and statement-II are true

(2) Both statement-I and statement-II are false

(3) Statement-I is true but statement-II is false

(4) Statement-I is false but statement-II is true

Ans. **(1)**

Sol. Paramagnetic species :

$[CoF_6]^{3-}$, $[TiF_6]^{3-}$, $[Fe(CN)_6]^{3-}$

Diamagnetic species : V_2O_5

In $K_4[Fe(CN)_6] \Rightarrow$ No. of unpaired electron = 0

$K_3[Fe(CN)_6] \Rightarrow$ No. of unpaired electron = 1

$[Fe(H_2O)_6]SO_4.H_2O \Rightarrow$ No. of unpaired electron = 4

$[Fe(H_2O)_6]Cl_3 \Rightarrow$ No. of unpaired electron = 5

60. Consider three metal chlorides x, y and z, where x is water soluble at room temperature, y is sparingly soluble in water at room temperature and z is soluble in hot water. x, y and z are respectively

(1) $MgCl_2$, AgCl and $AlCl_3$

(2) AgCl, Hg_2Cl_2 and $PbCl_2$

(3) $AlCl_3$, $PbCl_2$ and $BaCl_2$

(4) $CuCl_2$, AgCl and $PbCl_2$

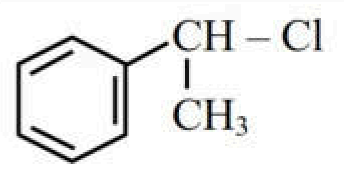
Ans. **(4)**

Sol. $MgCl_2$, $AlCl_3$, $CuCl_2$ are water soluble at room temperature.

AgCl, Hg_2Cl_2 are sparingly soluble in water

$PbCl_2$ is soluble in hot water.

61. Match the **List-I** with **List-II**

List-I (Chloro derivative)		**List-II (Example)**	
A.	Vinyl Chloride	I.	$CH_2 = CH - CH_2Cl$
B.	Benzyl chloride	II.	$CH_3 = CH(Cl)CH_3$
C.	Alkyl chloride	III.	$CH_2 = CHCl$
D.	Allyl chloride	IV.	

Choose the correct answer from the options given below :

(1) A-IV, B-I, C-III, D-II

(2) A-III, B-IV, C-I, D-II

(3) A-III, B-IV, C-II, D-I

(4) A-I, B-II, C-IV, D-III

Ans. **(3)**

Sol. Common Name (Theory based)

62. 'W' g of a non-volatile electrolyte solid solute of molar mass 'M' g mol^{-1} when dissolved in 100 mL water, decreases vapour pressure of water from 640 mm Hg to 600 mm Hg. If aqueous solution of the electrolyte boils at 375 K and K_b for water is 0.52 K kg mol^{-1}, then the mole fraction of the electrolyte solute (x_2) in the solution can be expressed as

(Given : density of water = 1 g/mL and boiling point of water = 373 K)

(1) $\frac{1.3}{8} \times \frac{W}{M}$ (2) $\frac{16}{2.6} \times \frac{W}{M}$

(3) $\frac{2.6}{16} \times \frac{M}{W}$ (4) $\frac{1.3}{8} \times \frac{M}{W}$

Ans. **(1)**

Sol. P° = 640 mm Hg

P_S = 600 mm Hg

ΔP = 40 mm Hg

moles of solute = $\frac{W}{M}$

$\frac{\Delta P}{P^o} = i.X_{solute}$

Again :

$\Delta T_b = i \times k_b \times m$

$2 = i \times 0.52 \times \frac{W/M}{100} \times 1000$

$i = \frac{2}{5.2} \times \frac{M}{W}$

$X_{solute} = \frac{40}{640} \times \frac{1}{i} = \frac{1}{16} \times \frac{5.2}{2} \times \frac{W}{M}$

$X_{solute} = \frac{1.3}{8} \times \frac{W}{M}$

63. Match the List-I with List-II

List-I (Isothermal process for ideal gas system)		**List-II Work done ($V_f > V_i$)**	
A.	Reversible expansion	I.	w = 0
B.	Free expansion	II.	$w = -nRT \ln \frac{V_f}{V_i}$
C.	Irreversible expansion	III.	$w = -p_{ex}(V_f - V_i)$
D.	Irreversible compression	IV.	$w = -p_{ex}(V_i - V_f)$

Choose the **correct** answer from the options given below :

(1) A-IV, B-I, C-III, D-II
(2) A-IV, B-II, C-III, D-I
(3) A-I, B-III, C-II, D-IV
(4) A-II, B-I, C-III, D-IV

Ans. (4)

Sol. (A) $W_{Rev.} = -\int P_{gas}\, dV$

$W_{Rev.\ Isot.\ Exp.} = -nRT \ln \left[\frac{V_f}{V_i}\right]$

$(A) \rightarrow (II)$

(B) Free expansion

$W_{irrev.} = -P_{ext}\,\Delta V$

$P_{ext} = 0$

$W = 0$

$(B) \rightarrow (I)$

(C) Irreversible expansion

$W_{irrev.} = -P_{ext}\,\Delta V$

$W_{irrev.} = -P_{ext}(V_f - V_i)$

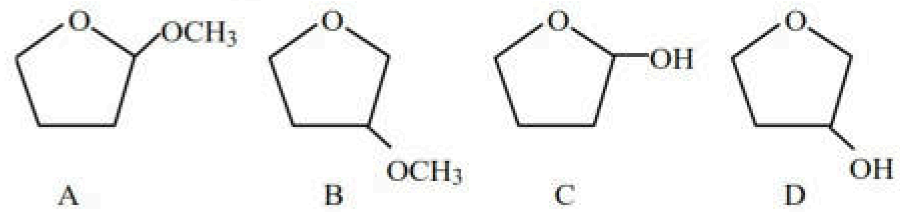
$(C) \rightarrow (III)$

(D) Irreversible compression

$W_{irrev.} = -P_{ext}\,\Delta V$

$W_{irrev.} = -P_{ext}(V_i - V_f)$

64. A student is given one compound among the following compounds that gives positive test with Tollen's reagent.



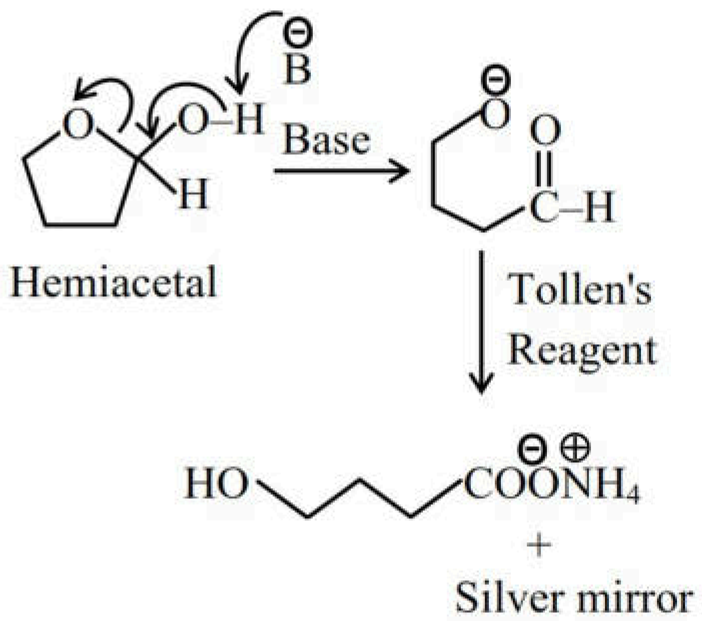
A B C D

The compound is :

(1) D (2) A
(3) B (4) C

Ans. (4)

Sol.

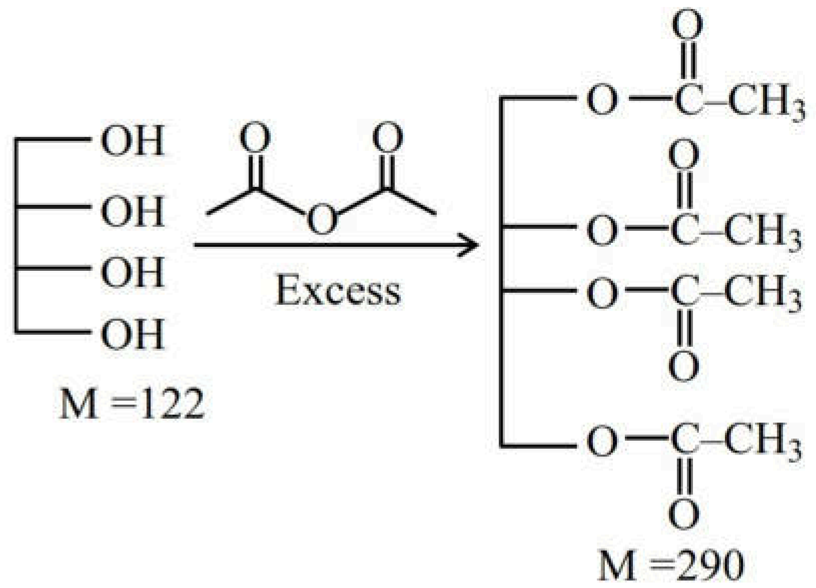


65. A hydroxy compound (X) with molar mass 122 g mol^{-1} is acetylated with acetic anhydride, using a large excess of the reagent ensuring complete acetylation of all hydroxyl groups. The product obtained has a molar mass of 290 g mol^{-1}. The number of hydroxyl groups present in compound (X) is :

(1) 3 (2) 5
(3) 2 (4) 4

Ans. (4)

Sol.



No. of OH groups $= \dfrac{290-122}{42} = 4$

66. At 27°C in presence of a catalyst, activation energy of a reaction is lowered by 10 kJ mol^{-1}. The logarithm ratio of $\dfrac{k(\text{catalysed})}{k(\text{uncatalysed})}$ is ....

(Consider that the frequency factor for both the reactions is same)

(1) 17.41

(2) 1.741

(3) 3.482

(4) 0.1741

Ans. **(2)**

Sol. $\dfrac{K_{catalyst}}{K_{uncatalyst}} = e^{\frac{\Delta E_a}{RT}}$

$\ln \dfrac{K_{catalyst}}{K_{uncatalyst}} = \dfrac{\Delta E_a}{RT}$

$\log \dfrac{K_{catalyst}}{K_{uncatalyst}} = \dfrac{\Delta E_a}{2.303RT}$

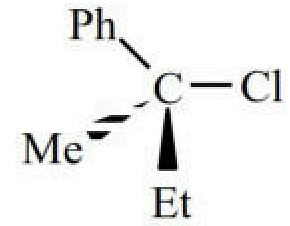
$= \dfrac{10 \times 1000}{2.303 \times 8.314 \times 300}$

$\log \dfrac{K_{catalyst}}{K_{uncatalyst}} = 1.741$

67. Given below are two statements :

Statement I : 'C–Cl' bond is stronger in $CH_2 = CH - Cl$ than $CH_3 - CH_2 - Cl$

Statement II : The given optically active molecule,



on hydrolysis gives a solution that can rotate the plane polarized light.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Statement I is false but Statement II is true

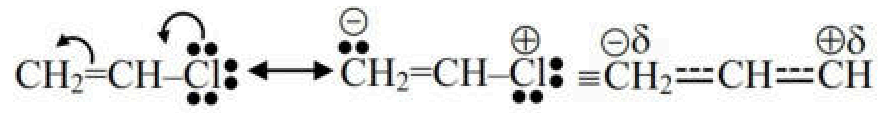
(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

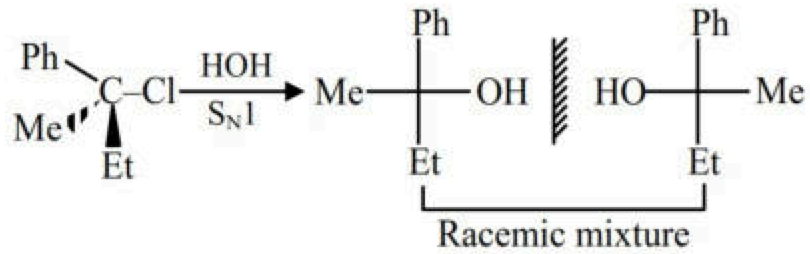
Ans. **(4)**

Sol. **Statement-I :**



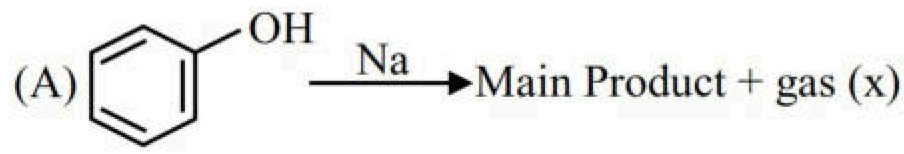
C–Cl bond is strong in vinyl chloride because of double bond character.

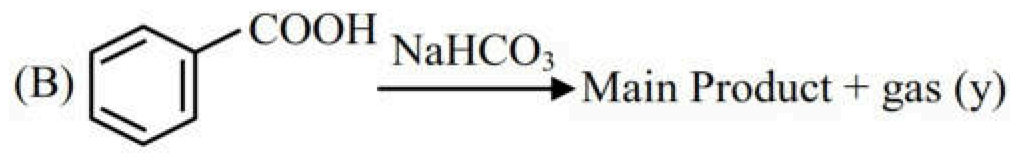
Statement-II :



Racemic mixture is optically inactive, which can not rotate PPL.

68. Consider the following two reactions A and B.





Numerical value of [molar mass of x+ molar mass of y] is _____ .

(1) 4 (2) 88

(3) 46 (4) 160

Ans. **(3)**

Sol. $x = H_2$ (gas), $y = CO_2$ (gas)

Sum of molar mass = 2 + 44 = 46

69. Arrange the following carbanions in the decreasing order of stability

I. $p - Br - C_6H_4 - \overset{\ominus}{\ddot{C}}H_2$

II. $C_6H_5 - \overset{\ominus}{\ddot{C}}H_2$

III. $p - CH_3O - C_6H_4 - \overset{\ominus}{\ddot{C}}H_2$

IV. $p - CHO - C_6H_4 - \overset{\ominus}{\ddot{C}}H_2$

V. $p - CH_3 - C_6H_4 - \overset{\ominus}{\ddot{C}}H_2$

Choose the **correct** answer from the options given below :

(1) I > II > IV > V > III

(2) I > IV > II > V > III

(3) IV > I > II > V > III

(4) IV > II > I > III > V

Ans. **(3)**

Sol. Electron withdrawing groups increases stability of carbanion and electron donating groups decreases stability of carbanion.

70. Given below are two statements :

Statement I : K > Mg > Al > B is the correct order in terms of metallic character.

Statement II : Atomic radius is always greater than the ionic radius for any element.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Ans. **(4)**

Sol. Metallic character of s-block elements is greater than p-block elements.

Anionic radius is greater than atomic radius but cationic radius is always less than atomic radius for any element.

SECTION-B

71. Electricity is passed through an acidic solution of Cu^{2+} till all the cu^{2+} was exhausted, leading to the deposition of 300 mg of Cu metal. However, a current of 600 mA was continued to pass through the same solution for another 28 minutes by keeping the total volume of the solution fixed at 200 mL. The total volume of oxygen evolved at STP during the entire process is ______ mL. (Nearest integer)

[Given :

$Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)\ E^0_{red} = +0.34V$

$O_2(g) + 4H^+ + 4e^- \rightarrow 2H_2O\ E^0_{red} = +1.23V$

Molar mass of Cu = 63.54 g mol^{-1}

Molar mass of O_2 = 32 g mol^{-1}

Faraday Constant = 96500 C mol^{-1}

Molar volume at STP = 22.4 L]

Ans. **(111)**

Sol. Eq of Cu = Eq of O_2

$$\frac{300 \times 10^{-3} \times 2}{63.54} = n_{O_2} \times 4$$

$2.36 \times 10^{-3} = n_{O_2}$

When current is further passed

$$n_{O_2} \times 4 = \frac{600 \times 28 \times 60}{96500 \times 1000}$$

$n_{O_2} = 2.611 \times 10^{-3}$

Total O_2 released

$= [10^{-3} \times (2.36 + 2.611)] \times 22400$ ml

= 111.35 ml

72. Consider two Group IV metal ions X^{2+} and Y^{2+}.

A solution containing 0.01 MX^{2+} and 0.01 MY^{2+} is saturated with H_2S. The pH at which the metal sulphide YS will form as a precipitate is ______ (Nearest integer)

(Given :

$K_{sp}(XS) = 1 \times 10^{-22}$ at 25°C, $K_{sp}(YS) = 4 \times 10^{-16}$ at 25°C,

$[H_2S]$ = 0.1M in solution, $K_{a1} \times K_{a2}\ (H_2S) = 1.0 \times 10^{-21}$,

log 2 = 0.30, log 3 = 0.48, log 5 = 0.70)

Ans. **(4)**

Sol. $XS(s) \rightleftharpoons X^{+2}(aq.) + S^{2-}(aq.)$

For precipitation of XS(s)

$[X^{+2}][S^{2-}] \geq K_{SP}(XS)$

$[S^{2-}] \geq \dfrac{1\times10^{-22}}{0.01} = 10^{-20}$

$YS(s) \rightleftharpoons Y^{+2}(aq) + S^{2-}(aq)$

For precipitation of YS(s)

$[Y^{+2}][S^{2-}] \geq K_{SP}(YS)$

$[S^{2-}] \geq \dfrac{4\times10^{-16}}{10^{-2}} = 4 \times 10^{-14}$

Now, $H_2S(aq) \rightleftharpoons 2H^+(aq) + S^{2-}(aq)$

$$\frac{[S^{2-}][H^+]^2}{H_2S} = K_{a_1} \times K_{a_2} = 1\times10^{-21}$$

$$[S^{2-}] = \frac{1\times10^{-21}\times[H_2S]}{[H^+]^2} \geq 4 \times 10^{-14}$$

$$[H^+]^2 \leq \frac{1}{4} \times 10^{-7} \times 10^{-1}$$

$$[H^+] \leq \frac{1}{2} \times 10^{-4} \Rightarrow pH \geq 4.3$$

73. The hydrogen spectrum consists of several spectral lines in Lyman series (L_1, L_2, L_3 ….; L_1 has lowest energy among Lyman series). Similarly it consists of several spectral lines in Balmer series (B_1, B_2, B_3……. ; B_1 has lowest energy among Balmer lines). The energy of L_1 is x times the energy of B_1. The value of x is ______ $\times 10^{-1}$ (Nearest inteter)

Ans. **(54)**

Sol. $\Delta E(L_1) = 13.6\times Z^2\left(\dfrac{1}{1^2}-\dfrac{1}{2^2}\right) = 13.6Z^2\times\dfrac{3}{4}$

$$\Delta E(B_1) = 13.6\times Z^2\times\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = 13.6\times Z^2\times\frac{5}{4\times9}$$

$$\frac{\Delta E(L_1)}{\Delta E(B_1)} = \frac{3}{5}\times9 = \frac{27}{5} = x$$

$$x = \left(\frac{27}{5}\times10\right)\times10^{-1} = 54 \times 10^{-1}$$

74. In Dumas method for estimation of nitrogen, 0.50 g of an organic compound gave 70 mL of nitrogen collected at 300 K and 715 mm pressure. The percentage of nitrogen in the organic compound is ______%

(Aqueous tension at 300 K is 15 mm).

Ans. **(15)**

Sol. $P_{N_2} = (715-15)mm = \dfrac{700}{760}atm$

$$V_{N_2} = 70\,ml = \frac{70}{1000}1$$

$$n_{N_2} = \frac{PV}{RT} = \frac{\left(\frac{700}{760}\right)\times\left(\frac{70}{1000}\right)}{0.0821\times300}$$

$$W_{N_2} = \frac{700}{760}\times\frac{\frac{70}{1000}}{0.0821\times300}\times28$$

$$\%\,N = \frac{W_{N_2}}{0.5}\times100 = \frac{700}{760}\times\frac{\frac{70/1000}{0.0821\times300}\times28}{0.5}\times100$$

$= 14.65\% \approx 15$

75. X and Y are the number of electrons involved, respectively during the oxidation of I^- to I_2 and S^{2-} to S by acidified $K_2Cr_2O_7$. The value of X + Y is ______.

Ans. **(12)**

Sol. $Cr_2O_7^{2-} + 14H^+ + 6I^- \rightarrow 2Cr^{3+} + 3I_2 + 7H_2O$

no. of moles e^- involved = x = 6

$Cr_2O_7^{2-} + 3S^{2-} + 14H^+ \rightarrow +S + 2Cr^{3+} + 7H_2O$

No. of moles e^- involved = y = 6

Sum of x + y = 6 + 6 = 12