

JEE (Main)-2026 Session-1
Question Paper with Solutions
(Mathematics, Physics, And Chemistry)
28 January 2026 Shift – 2

Time: 3 hrs.

M.M: 300

IMPORTANT INSTRUCTIONS:

- (1) The test is of 3 hours duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
- (4) Section - A: Attempt all questions.
- (5) Section - B: Attempt all questions.
- (6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.
- (7) Section - B (21 - 25) contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS

SECTION-A

1. Given below two statements :

Statement I : $25^{13} + 20^{13} + 8^{13} + 3^{13}$ is divisible by 7.

Statement II : The integral part of $(7 + 4\sqrt{3})^{25}$ is an odd number.

In the light of the above statements, choose the **correct answer** from the options given below :

- (1) Both **Statement I** and **Statement II** are false.
- (2) Both **Statement I** and **Statement II** are true.
- (3) **Statement I** is false but **Statement II** is true.
- (4) **Statement I** is true but **Statement II** is false.

Ans. (2)

Sol. Statement I :

$$\begin{array}{ccc} 25^{13} + 3^{13} & + & 20^{13} + 8^{13} \\ \downarrow & & \downarrow \\ \text{divisible by} & & \text{divisible by} \\ (25+3) & & (20+8) \\ \swarrow & & \searrow \\ \therefore \text{divisible by } 7 \end{array}$$

Statement II : $R = (7 + 4\sqrt{3})^{25} = I + f$

$$R' = (7 - 4\sqrt{3})^{25} = f'$$

$$\therefore R + R' = 2 \left[{}^{25}C_0 7^{25} + {}^{25}C_2 7^{23} (4\sqrt{3})^2 + \dots \right]$$

$$I + f + f' = \text{even integer}$$

$$\therefore I = \text{odd integer}$$

$$\because 0 < f + f' < 2 \Rightarrow f + f' = 1$$

$\Rightarrow$ Both the statements are correct

2. The sum of the coefficients of x^{499} and x^{500} in

$$(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \dots + x^{1000}$$
 is

(1) ${}^{1001}C_{501}$

(2) ${}^{1002}C_{500}$

(3) ${}^{1002}C_{501}$

(4) ${}^{1000}C_{501}$

Ans. (2)

Sol. $S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \dots + x^{1000}$

$$= (1+x)^{1000} \frac{1 - \left(\frac{x}{1+x}\right)^{1001}}{1 - \frac{x}{1+x}}$$

$$= (1+x)^{1001} - x^{1001}$$

$$\text{Required sum} = {}^{1001}C_{499} + {}^{1001}C_{500} = {}^{1002}C_{500}$$

3. Let A be the focus of the parabola $y^2 = 8x$. Let the line $y = mx + c$ intersect the parabola at two distinct points B and C. If the centroid of the triangle ABC is $\left(\frac{7}{3}, \frac{4}{3}\right)$, then $(BC)^2$ is equal to :

(1) 41

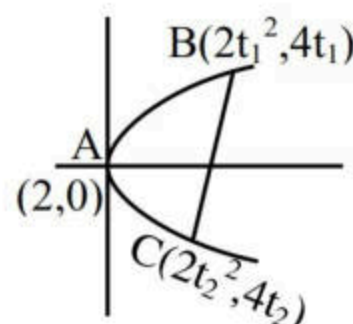
(2) 80

(3) 89

(4) 32

Ans. (2)

Sol.



Coordinates of centroid of triangle ABC are

$$\frac{2}{3}(t_1^2 + t_2^2 + 1) = \frac{7}{3} \Rightarrow t_1^2 + t_2^2 = \frac{5}{2}$$

$$\frac{4}{3}(t_1 + t_2) = \frac{4}{3} \Rightarrow t_1 + t_2 = 1$$

$$(t_1 + t_2)^2 = t_1^2 + t_2^2 + 2t_1t_2 \Rightarrow t_1t_2 = \frac{-3}{4}$$

$$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2 = 4$$

$$(BC)^2 = 4(t_1^2 - t_2^2)^2 + 16(t_1 - t_2)^2$$

$$\Rightarrow (BC)^2 = 80$$

4. The probability distribution of a random variable X is given below :

X	$4k$	$\frac{30}{7}k$	$\frac{32}{7}k$	$\frac{34}{7}k$	$\frac{36}{7}k$	$\frac{38}{7}k$	$\frac{40}{7}k$	$6k$
$P(X)$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

If $E(X) = \frac{263}{15}$, then $P(X < 20)$ is equal to :

- (1) $\frac{3}{5}$ (2) $\frac{8}{15}$
 (3) $\frac{11}{15}$ (4) $\frac{14}{15}$

Ans. (3)

Sol. $E(X) = \sum X_i P(X_i) = \frac{526k}{15 \times 7} = \frac{263}{15} \Rightarrow k = \frac{7}{2}$

X	14	15	16	17	18	19	20	21
$P(X)$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

$$P(X < 20) = \sum_{X=14}^{19} P(X) = \frac{11}{15}$$

5. Let $f(x) = \lim_{\theta \rightarrow 0} \left(\frac{\cos \pi x - x^{\left(\frac{2}{\theta}\right)} \sin(x-1)}{1 + x^{\left(\frac{2}{\theta}\right)} (x-1)} \right), x \in \mathbb{R}.$

Consider the following two statements :

- (I) $f(x)$ is discontinuous at $x = 1$.
 (II) $f(x)$ is continuous at $x = -1$.

Then,

- (1) Neither (I) nor (II) is True
 (2) Both (I) and (II) are True
 (3) Only (II) is True
 (4) Only (I) is True

Ans. (1)

Sol. $f(x) = \begin{cases} \cos \pi x & x \rightarrow 1^- \\ \frac{-\sin(x-1)}{(x-1)} & x \rightarrow 1^+ \end{cases}$

$$\text{RHL} = \lim_{x \rightarrow 1} \frac{-\sin(x-1)}{(x-1)} = -1$$

$$\text{LHL} = \lim_{x \rightarrow 1} \cos \pi x = -1, f(1) = -1$$

$f(x)$ is continuous at $x = 1$

$$f(x) = \begin{cases} \frac{-\sin(x-1)}{-(x-1)} & x \rightarrow -1^- \\ \cos \pi x & x \rightarrow -1^+ \end{cases}$$

$$\text{RHL} = \lim_{x \rightarrow -1} \cos \pi x = -1$$

$$\text{LHL} = \lim_{x \rightarrow -1} \frac{-\sin(x-1)}{(x-1)} = \frac{\sin 2}{-2}$$

$f(x)$ is discontinuous at $x = -1$

6. Considering the principal values of inverse trigonometric functions, the value of the expression

$$\tan \left(2 \sin^{-1} \left(\frac{2}{\sqrt{13}} \right) - 2 \cos^{-1} \left(\frac{3}{\sqrt{10}} \right) \right) \text{ is equal to :}$$

- (1) $-\frac{33}{56}$ (2) $\frac{33}{56}$
 (3) $\frac{16}{63}$ (4) $-\frac{16}{63}$

Ans. (2)

Sol. Let $\sin^{-1} \frac{2}{\sqrt{13}} = \theta$ & $\cos^{-1} \frac{3}{\sqrt{10}} = \phi$

$$\sin \theta = \frac{2}{\sqrt{13}} \text{ \& \> } \cos \phi = \frac{3}{\sqrt{10}}$$

$$\tan(2\theta - 2\phi) = \frac{\tan 2\theta - \tan 2\phi}{1 + \tan 2\theta \tan 2\phi}$$

$$\left(\because \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \right)$$

$$= \frac{\frac{12}{5} - \frac{3}{4}}{1 + \frac{12}{5} \cdot \frac{3}{4}}$$

$$= \frac{33}{56}$$

7. Let the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ be $\frac{5}{16}$, $a > 2$.

If α is such that $a, 4, \alpha, b$ are in A.P., then the equation $\alpha x^2 - ax + 2(\alpha - 2b) = 0$ has :

- (1) One root in $(1, 4)$ and another in $(-2, 0)$
- (2) One root in $(0, 2)$ and another in $(-4, -2)$
- (3) Complex roots of magnitude less than 2
- (4) Both roots in the interval $(-2, 0)$

Ans. (1)

Sol. $a = 4 - d, \alpha = 4 + d, b = 4 + 2d$

$$\Rightarrow (4 + d)x^2 - (4 - d)x + 2(4 + d - 8 - 4d) = 0$$

$$\Rightarrow (4 + d)x^2 - (4 - d)x + 2(-4 - 3d) = 0$$

$$\text{Also } \frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{5}{16}$$

$$\Rightarrow \frac{\frac{1}{4-d} + \frac{1}{4+2d}}{2} = \frac{5}{16}$$

$$\Rightarrow d = 2$$

Equation becomes

$$6x^2 - 2x - 20 = 0$$

$$3x^2 - x - 10 = 0$$

$$x = 2, \frac{-5}{3}$$

8. Given below are two statements :

Statement I : The function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by

$$f(x) = \frac{x}{1+|x|} \text{ is one-one.}$$

Statement II : The function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by

$$f(x) = \frac{x^2 + 4x - 30}{x^2 - 8x + 18} \text{ is many-one.}$$

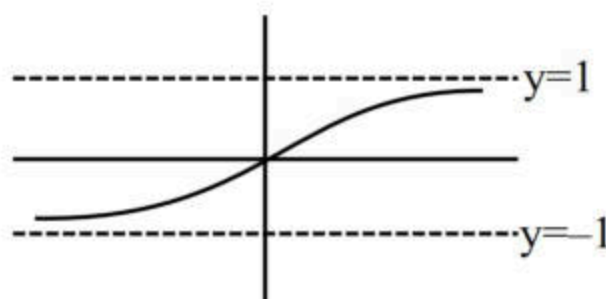
In the light of the above statements, choose the **correct answer** from the options given below :

- (1) Both **Statement I** and **Statement II** are false.
- (2) Both **Statement I** and **Statement II** are true.
- (3) **Statement I** is false but **Statement II** is true.
- (4) **Statement I** is true but **Statement II** is false.

Ans. (2)

Sol. **Statement 1:** $f(x) = \frac{x}{1+|x|}$

$$f(x) = \begin{cases} \frac{x}{1+x} & x \geq 0 \\ \frac{x}{1-x} & x < 0 \end{cases}$$



$f(x)$ is one-one

$$\text{Statement 2: } f(x) = \frac{x^2 + 4x - 30}{x^2 - 8x + 18}, f(0) = \frac{-30}{18} = \frac{-5}{3}$$

$$\frac{-5}{3} = \frac{x^2 + 4x - 30}{x^2 - 8x + 18}$$

On solving $x = 0, -1$

$$\Rightarrow f(0) = f(-1) = \frac{-5}{3}$$

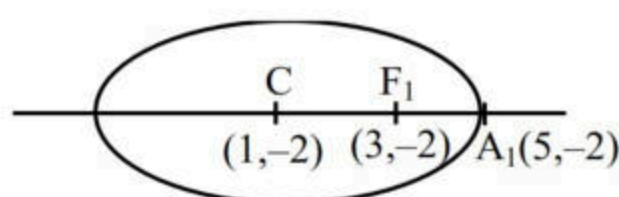
$\therefore f(x)$ is many-one

9. An ellipse has its center at $(1, -2)$, one focus at $(3, -2)$ and one vertex at $(5, -2)$. Then the length of its latus rectum is :

- (1) $\frac{16}{\sqrt{3}}$
- (2) 6
- (3) $4\sqrt{3}$
- (4) $6\sqrt{3}$

Ans. (2)

Sol.



$$CA_1 = a = 4$$

$$CF_1 = ae = 2$$

$$e = \frac{1}{2}$$

$$LR = 2e \left(\frac{a}{e} - ae \right)$$

$$= 2 \times \frac{1}{2} \times \left(\frac{4}{1/2} - 2 \right)$$

$$= 6$$

10. Let the ellipse $E: \frac{x^2}{144} + \frac{y^2}{169} = 1$ and the hyperbola

$$H: \frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1 \text{ have the same foci. If } e \text{ and } L$$

respectively denote the eccentricity and the length of the latus rectum of H , then the value of $24(e + L)$ is :

- (1) 296 (2) 126
(3) 148 (4) 67

Ans. (1)

Sol. Equation of hyperbola : $\frac{y^2}{\lambda^2} - \frac{x^2}{16} = 1$

$$\text{Equation of ellipse : } \frac{x^2}{144} + \frac{y^2}{169} = 1$$

$$e' = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$$

$$\text{focus} \Rightarrow (0, 5)$$

$$\Rightarrow \lambda \sqrt{1 + \frac{16}{\lambda^2}} = 5$$

$$\Rightarrow \lambda^2 + 16 = 25$$

$$\lambda = 3$$

$$\text{Eccentricity of hyperbola} = \sqrt{1 + \frac{16}{\lambda^2}} = \frac{5}{3}$$

$$\text{Length of latus rectum of hyperbola} = \frac{2(16)}{3} = \frac{32}{3}$$

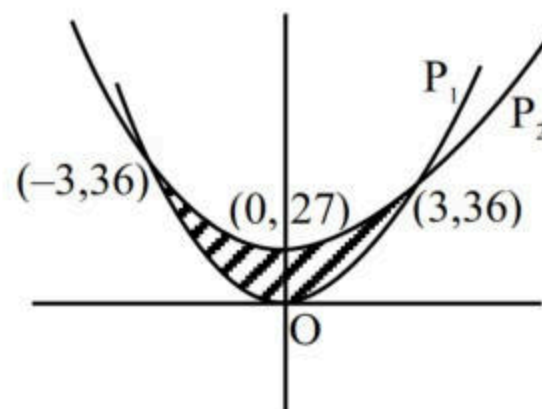
$$24(e + \ell) = 24 \left[\frac{5}{3} + \frac{32}{3} \right] = 8 \times 37 = 296$$

11. Let $P_1 : y = 4x^2$ and $P_2 : y = x^2 + 27$ be two parabolas. If the area of the bounded region enclosed between P_1 and P_2 is six times the area of the bounded region enclosed between the line $y = \alpha x$, $\alpha > 0$ and P_1 , then α is equal to :

- (1) 8 (2) 15
(3) 12 (4) 6

Ans. (3)

Sol.



Area bounded between P_1 & P_2 is

$$\int_{-3}^3 ((x^2 + 27) - (4x^2)) dx$$

(P.O.I. of P_1 & P_2 is $x = \pm 3$)

$$= 2 \int_0^3 (27 - 3x^2) dx = 2 [27x - x^3]_0^3$$

$$= 2[81 - 27] = 108$$

$\therefore$ Area bounded between P_1 & L is 18 sq. units

(Area between $x^2 = 4ay$ & line $x = my$) is $\frac{8a^2}{3m^3}$

$\therefore$ Area between $x^2 = \frac{y}{4}$ & $x = \frac{y}{\alpha}$ is

$$\frac{8 \cdot \left(\frac{1}{16}\right)^2}{3 \cdot \left(\frac{1}{\alpha}\right)^3} = 18$$

$$\Rightarrow \frac{8}{\frac{16 \cdot 16}{3 \alpha^3}} = 18 \Rightarrow \alpha^3 = 2^6 \cdot 3^3$$

$$\Rightarrow \alpha = 12$$

12. Let the circle $x^2 + y^2 = 4$ intersect x -axis at the points $A(a, 0)$, $a > 0$ and $B(b, 0)$. Let $P(2 \cos \alpha, 2 \sin \alpha)$, $0 < \alpha < \frac{\pi}{2}$ and $Q(2 \cos \beta, 2 \sin \beta)$ be two

points such that $(\alpha - \beta) = \frac{\pi}{2}$. Then the point of

intersection of AQ and BP lies on :

- (1) $x^2 + y^2 - 4y - 4 = 0$
(2) $x^2 + y^2 - 4x - 4 = 0$
(3) $x^2 + y^2 - 4x - 4y = 0$
(4) $x^2 + y^2 - 4x - 4y - 4 = 0$

Ans. (1)

Sol. Let point of intersection R(h,k)

$$m_{BR} = m_{BP} \Rightarrow \frac{k}{h+2} = \frac{2 \sin \alpha}{2 \cos \alpha + 2} \Rightarrow \frac{k}{h+2} = \tan \frac{\alpha}{2}$$

$$m_{AR} = m_{AQ} \Rightarrow \frac{k}{h-2} = \frac{2 \sin \beta}{2 \cos \beta - 2} = \frac{\sin \beta}{\cos \beta - 1} = -\cot \frac{\beta}{2}$$

$$\frac{\alpha}{2} - \frac{\beta}{2} = \frac{\pi}{4}$$

$$\tan\left(\frac{\alpha}{2} - \frac{\beta}{2}\right) = \tan \frac{\pi}{4} = 1$$

$$\frac{\tan \frac{\alpha}{2} - \tan \frac{\beta}{2}}{1 + \tan \frac{\alpha}{2} \tan \frac{\beta}{2}} = 1$$

$$\frac{\frac{k}{h+2} + \frac{h-2}{k}}{1 + \left(\frac{k}{h+2}\right)\left(\frac{2-h}{k}\right)} = 1 \Rightarrow \frac{k^2 + h^2 - 4}{\frac{4}{h+2}} = 1$$

$$\frac{h^2 + k^2 - 4}{4k} = 1$$

$$x^2 + y^2 - 4y - 4 = 0$$

13. Let $[\cdot]$ denote the greatest integer function. Then

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{12(3 + [x])}{3 + [\sin x] + [\cos x]} \right) dx \text{ is equal to:}$$

- (1) $15\pi + 4$ (2) $11\pi + 2$
(3) $13\pi + 1$ (4) $12\pi + 5$

Ans. (2)

$$\text{Sol. } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{12(3 + [x])dx}{3 + [\sin x] + [\cos x]}$$

$$I = \int_{-\frac{\pi}{2}}^{-1} \frac{12(1)dx}{2} + \int_{-1}^0 \frac{12(2)dx}{2} + \int_0^1 \frac{12(3)dx}{3} + \int_1^{\frac{\pi}{2}} \frac{12(4)dx}{3}$$

$$I = 6\left(\frac{\pi}{2} - 1\right) + 12(0 + 1) + 12(1 - 0) + 16\left(\frac{\pi}{2} - 1\right)$$

$$I = 3\pi - 6 + 12 + 12 + 8\pi - 16$$

$$I = 11\pi + 2$$

14. Let $y = y(x)$ be the solution of the differential equation $x \frac{dy}{dx} - y = x^2 \cot x$, $x \in (0, \pi)$.

If $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then $6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$ is equal to :

- (1) 3π (2) -3π
(3) $-\pi$ (4) π

Ans. (3)

$$\text{Sol. } xdy - ydx = x^2 \cot x dx$$

$$x^2 d\left(\frac{y}{x}\right) = x^2 \cot x dx$$

$$d\left(\frac{y}{x}\right) = \cot x dx$$

$$\int d\left(\frac{y}{x}\right) = \int \cot x dx$$

$$\frac{y}{x} = \log_e \sin x + C$$

$$\text{given } y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$\Rightarrow c = 1$$

$$y = x(\log_e \sin x + 1)$$

$$y\left(\frac{\pi}{6}\right) = \frac{\pi}{6}[-\log_e 2 + 1]$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\left[-\frac{1}{2}\log_e 2 + 1\right]$$

$$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$$

$$= \pi \left[(-\log_e 2 + 1) + 2\left(\frac{1}{2}\log_e 2 - 1\right) \right]$$

$$= \pi[1 - 2] = -\pi$$

15. The sum of all the elements in the range of $f(x) = \text{Sgn}(\sin x) + \text{Sgn}(\cos x) + \text{Sgn}(\tan x) + \text{Sgn}(\cot x)$,

$$x \neq \frac{n\pi}{2}, n \in \mathbf{Z},$$

$$\text{where } \text{Sgn}(t) = \begin{cases} 1, & \text{if } t > 0 \\ -1 & \text{if } t < 0 \end{cases}, \text{ is}$$

- (1) 4 (2) 2
(3) -2 (4) 0

Ans. (2)

Sol. $x \in (0, \pi/2) \Rightarrow y = 1 + 1 + 1 + 1 = 4$
 $x \in (\pi/2, \pi) \Rightarrow y = 1 - 1 - 1 - 1 = -2$
 $x \in (\pi, 3\pi/2) \Rightarrow y = -1 - 1 + 1 + 1 = 0$
 $x \in (3\pi/2, 2\pi) \Rightarrow y = -1 + 1 - 1 - 1 = -2$
 $\therefore$ Range of y is $\{-2, 0, 4\}$

Required sum $= -2 + 0 + 4 = 2$

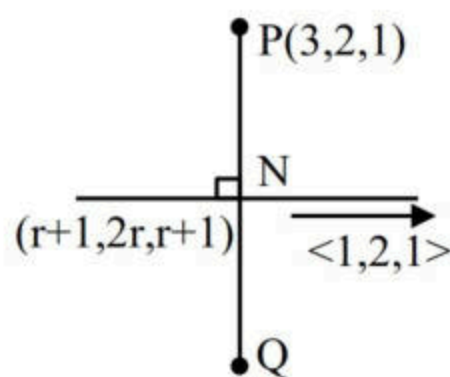
16. Let $Q(a, b, c)$ be the image of the point $P(3, 2, 1)$ in the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$. Then the distance of Q

from the line $\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$ is

- (1) 6
 (2) 8
 (3) 7
 (4) 5

Ans. (3)

Sol.



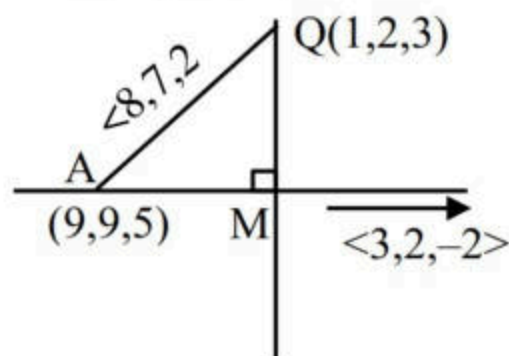
drs of $PN = \langle r-2, 2r-2, r \rangle$

$$1 \cdot (r-2) + 2(2r-2) + 1 \cdot (r) = 0$$

$$6r = 6 \Rightarrow r = 1$$

$$\therefore N \equiv (2, 2, 2)$$

$$\Rightarrow Q \equiv (1, 2, 3)$$



$$AQ = \sqrt{64 + 49 + 4} = \sqrt{117}$$

$$AM = \left| \frac{24 + 14 - 4}{\sqrt{9 + 4 + 4}} \right| = \frac{34}{\sqrt{17}} = 2\sqrt{17}$$

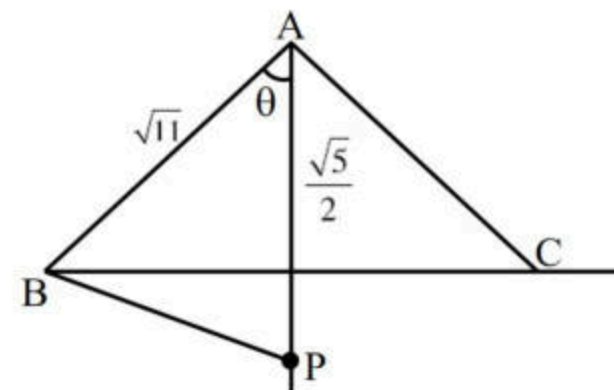
$$\therefore QM = \sqrt{117 - 68} = \sqrt{49} = 7$$

17. Let P be a point in the plane of the vector $\overrightarrow{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{AC} = \hat{i} - \hat{j} + 3\hat{k}$ such that P is equidistant from the lines AB and AC . If $|\overrightarrow{AP}| = \frac{\sqrt{5}}{2}$, then the area of the triangle ABP is :

- (1) 2
 (2) $\frac{3}{2}$
 (3) $\frac{\sqrt{30}}{4}$
 (4) $\frac{\sqrt{26}}{4}$

Ans. (3)

Sol. $\cos 2\theta = \frac{3-1-3}{\sqrt{11} \cdot \sqrt{11}} = -\frac{1}{11}$



$$1 - 2\sin^2 \theta = -\frac{1}{11} \Rightarrow 2\sin^2 \theta = \frac{12}{11} \Rightarrow \sin \theta = \sqrt{\frac{6}{11}}$$

$$\therefore \text{Area}(\triangle APB) = \frac{1}{2} \times \sqrt{11} \cdot \frac{\sqrt{5}}{2} \cdot \sqrt{\frac{6}{11}} = \frac{\sqrt{30}}{4}$$

18. Let

$$A = \{z \in \mathbb{C} : |z-2| \leq 4\} \text{ and}$$

$$B = \{z \in \mathbb{C} : |z-2| + |z+2| = 5\}.$$

Then the max $\{|z_1 - z_2| : z_1 \in A \text{ and } z_2 \in B\}$ is

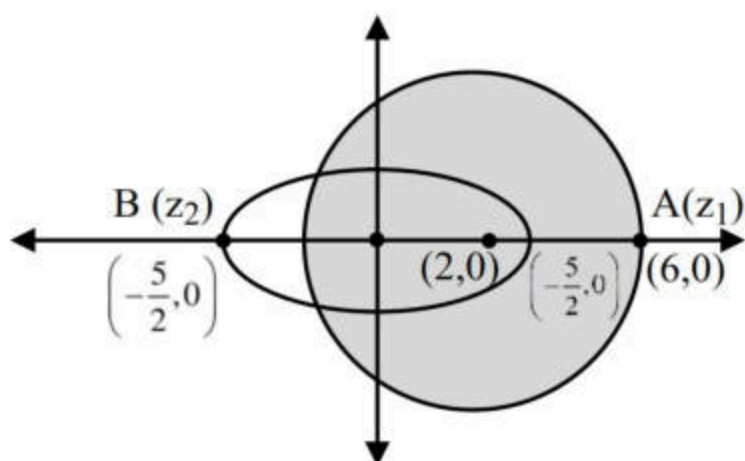
- (1) $\frac{15}{2}$
 (2) 8
 (3) $\frac{17}{2}$
 (4) 9

Ans. (3)

Sol. $|z-2| \leq 4 \Rightarrow (x-2)^2 + y^2 \leq 16$

$$|z-2| + |z+2| = 5 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{4x^2}{25} + \frac{4y^2}{9} = 1$$



Maximum value of $|z_1 - z_2| = 6 + \frac{5}{2} = \frac{17}{2}$

19. Let $f(x) = \int \frac{dx}{x^{\frac{2}{3}} + 2x^{\frac{1}{2}}}$ be such that $f(0) = -26 + 24$

$\log_e(2)$. If $f(1) = a + b \log_e(3)$, where $a, b \in \mathbb{Z}$, then $a + b$ is equal to:

- (1) -18 (2) -5
(3) -11 (4) -26

Ans. (3)

Sol. $f(x) = \int \frac{dx}{x^{\frac{2}{3}} + 2x^{\frac{1}{2}}}$

Put $x = t^6 \Rightarrow dx = 6t^5 dt$

$$= \int \frac{6t^5 dt}{t^4 + 2t^3} = 6 \int \frac{(t^2 - 4) + 4}{t + 2} dt$$

$$= 6 \left[\int (t - 2) dt + 4 \int \frac{1}{t + 2} dt \right]$$

$$= 6 \left[\frac{t^2}{2} - 2t + 4 \ln(t + 2) \right] + C$$

$$= 3x^{1/3} - 12x^{1/6} + 24 \ln(x^{1/6} + 2) + C$$

$$f(0) = 24 \ln 2 + C = -26 + 24 \ln 2 \text{ (given)}$$

$$\Rightarrow C = -26$$

Now

$$f(1) = -35 + 24 \ln 3 = a + b \ln 3 \text{ (as given in ques.)}$$

$$\Rightarrow a = -35 \text{ \& } b = 24$$

$$\Rightarrow a + b = -11$$

20. $\frac{6}{3^{26}} + \frac{10.1}{3^{25}} + \frac{10.2}{3^{24}} + \frac{10.2^2}{3^{23}} + \dots + \frac{10.2^{24}}{3}$ is

equal to

- (1) 2^{25} (2) 2^{26}
(3) 3^{25} (4) 3^{26}

Ans. (2)

Sol. $S = \frac{6}{3^{26}} + \frac{10}{3^{25}} \left[\frac{(6)^{25} - 1}{6 - 1} \right]$

$$S = \frac{6}{3^{26}} + \frac{10}{3^{25}} \left[\frac{6^{25} - 1}{5} \right]$$

$$S = \frac{2}{3^{25}} + 2 \left[2^{25} - \frac{1}{3^{25}} \right]$$

$$S = 2^{26}$$

SECTION-B

21. If $\sum_{r=1}^{25} \left(\frac{r}{r^4 + r^2 + 1} \right) = \frac{p}{q}$, where p and q are positive integers such that $\gcd(p, q) = 1$, then $p + q$ is equal to _____.

Ans. (976)

Sol. $S = \sum \frac{r}{(r^2 + r + 1)(r^2 - r + 1)}$

$$= \frac{1}{2} \sum_{r=1}^{25} \left(\frac{1}{r^2 - r + 1} - \frac{1}{r^2 + r + 1} \right)$$

$$= \frac{1}{2} \left[\left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{7} \right) + \dots + \left(\frac{1}{601} - \frac{1}{651} \right) \right]$$

$$= \frac{1}{2} \left[1 - \frac{1}{651} \right]$$

$$= \frac{1}{2} \left[\frac{650}{651} \right] = \frac{325}{651}$$

$$\frac{p}{q} = \frac{325}{651} \Rightarrow p + q = 976$$

22. Three persons enter in a lift at the ground floor. The lift will go upto 10th floor. The number of ways, in which the three persons can exit the lift at three different floors, if the lift does not stop at first, second and third floors, is equal to _____.

Ans. (210)

Sol. ${}^7C_3 \times 3$
 $= 210$

23. Let f be a differentiable function satisfying

$$f(x) = 1 - 2x + \int_0^x e^{(x-t)} f(t) dt, x \in \mathbf{R} \text{ and let}$$

$$g(x) = \int_0^x (f(t) + 2)^{15} (t - 4)^6 (t + 12)^{17} dt, x \in \mathbf{R}.$$

If p and q are respectively the points of local minima and local maxima of g , then the value of $|p + q|$ is equal to _____.

Ans. (9)

Sol. $f(x) = 1 - 2x + e^x \int_0^x e^{-t} f(t) dt$

$$e^{-x} f(x) = (1 - 2x)e^{-x} + \int_0^x e^{-t} f(t) dt$$

$$e^{-x} f'(x) - e^{-x} f(x) = -2e^{-x} + (1 - 2x)e^{-x}(-1) + e^{-x} f(x)$$

$$f'(x) - 2f(x) = 2x - 3$$

$$\frac{dy}{dx} - 2y = 2x - 3$$

$$\Rightarrow y \cdot e^{-2x} = \int e^{-2x} (2x - 3) dx$$

On solving we get

$$f(x) = 1 - x$$

$$g(x) = \int_0^x (3 - t)^{15} (t - 4)^6 (t + 12)^{17} dt$$

$$g'(x) = (3 - x)^{15} (x - 4)^6 (x + 12)^{17}$$

$$= -(x - 3)^{15} (x - 4)^6 (x + 12)^{17}$$

$$\begin{array}{c} - \quad + \quad - \quad - \\ -12 \quad 3 \quad 4 \end{array}$$

Local maxima $\Rightarrow q = 3$

Local minima $\Rightarrow p = -12 = |p + q| = 9$

24. If the distance of the point $P(43, \alpha, \beta)$, $\beta < 0$, from the line $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$, $\mu \in \mathbf{R}$ along a line with direction ratios 3, -1, 0 is $13\sqrt{10}$, then $\alpha^2 + \beta^2$ is equal to _____.

Ans. (170)

Sol. $\frac{x - 43}{3} = \frac{y - \alpha}{-1} = \frac{z - \beta}{0} \Rightarrow P_1(43 + 3\lambda, \alpha - \lambda, \beta)$

$$\frac{x - 4}{2} = \frac{y}{0} = \frac{z + 1}{3} \Rightarrow P_1(2\mu + 4, 0, 3\mu - 1)$$

$$\therefore \mu = \frac{3\lambda + 39}{2}, \alpha = \lambda, \beta = \frac{9\lambda - 115}{2}$$

$$P(43, \alpha, \beta), P_1(43 + 3\alpha, 0, \beta)$$

$$(PP_1)^2 = 1690 = 10\alpha^2, \therefore \alpha = 13, \beta = 1$$

$$\therefore \alpha^2 + \beta^2 = 170$$

25. Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and B be two matrices such that $A^{100} = 100B + I$. Then the sum of all the elements of B^{100} is _____.

Ans. (0)

Sol. $A = I + \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$, let $M = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

$$M^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = M^3 = M^4 = \dots = M^{100}$$

$$A^{100} = (I + M)^{100} = \sum_{r=0}^{100} \binom{100}{r} M^r \cdot I$$

$$A^{100} = I + 100M = I + 100B$$

$$\therefore M = B \Rightarrow M^{100} = B^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

PHYSICS

SECTION-A

26. A nucleus has mass number α and radius R_α . Another nucleus has mass number β and radius R_β . If $\beta = 8\alpha$ then R_α/R_β is :

- (1) 2 (2) 8
(3) 1 (4) 0.5

Ans. (4)

Sol. $R_\alpha = R_0 \alpha^{1/3}$
 $R_\beta = R_0 \beta^{1/3}$
 $\frac{R_\alpha}{R_\beta} = \left(\frac{\alpha}{\beta}\right)^{1/3} = \frac{1}{2}$

27. A plane electromagnetic wave is moving in free space with velocity $c = 3 \times 10^8$ m/s and its electric field is given as $\vec{E} = 54 \sin(kz - \omega t) \hat{j}$ V/m, where

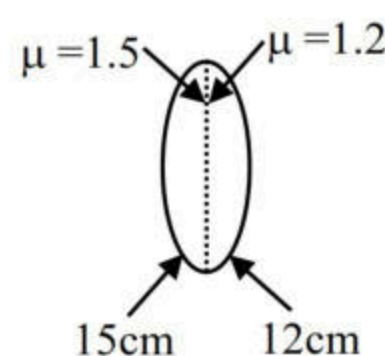
$\hat{j}$ is the unit vector along y-axis. The magnetic field vector $\vec{B}$ of the wave is :

- (1) $-1.8 \times 10^{-7} \sin(kz - \omega t) \hat{i}$ T
 (2) $1.4 \times 10^{-7} \sin(kz - \omega t) \hat{k}$ T
 (3) $1.4 \times 10^{-7} \sin(kz - \omega t) \hat{i}$ T
 (4) $+1.8 \times 10^{-7} \sin(kz - \omega t) \hat{i}$ T

Ans. (1)

Sol. $\hat{B} = \hat{C} \times \hat{E} = \hat{k} \times \hat{j} = -\hat{i}$
 $\therefore \vec{B} = \frac{54}{3 \times 10^8} \sin(kz - \omega t) (-\hat{i})$
 $= -1.8 \times 10^{-7} \sin(kz - \omega t) \hat{i}$

28. A biconvex lens is formed by using two thin planoconvex lenses, as shown in the figure. The refractive index and radius of curved surfaces are also mentioned in figure. When an object is placed on the left side of lens at a distance of 30 cm from the biconvex lens, the magnification of the image will be :



- (1) -2 (2) +2
(3) +2.5 (4) -2.5

Ans. (1)

Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_{\text{net}}} = \frac{1}{f_1} + \frac{1}{f_2}$
 $\frac{1}{v} + \frac{1}{30} = (1.5 - 1) \left(\frac{1}{15} - \frac{1}{\infty} \right) + (1.2 - 1) \left(\frac{1}{\infty} + \frac{1}{12} \right)$
 $\frac{1}{v} + \frac{1}{30} = \frac{1}{30} + \frac{1}{60}$
 $v = 60$
 $m = \frac{v}{u} = \frac{60}{-30} = -2$

29. The mean free path of a molecule of diameter 5×10^{-10} m at the temperature 41°C and pressure 1.38×10^5 Pa, is given as ____ m.

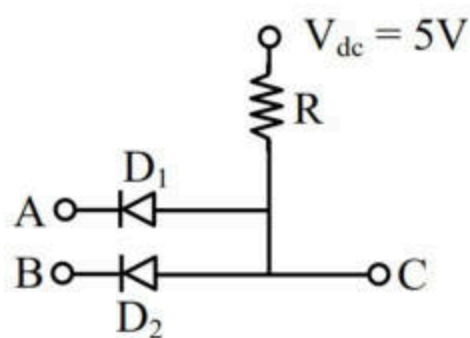
(Given $k_B = 1.38 \times 10^{-23}$ J/K).

- (1) $2\sqrt{2} \times 10^{-10}$
 (2) $10\sqrt{2} \times 10^{-8}$
 (3) $2\sqrt{2} \times 10^{-8}$
 (4) 2×10^{-8}

Ans. (3)

Sol. $\lambda = \frac{k_B T}{\sqrt{2} \pi \sigma^2 P}$
 $= \frac{1.38 \times 10^{-23} \times (273 + 41) \times 100}{\sqrt{2} \times 3.14 \times (5 \times 10^{-10})^2 \times 1.38 \times 10^5} = 2\sqrt{2} \times 10^{-8}$

30. Two p-n junction diodes D_1 and D_2 are connected as shown in figure. A and B are input signals and C is the output. The given circuit will function as a ____.



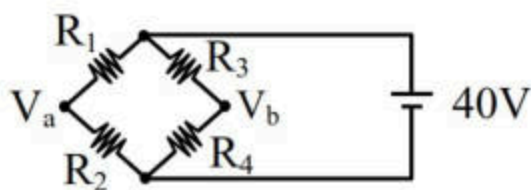
- (1) OR Gate (2) NOR Gate
(3) NAND Gate (4) AND Gate

Ans. (4)

Sol. If either A or B is zero, in that case current flow and $v_c = 0$.

Hence the Gate will be AND Gate

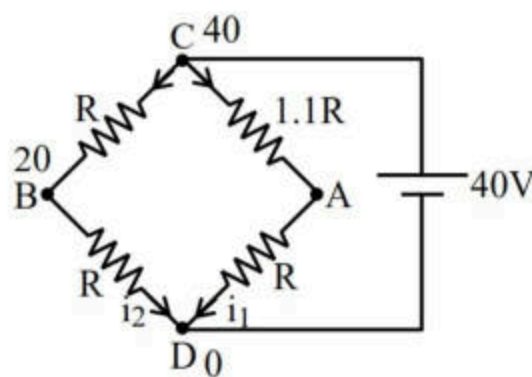
31. A wheatstone bridge is initially at room temperature and all arms of the bridge have same value of resistances ($R_1 = R_2 = R_3 = R_4$). When R_3 resistance is heated to some temperature, its resistance value has gone up by 10%. The potential difference ($V_a - V_b$) (after R_3 is heated) is ____ V.



- (1) 1.05 (2) 0 (3) 0.95 (4) 2

Ans. (3)

Sol.



$$V_A = \frac{V}{2}$$

$$V_B = \frac{V}{2.1R} \times R = \frac{V}{2.1}$$

$$\therefore V_A - V_B = V \left[\frac{1}{2} - \frac{1}{2.1} \right]$$

$$V_A - V_B = \frac{0.1}{2 \times 2.1} \times 40$$

$$V_A - V_B = \frac{4}{4.2} = 0.95$$

32. In an experiment, a set of reading are obtained -1.24 mm, 1.25 mm, 1.23 mm, 1.21 mm. The expected least count of the instrument used in recording these readings is ____ mm.

- (1) 0.01 (2) 0.001
(3) 0.1 (4) 0.05

Ans. (1)

Sol. Least count will be 0.01 mm.

33. A particle starts moving from time $t = 0$ and its coordinate is given as $x(t) = 4t^3 - 3t$.

- A. The particle returns to its original position (origin) 0.866 units later
B. The particle is 1 unit away from origin at its turning point.
C. Acceleration of the particle is non-negative.
D. The particle is 0.5 units away from origin at its turning point.
E. Particle never turns back as acceleration is non-negative.

Choose the **correct** answer from the options given below :

- (1) A,C,D only (2) A,B,C only
(3) C,E only (4) A,C only

Ans. (2)

Sol. $x = 0 \Rightarrow t = 0, \frac{\sqrt{3}}{2}$

$$v = 12t^2 - 3$$

At turning point, $v = 0$

$$t = \frac{1}{2} \Rightarrow x = \frac{4}{8} - \frac{3}{2} = -1$$

$$a = 24t \text{ (always positive)}$$

34. The speed of a longitudinal wave in a metallic bar is 400 m/s. If the density and Young's modulus of the bar material are increased by 0.5% and 1% respectively then the speed of the wave is changed approximately to ____ m/s.

- (1) 399 (2) 398 (3) 402 (4) 401

Ans. (4)

Sol. $V_{\text{sound}} = \sqrt{\frac{Y}{\rho}}$

$$\frac{\Delta V}{V} \times 100 = \frac{1}{2} \left(\frac{\Delta Y}{Y} \times 100 \right) - \frac{1}{2} \left(\frac{\Delta \rho}{\rho} \times 100 \right)$$

$$= \frac{1}{2} \times 1 - \frac{1}{2} \times 0.5$$

$$\frac{\Delta V}{V} \times 100 = \frac{1}{4}$$

$$\Delta V = \frac{1}{4} \times \frac{V}{100}$$

$$\Delta V = 1 \text{ m/s}$$

$$V_{\text{final}} = 400 + 1 = 401 \text{ m/s}$$

35. Identify the correct statements :

- A. Effective capacitance of a series combination of capacitors is always smaller than the smallest capacitance of the capacitor in the combination.
- B. When a dielectric medium is placed between the charged plates of a capacitor, displacement of charges cannot occur due to insulation property of dielectric.
- C. Increasing of area of capacitor plate or decreasing of thickness of dielectric is an alternate method to increase the capacitance.
- D. For a point charge, concentric spherical shells centered at the location of the charge are equipotential surfaces.

Choose the **correct** answer from the options given below.

- (1) A, B and C only
- (2) C and D only
- (3) A, C and D only
- (4) B and D only

Ans. (3)

Sol. For series combination

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$\therefore C_{\text{eq}}$ is less than C_1 & C_2 .

Note : In statement C, capacitor is assumed to be completely filled with dielectric then on decreasing thickness of dielectric capacitance will increase.

36. Number of photons of equal energy emitted per second by a 6 mW laser source operating at 663 nm is _____.

(Given : $h = 6.63 \times 10^{-34}$ J.s and $c = 3 \times 10^8$ m/s)

- (1) 5×10^{16}
- (2) 5×10^{15}
- (3) 10×10^{15}
- (4) 2×10^{16}

Ans. (4)

Sol. $P = \frac{nhc}{\lambda}$

$$6 \times 10^{-3} = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^8}{663 \times 10^{-9}}$$

$$n = 2 \times 10^{16} \text{ photons}$$

37. When the position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ changes sign as $-\vec{r}$, which one of the following vector will not flip under sign change ?

- (1) Linear momentum
- (2) Velocity
- (3) Acceleration
- (4) Angular momentum

Ans. (4)

Sol. $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\vec{p} = m\vec{v}$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$= (x\hat{i} + y\hat{j} + z\hat{k}) \times m(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$

When sign of $\vec{r}$ changes, $\vec{L}$ remains same.

38. Which one of the following is **not** a measurable quantity ?

- (1) Voltage difference
- (2) Resistance
- (3) Voltage
- (4) Displacement current

Ans. (3)

Sol. Here from voltage, question refers to potential. We can measure potential difference between two points but not potential at any point.

Note : If the potential of reference point is known then we can measure potential as well.

39. A long cylindrical conductor with large cross section carries an electric current distributed uniformly over its cross-section. Magnetic field due to this current is :

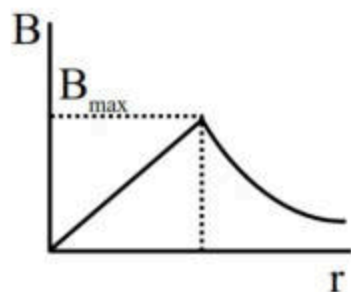
- A. maximum at either ends of the conductor and minimum at the midpoint
- B. maximum at the axis of the conductor
- C. minimum at the surface of the conductor
- D. minimum at the axis of the conductor
- E. same at all points in the cross-section of the conductor

Choose the **correct** answer from the options given below :

- (1) D Only
- (2) A, D Only
- (3) B, C Only
- (4) E Only

Ans. (1)

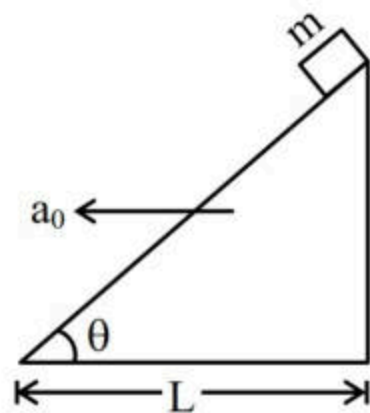
Sol. Solid cylinder



$B_{\max}$ at surface

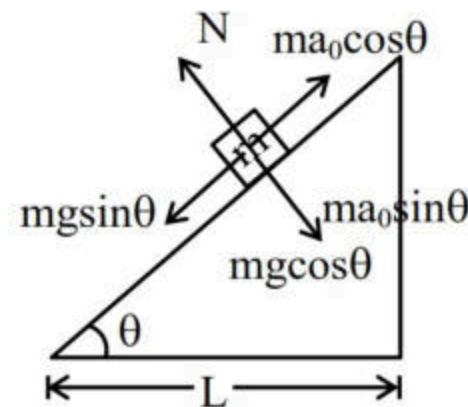
$B_{\min}$ at Axis

40. A small block of mass m slides down from the top of a frictionless inclined surface, while the inclined plane is moving towards left with constant acceleration a_0 . The angle between the inclined plane and ground is θ and its base length is L . Assuming that initially the small block is at the top of the inclined plane, the time it takes to reach the lowest point of the inclined plane is _____.



- (1) $\sqrt{\frac{2L}{g \sin 2\theta - a_0(1 + \cos 2\theta)}}$
- (2) $\sqrt{\frac{4L}{g \sin 2\theta - a_0(1 + \cos 2\theta)}}$
- (3) $\sqrt{\frac{4L}{g \cos^2 \theta - a_0 \sin \theta \cos \theta}}$
- (4) $\sqrt{\frac{2L}{g \sin \theta - a_0 \cos \theta}}$

Ans. (2)



Sol.

$$mg \sin \theta - ma_0 \cos \theta = ma$$

$$a = g \sin \theta - a_0 \cos \theta$$

Now using,

$$S = ut + \frac{1}{2} a_{\text{down}} t^2$$

$$\frac{L}{\cos \theta} = \frac{1}{2} (g \sin \theta - a_0 \cos \theta) t^2$$

$$t = \sqrt{\frac{2L}{g \sin \theta \cos \theta - a_0 \cos^2 \theta}}$$

$$t = \sqrt{\frac{4L}{g \sin 2\theta - a_0(1 + \cos 2\theta)}}$$

41. Identify the correct statements :

- A. Electrostatic field lines form closed loops.
- B. The electric field lines point radially outward when charge is greater than zero.
- C. The Gauss-Law is valid only for inverse-square force.
- D. The workdone in moving a charged particle in a static electric field around a closed path is zero.
- E. The motion of a particle under Coulomb's force must take place in a plane.

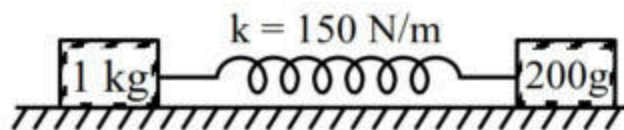
Choose the **correct** answer from the options given below :

- (1) A, B, D, E Only
- (2) A, B, C, D Only
- (3) B, C, D, E Only
- (4) A, C, E Only

Ans. (3)

Sol. Theoretical

42. As shown in the figure, a spring is kept in a stretched position with some extension by holding the masses 1 kg and 0.2 kg with a separation more than spring natural length and are released. Assuming the horizontal surface to be frictionless, the angular frequency (in SI unit) of the system is :



- (1) 30 (2) 27 (3) 20 (4) 5

Ans. (1)

Sol. $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{1 \times 0.2}{1.2}$

$$\mu = \frac{1}{6}$$

$$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{150}{1/6}} = 30$$

43. For a transparent prism, if the angle of minimum deviation is equal to its refracting angle, the refractive index n of the prism satisfies.

- (1) $\sqrt{2} < n < 2\sqrt{2}$ (2) $1 < n < 2$
(3) $n \geq 2$ (4) $\sqrt{2} < n < 2$

Ans. (4)

Sol. $\delta_{\min} = 2i - A \Rightarrow i = \delta_{\min} = A$

$$\text{Also, } \mu = \frac{\sin\left(\frac{\delta_{\min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \mu = \frac{\sin A}{\sin \frac{A}{2}} = 2 \cos\left(\frac{A}{2}\right)$$

$$1 < \mu < 2 \quad \dots (1)$$

$$\delta_{\min} = 2i - A$$

$$A = 2i - A \Rightarrow i = A$$

$$i < 90^\circ \text{ (grazing incidence)}$$

$$A < 90^\circ$$

$$\mu = 2 \cos(A/2)$$

$$\& A < 90^\circ$$

$$\mu > \sqrt{2} \quad \dots (2)$$

$$\text{from (i) \& (2)}$$

$$\sqrt{2} < \mu < 2$$

44. The time period of a simple harmonic oscillator is $T = 2\pi\sqrt{\frac{k}{m}}$. The measured value of mass (m) of

the object is 10 g with an accuracy of 10 mg, and time for 50 oscillations of the spring is found to be 60 s using a watch of 2 s resolution. Percentage error in determination of spring constant(k) is

_____ %.

- (1) 3.43 (2) 3.35 (3) 7.60 (4) 6.76

Ans. (4)

Sol. $\frac{\Delta K}{K} = \frac{2\Delta T}{T} + \frac{\Delta m}{m}$

$$T = \frac{60}{50} = 1.2 \text{ sec}$$

$$\Delta T = \frac{2}{50}$$

$$\therefore \frac{\Delta K}{K} = \frac{2 \times 2}{50 \times 1.2} + \frac{10 \times 10^{-3}}{10} = 0.0676$$

$$\therefore \% \text{ Error} = 6.76\%$$

45. Match List-I with List-II.

	List-I		List-II
A.	Coefficient of viscosity	I.	$[ML^{-1}T^{-2}]$
B.	Surface tension	II.	$[ML^2T^{-2}]$
C.	Pressure	III.	$[ML^0T^{-2}]$
D.	Surface energy	IV.	$[ML^{-1}T^{-1}]$

Choose the **correct** answer from the options given below :

- (1) A-I, B-II, C-IV, D-III
(2) A-IV, B-III, C-I, D-II
(3) A-I, B-III, C-II, D-IV
(4) A-IV, B-I, C-II, D-III

Ans. (2)

Sol. (A) $\eta = \frac{F_{dr}}{Adv} = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]} = [ML^{-1}T^{-1}]$

(B) $S = \frac{F}{L} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$

(C) $P = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

(D) $E = S \times A = [MT^{-2}][L^2] = [ML^2T^{-2}]$

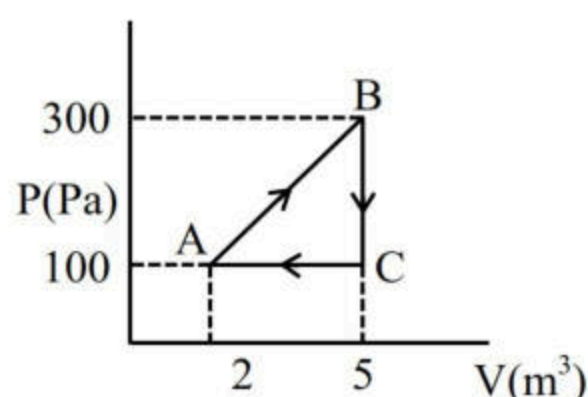
SECTION-B

46. Two tuning forks A and B are sounded together giving rise to 8 beats in 2 s. When fork A is loaded with wax, the beat frequency is reduced to 4 beats in 2 s. If the original frequency of tuning fork B is 380 Hz, then the original frequency of tuning fork A is _____ Hz.

Ans. (384)

Sol. $|f_A - f_B| = 4$
 $|f_A - 380| = 4$
 So, $f_A = 384$ Hz or 376 Hz
 on loading with wax f_A decreases
 So, $f_A = 384$ Hz

47. A thermodynamic system is taken through the cyclic process ABC as shown in the figure. The total work done by the system during the cycle ABC is _____ J.



Ans. (300)

Sol. Work done = Area bounded by cycle
 $= \frac{1}{2} \times 3 \times 200 = 300$ J

48. An inductor stores 16 J of magnetic field energy and dissipates 32 W of thermal energy due to its resistance when an a.c. current of 2 A (rms) and frequency 50 Hz flows through it. The ratio of inductive reactance to its resistance is _____. ($\pi = 3.14$)

Ans. (314)

Sol. $\frac{1}{2} Li_{rms}^2 = 16 \Rightarrow L = 8$
 $i^2 R = 32 \Rightarrow R = 8$
 $x_L = \omega L \Rightarrow 2 \times 3.14 \times 50 \times 8$
 $\Rightarrow 800 \times 3.14$
 $R = 8$
 $\frac{x_L}{R} = 314$

49. A beam of light consisting of wavelengths 650 nm and 550 nm illuminates the Young's double slits with separation of 2 mm such that the interference fringes are formed on a screen, placed at a distance of 1.2 m from the slits. The least distance of a point from the central maximum, where the bright fringes due to both the wavelengths coincide, is _____ $\times 10^{-5}$ m.

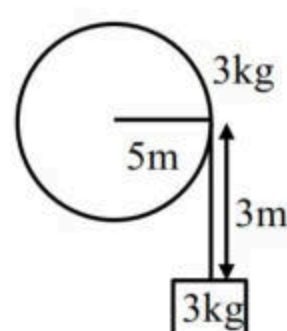
Ans. (429)

Sol. $y = n \frac{\lambda D}{d}$
 $y_1 = y_2$
 $n_1 \lambda_1 \frac{D}{d} = n_2 \lambda_2 \frac{D}{d}$
 $\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{550}{650} = \frac{11}{13}$
 $y = 11 \times \frac{\lambda_1 D}{d} = \frac{11 \times 650 \times 10^{-9} \times 1.2}{2 \times 10^{-3}}$
 $y = 429 \times 10^{-5}$

50. A fly wheel having mass 3 kg and radius 5 m is free to rotate about a horizontal axis. A string having negligible mass is wound around the wheel and the loose end of the string is connected to a 3 kg mass. The mass is kept at rest initially and released. Kinetic energy of the wheel when the mass descends by 3 m is _____ J. ($g = 10$ m/s²)

Ans. (30)

Sol.



$$mg \times 3 = \frac{1}{2} \cdot \frac{mR^2}{2} \omega^2 + \frac{1}{2} mv^2 \quad \dots(i)$$

$$\& v = \omega R \quad \dots(ii)$$

From equation (i) & (ii)

$$g \times 3 = \frac{3}{4} \cdot v^2$$

$$\text{K.E. of flywheel} = \frac{1}{2} \times \frac{mR^2}{2} \times \omega^2 = \frac{1}{4} mv^2$$

$$= \frac{1}{4} \times 3 \times 40 = 30 \text{ Joule}$$

CHEMISTRY

SECTION-A

51. Identify the **correct** statements :

The presence of $-\text{NO}_2$ group in benzene ring

A. activates the ring towards electrophilic substitutions.

B. deactivates the ring towards electrophilic substitutions.

C. activates the ring towards nucleophilic substitutions.

D. deactivates the ring towards nucleophilic substitutions.

(1) B and D Only

(2) C and A Only

(3) A and D Only

(4) B and C Only

Ans. (4)

Sol. Presence of NO_2 group in Benzene ring deactivate ring towards electrophilic substitution reaction due to $-\text{M}$ effective & activate ring towards nucleophilic substitution.

Ans. $\rightarrow$ (4) B & C

52. Given below are two statements :

Statement I : The increasing order of boiling point of hydrogen halides is $\text{HCl} < \text{HBr} < \text{HI} < \text{HF}$.

Statement II : The increasing order of melting point of hydrogen halides is $\text{HCl} < \text{HBr} < \text{HF} < \text{HI}$.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Ans. (1)

Sol. Correct order of

(i) Boiling point : $\text{HF} > \text{HI} > \text{HBr} > \text{HCl}$

(ii) Melting point : $\text{HI} > \text{HF} > \text{HBr} > \text{HCl}$

53. Consider the elements N, P, O, S, Cl and F. The number of valence electrons present in the elements with most and least metallic character from the above list is respectively.

(1) 7 and 5

(2) 5 and 6

(3) 5 and 7

(4) 6 and 7

Ans. (3)

Sol. Least metallic = F, valence electrons = 7

Most metallic = P, valence electrons = 5

54. Observe the following equilibrium in a 1 L flask.



At T(K), the equilibrium concentrations of A and B are 0.5 M and 0.375 M respectively. 0.1 moles of A is added into the flask and heated to T(K) to establish the equilibrium again. The new equilibrium concentrations (in M) of A and B are respectively.

(1) 0.367, 0.275

(2) 0.53, 0.4

(3) 0.742, 0.557

(4) 0.557, 0.418

Ans. (4)

Sol. $\text{A} \rightleftharpoons \text{B}$
0.5M 0.375 M (At equilibrium)

$$K_{\text{eq}} = \frac{[\text{B}]_{\text{eq}}}{[\text{A}]_{\text{eq}}} = \frac{0.375}{0.5} = 0.75$$

Now 0.1 mole of A is added so reaction will move in forward direction.

$$\begin{aligned} &\text{A} \rightleftharpoons \text{B} \\ &0.6-x \quad 0.375+x \\ K_{\text{eq}} = 0.75 &= \frac{0.375+x}{0.6-x} \\ 0.45 - 0.75x &= 0.375 + x \\ 1.75x &= 0.075 \end{aligned}$$

$$X = \frac{0.075}{1.75} = \frac{3}{70} = 0.043$$

Moles of A = 0.043 = 0.557

Moles of B = 0.418

Ans. (4) is correct.

55. The plot of $\log_{10} K$ vs $\frac{1}{T}$ gives a straight line. The intercept and slope respectively are (where K is equilibrium constant).

(1) $\frac{2.303R}{\Delta H^\circ}, \frac{2.303R}{\Delta S^\circ}$ (2) $\frac{\Delta S^\circ}{2.303R}, -\frac{\Delta H^\circ}{2.303R}$
 (3) $-\frac{\Delta S^\circ R}{2.303}, \frac{\Delta H^\circ R}{2.303}$ (4) $-\frac{\Delta H^\circ}{2.303R}, \frac{\Delta S^\circ}{2.303R}$

Ans. (2)

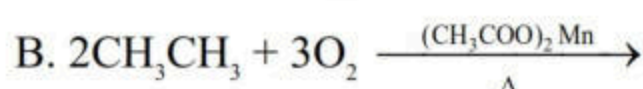
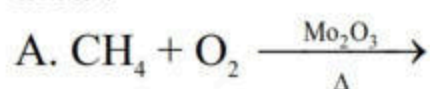
Sol. $\log_{10} K = -\frac{\Delta H^\circ}{2.303RT} + \frac{\Delta S^\circ}{2.303R}$

y-intercept = $\frac{\Delta S^\circ}{2.303R}$

Slope = $-\frac{\Delta H^\circ}{2.303R}$

Ans. (2) is correct.

56. The reactions which produce alcohol as the product are :

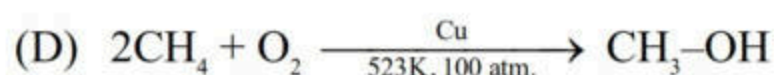
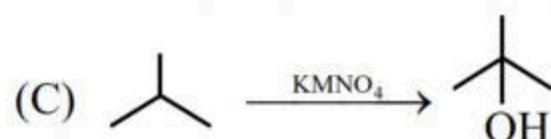
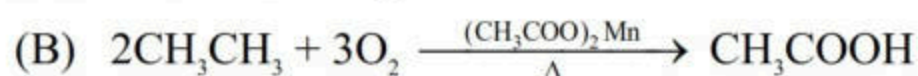
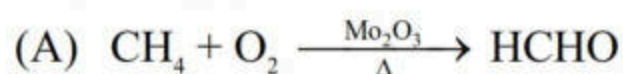


Choose the **correct** answer from the options given below :

- (1) A and D Only (2) A, C and E Only
 (3) C and D Only (4) B, D and E Only

Ans. (3)

Sol. Reaction given Alcohol



And. $\rightarrow$ (3) C, D

57. Consider the following statements about manganate and permanganate ions. Identify the **correct** statements :

- A. The geometry of both manganate and permanganate ions is tetrahedral.
 B. The oxidation states of Mn in manganate and permanganate are +7 and +6, respectively.
 C. Oxidation of Mn(II) salt by peroxodisulphate gives manganate ion as the final product.
 D. Manganate ion is paramagnetic and permanganate ions is diamagnetic.
 E. Acidified permanganate ion reduces oxalate, nitrite and iodide ions.

Choose the **correct** answer from the options given below:

- (1) A, C and D Only (2) A, B and C Only
 (3) A, D and E Only (4) A and D Only

Ans. (4)

Sol. Manganate ion $\rightarrow MnO_4^{2-}$

Permanganate ion $\rightarrow MnO_4^-$

(A) Both are tetrahedral (d^3 Hybridisation)

(B) MnO_4^- (+7 oxidation state)

MnO_4^{2-} (+6 oxidation state)

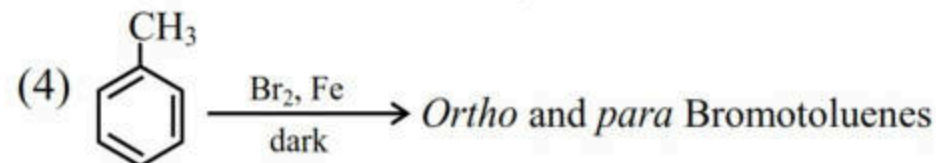
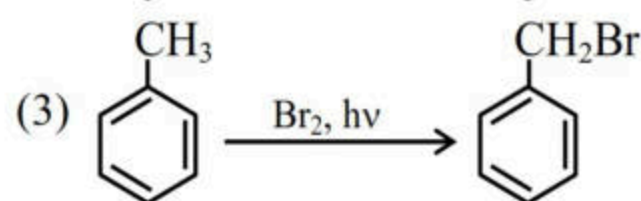
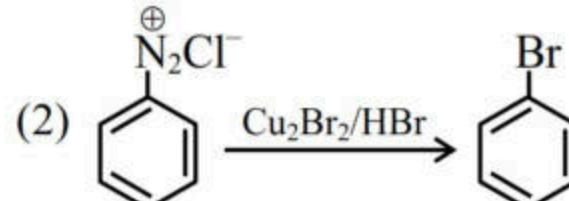
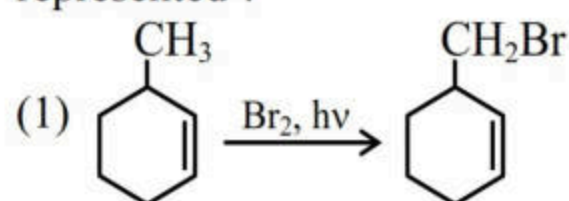
(C) $Mn^{2+} + S_2O_8^{2-} \rightarrow MnO_4^-$ (Permanganate ion)

(D) $MnO_4^- \rightarrow$ Diamagnetic

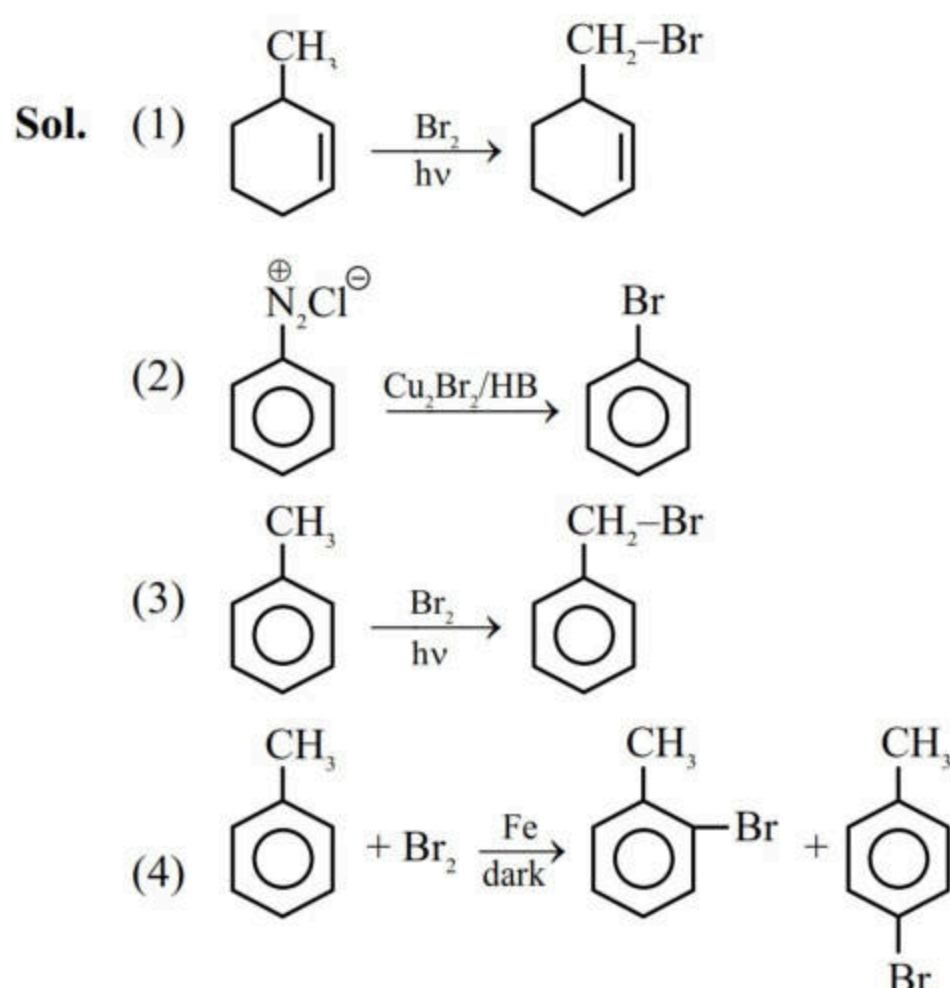
$MnO_4^{2-} \rightarrow$ Paramagnetic

(E) It is oxidising agent

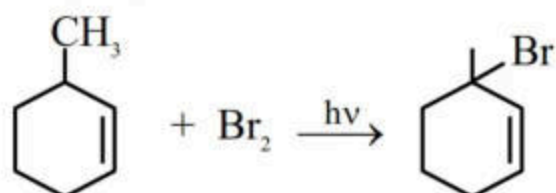
58. Which of the following reaction is NOT correctly represented ?



Ans. (1)



Major product of reaction (1) will be



As 3° radical more stable

Ans. (1)

- 59.** The wavelength of photon 'A' is 400 nm. The frequency of photon 'B' is 10^{16} s^{-1} . The wave number of photon 'C' is 10^4 cm^{-1} . The correct order of energy of these photons is :

- (1) $C > B > A$ (2) $B > A > C$
 (3) $A > B > C$ (4) $A > C > B$

Ans. (2)

Sol. (1) Wavelength of A = 400 nm.

$$(2) \text{ Wavelength of B } (\lambda) = \frac{c}{\nu} = \frac{3 \times 10^8}{10^{16}} \\ = 3 \times 10^{-8} = 30 \times 10^{-9} = 30 \text{ nm.}$$

$$(3) \text{ Wavelength of C } (\lambda) = \frac{1}{\bar{\nu}} = \frac{1}{10^4} = 10^{-4} \text{ cm} \\ = 10^{-6} \text{ m} = 1000 \text{ nm}$$

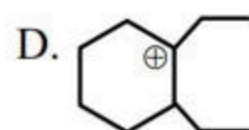
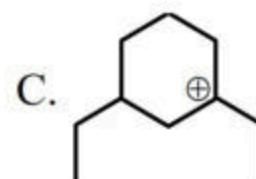
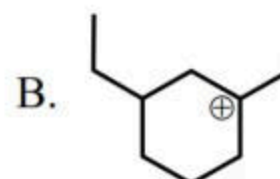
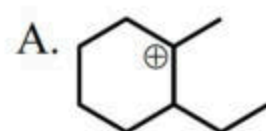
Here $\lambda_C > \lambda_A > \lambda_B$

$$\text{Energy}(E) \propto \frac{1}{\lambda}$$

So $E_B > E_A > E_C$

Ans. (2) is correct.

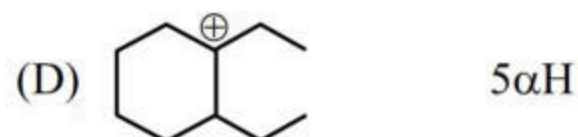
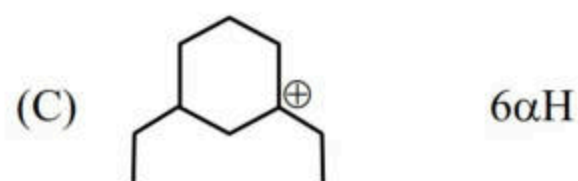
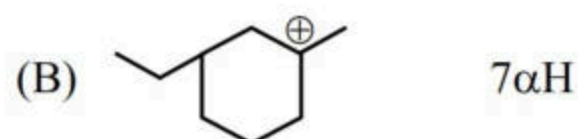
- 60.** The cyclic cations having the same number of hyperconjugation are :



Choose the **correct** answer from the options given below :

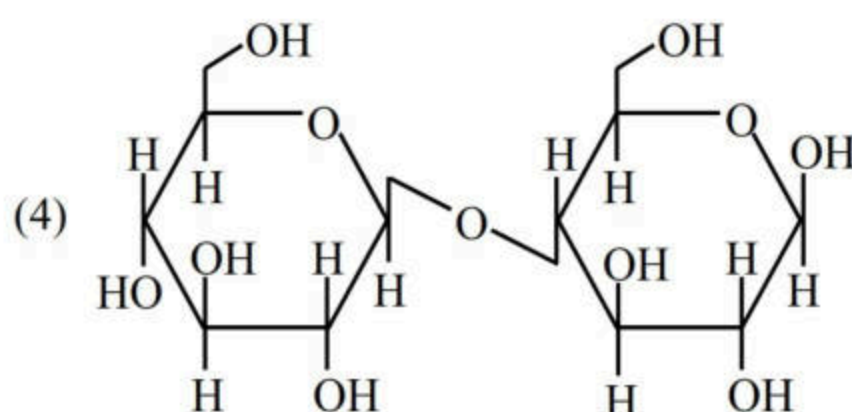
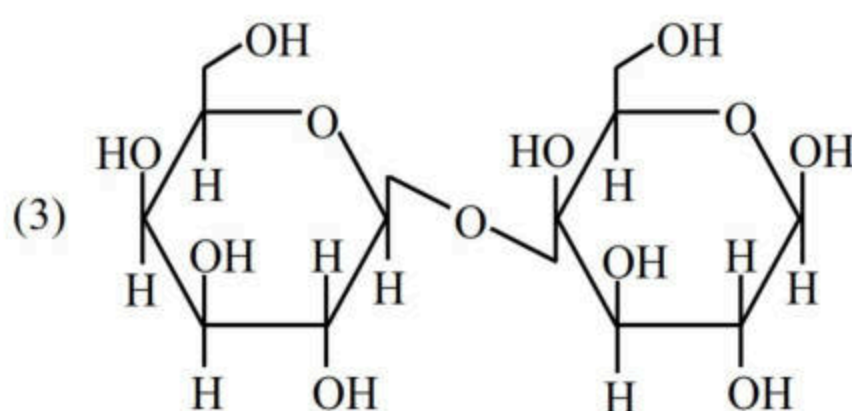
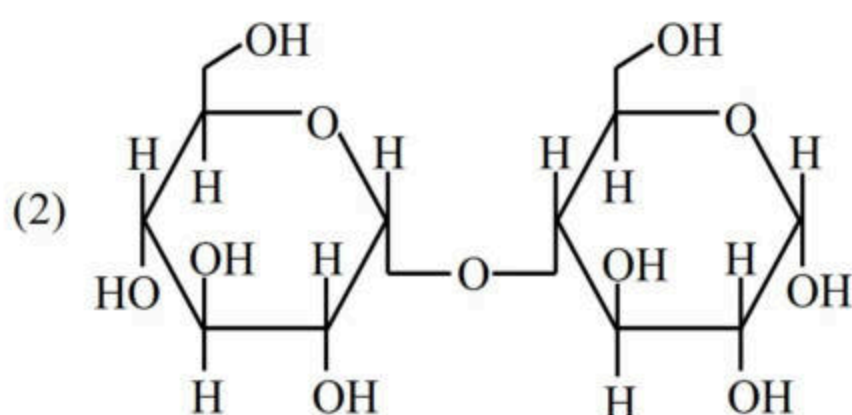
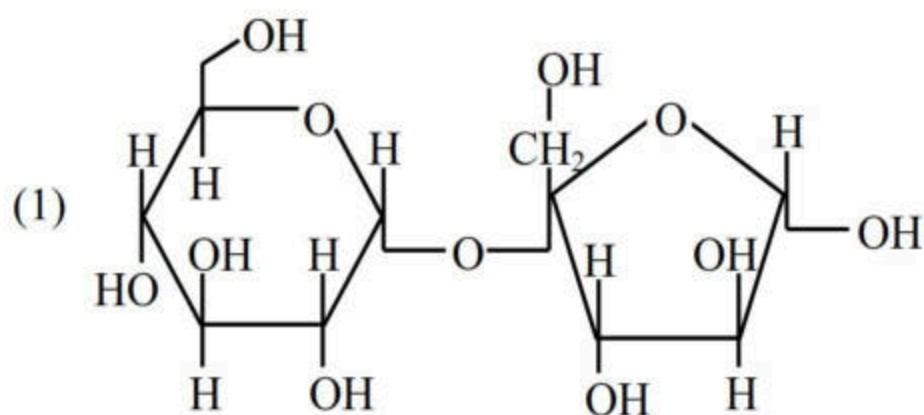
- (1) A and C Only
 (2) B and C Only
 (3) A and B Only
 (4) A, C and D only

Ans. (1)



Ans. – (1) A & C

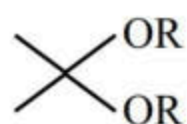
61. Structures of four disaccharides are given below. Among the given disaccharides, the non-reducing sugar is :



Ans. (1)

Sol. Structure (1) given is of sucrose which is non reducing.

For non reducing sugar compound should have acetal linkage not hemi acetal linkage.



62. Match List-I with List-II according to shape.

List-I	List-II
A. XeO_3	I. BrF_5
B. XeF_2	II. NH_3
C. XeO_2F_2	III. $[\text{I}_3]^-$
D. XeOF_4	IV. SF_4

Choose the **correct** answer from the options given below :

- (1) A-II, B-I, C-III, D-IV
 (2) A-II, B-III, C-IV, D-I
 (3) A-II, B-III, C-I, D-IV
 (4) A-III, B-II, C-IV, D-I

Ans. (2)

Sol. XeF_2 & I_3^- : 2 bond pair 3 lone pair ; Linear

XeOF_4 & BrF_5 : 5 bond pair 1 lone pair ; Square pyramidal

XeO_2F_2 & SF_4 : 4 bond pair 1 lone pair ; See saw

XeO_3 & NH_3 : 3 bond pair 1 lone pair ; Pyramidal

63. A student performed analysis of aliphatic organic compound 'X' which on analysis gave C = 61.01%, H=15.25%, N=23.74%.

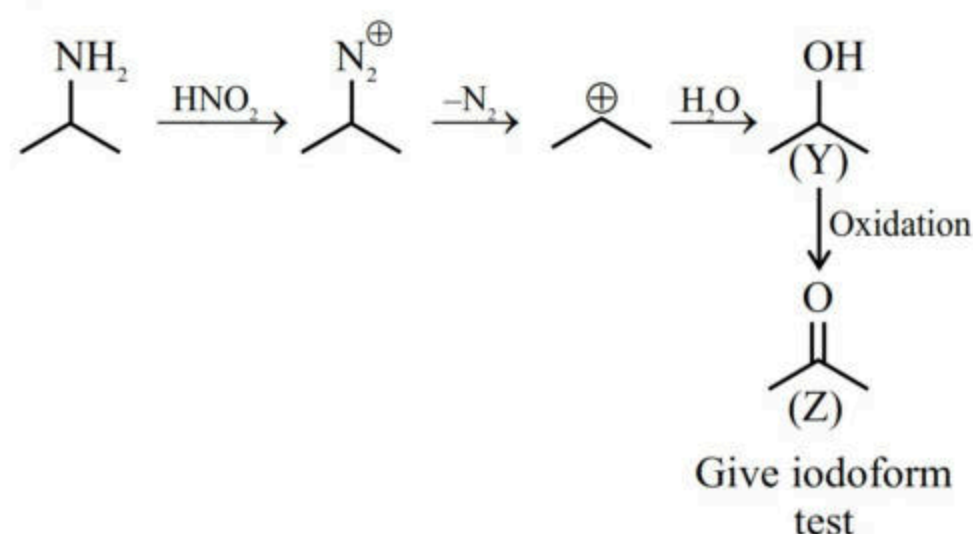
This compound, on treatment with $\text{HNO}_2/\text{H}_2\text{O}$ produced another compound 'Y' which did not contain any nitrogen atom. However, the compound 'Y' upon controlled oxidation produced another compound 'Z' that responded to iodoform test.

The structure of 'X' is:

- (1) $\text{CH}_3\text{CH}_2\text{CH}_2\text{NH}_2$ (2) $\text{Ph}-\underset{\text{CH}_3}{\text{CH}}-\text{NH}_2$
 (3) $\text{CH}_3-\underset{\text{CH}_3}{\text{CH}}-\text{NH}_2$ (4) $\text{CH}_3-\text{CH}_2\underset{\text{NH}_2}{\text{CH}}-\text{CH}_3$

Ans. (3)

Sol.



64. Consider the following aqueous solutions.
 I. 2.2 g Glucose in 125 mL of solution.
 II. 1.9 g Calcium chloride in 250 mL of solution.
 III. 9.0 g Urea in 500 mL of solution.
 IV. 20.5 g Aluminium sulphate in 750 mL of solution.

The **correct** increasing order of boiling point of these solutions will be:

[Given: Molar mass in g mol^{-1} : H=1, C=12, N=14, O=16, Cl=35.5, Ca=40, Al=27 and S=32]

- (1) I < II < III < IV (2) III < I < II < IV
 (3) II < III < I < IV (4) II < III < IV < I

Ans. (1)

Sol. $\Delta T_b = i \cdot k_b \cdot m$

For dilute solution ($M = m$)

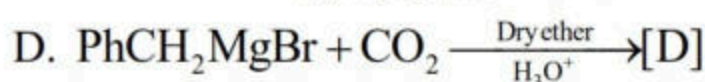
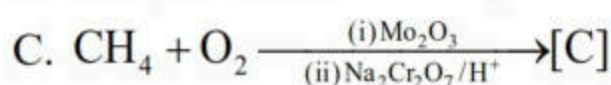
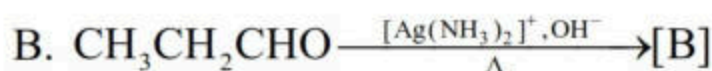
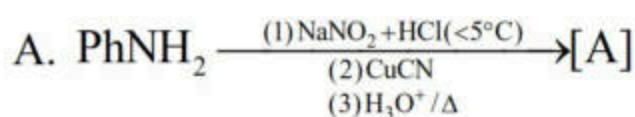
Molarity	$i \times m$
(I) $M_{\text{glucose}} = \frac{2.2}{180} \times \frac{1000}{125} = 0.098$	0.098×1
(II) $M_{\text{CaCl}_2} = \frac{1.9}{111} \times \frac{1000}{250} = 0.068$	0.068×3
(III) $M_{\text{urea}} = \frac{9}{60} \times \frac{1000}{500} = 0.3$	0.3×1
(IV) $M_{\text{Al}_2(\text{SO}_4)_3} = \frac{20.5}{342} \times \frac{1000}{750} = 0.08$	0.08×5

Order of $\Delta T_b = \text{Al}_2(\text{SO}_4)_3 > \text{Urea} > \text{CaCl}_2 > \text{Glucose}$

So order of BP = $\text{Al}_2(\text{SO}_4)_3 > \text{Urea} > \text{CaCl}_2 > \text{Glucose}$

So Answer will be I < II < III < IV

65. The correct order of acidic strength of the major products formed in the given reactions, is :

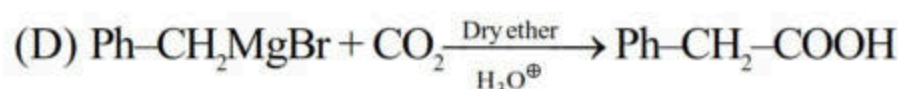
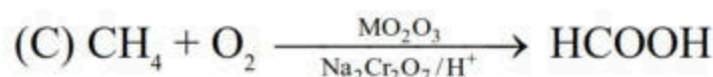
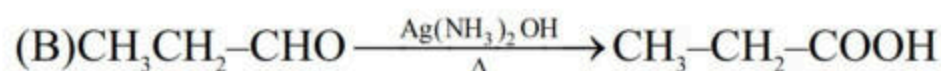
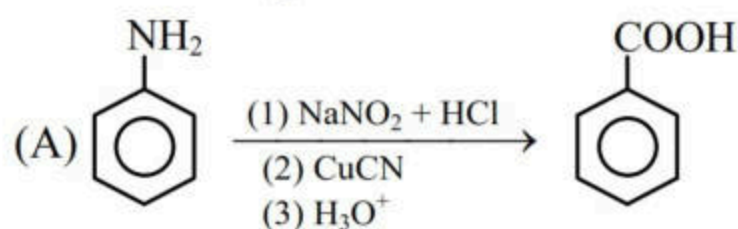


Choose the **correct** answer from the options given below :

- (1) C > B > A > D (2) A > D > C > B
 (3) A > D > B > C (4) C > A > D > B

Ans. (4)

- Sol. Correct order of acidic strength of major product formed in the given reaction is



Ans. (4) C > A > D > B

66. Total number of alkali insoluble solid sulphonamides obtained by reaction of given amines with Hinsberg's reagent is

Aniline, N-Methylaniline, Methanamine,

N, N-Dimethylmethanamine,

N-Methyl methanamine, Phenylmethanamine,

N-propylaniline, N-phenylaniline,

N, N-Dimethylaniline, Allyl amine,

Isopropyl amine

- (1) 4 (2) 2
 (3) 8 (4) 5

Ans. (1)

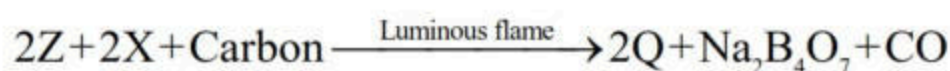
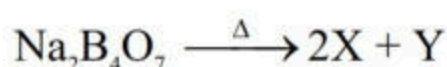
- Sol. 2° Amine are insoluble with hinsberg reagent.

Ph-NH-CH_3 , Me-NH-Me ,

$\text{Ph-NH-CH}_2\text{-CH}_2\text{-CH}_3$, Ph-NH-Ph

Ans. (1) 4

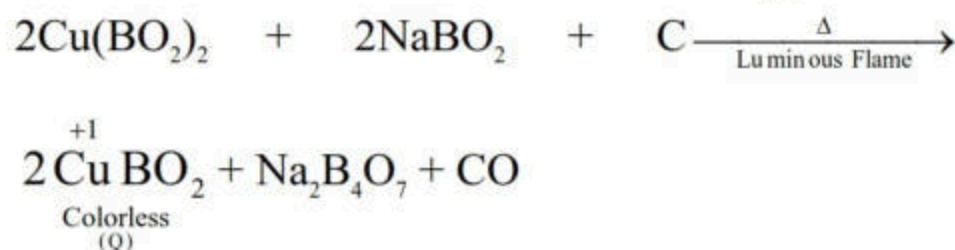
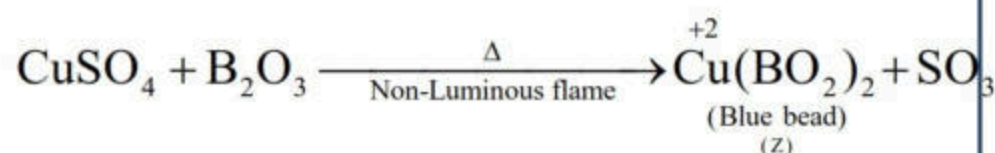
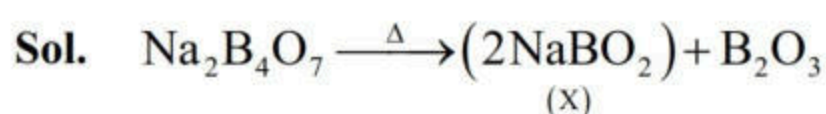
67. Consider the following reactions.



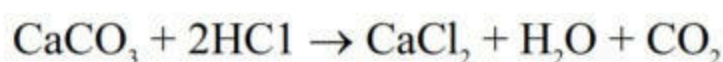
The oxidation states of Cu in Z and Q, respectively are :

- (1) +2 and +2 (2) +2 and +1
 (3) +1 and +2 (4) +1 and +1

Ans. (2)



68. For the given reaction;



If 90 g CaCO_3 is added to 300 mL of HCl which contains 38.55% HCl by mass and has density 1.13 g mL^{-1} , then which of the following option is correct?

Given molar mass of H, Cl, Ca and O are 1, 35.5, 40 and 16 g mol^{-1} respectively.

- (1) 64.97 g of HCl remains unreacted
- (2) 32.85 g of CaCO_3 remains unreacted
- (3) 97.30 g of HCl reacted
- (4) 60.32 g of HCl remains unreacted

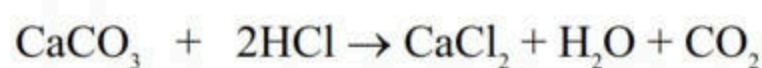
Ans. (1)

Sol. Density of HCl solution (d) = 1.13 g/mL
V = 300 mL

Wt. of HCl solution = 339 g

$$\text{Wt. of HCl} = 339 \times \frac{38.55}{100} = 130.68 \text{ g}$$

(LR)



$$\frac{90}{100} \quad \frac{130.68}{36.5}$$

$$= 0.90 \text{ mole} \quad = 3.58 \text{ mole}$$

Moles of HCl remained = 1.78 mole.

Mass of HCl remained = 64.97 g.

69. The correct increasing order of spin-only magnetic moment values of the complex ions $[\text{MnBr}_4]^{2-}$ (A), $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$ (B), $[\text{Ni}(\text{CN})_4]^{2-}$ (C) and $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$ (D) is:

$$(1) A = B < C < D \quad (2) A = B < D < C$$

$$(3) C = D < B < A \quad (4) C < B < D < A$$

Ans. (4)

Sol. $\text{Mn}^{2+} 3d^5 n = 5$

$$\text{Cu}^{2+} 3d^9 t_{2g}^{2,2,2} e_g^{2,1} n = 1$$

$$\text{Ni}^{2+} 3d^8 \text{ square planar } n = 0$$

$$\text{Ni}^{2+} 3d^8 \text{ tetrahedral } e^{2,2} t_2^{2,1,1} n = 2$$

70. A student has been given 0.314 g of an organic compound and asked to estimate Sulphur. During the experiment, the student has obtained 0.4813 g of barium sulphate. The percentage of sulphur present in the compound is _____.

(Given Molar mass in g mol^{-1} S:32, BaSO_4 : 233)

$$(1) 42.10\% \quad (2) 63.15\%$$

$$(3) 21.05\% \quad (4) 48.24\%$$

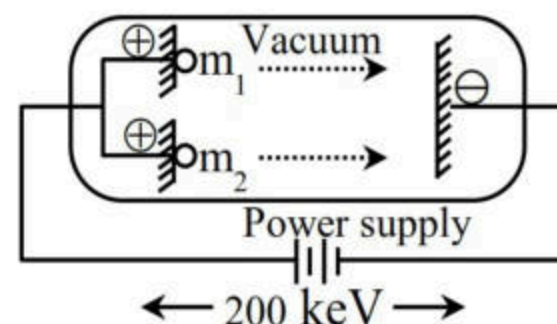
Ans. (3)

$$\begin{aligned} \text{Sol. } \%S &= \frac{32}{233} \times \frac{0.4813}{0.314} \times 100 \\ &= 21.052\% \end{aligned}$$

Ans. (3) 21.05%

SECTION-B

71. Two positively charged particles m_1 and m_2 have been accelerated across the same potential difference of 200 keV as shown below.



[Given mass of $m_1 = 1 \text{ amu}$ and $m_2 = 4 \text{ amu}$]

The deBroglie wavelength of m_1 will be x times of m_2 . The value of x is _____. (nearest integer)

Ans. (2)

Sol. $\lambda_d = \frac{h}{\sqrt{2mK.E.}}$

Here KE is same i.e. 200 k eV

So $\lambda_d \propto \frac{1}{\sqrt{m}}$

$$\frac{(\lambda_d)_{m_1}}{(\lambda_d)_{m_2}} = \sqrt{\frac{m_2}{m_1}} = \sqrt{4} = 2$$

$$(\lambda_d)_{m_1} = 2(\lambda_d)_{m_2}$$

So $x = 2$.

72. $A \rightarrow B$ (first reaction)

$C \rightarrow D$ (second reaction)

Consider the above two first-order reactions. The rate constant for first reaction at 500 K is double of the same at 300 K. At 500 K, 50% of the reaction becomes complete in 2 hour. The activation energy of the second reaction is half of that of first reaction. If the rate constant at 500 K of the second reaction becomes double of the rate constant of first reaction at the same temperature; then rate constant for the second reaction at 300 K is $\times 10^{-1} \text{ hour}^{-1}$ (nearest integer).

Ans. (5)

Sol. For $A \xrightarrow{K_1} B$

$$\ln(2) = \frac{E_{a_1}}{R} \left[\frac{1}{300} - \frac{1}{500} \right]$$

$$E_{a_1} = \frac{\ln 2 \times R \times 1500}{2}$$

$$E_{a_2} = \frac{E_{a_1}}{2} = \frac{\ln 2 \times R \times 1500}{4}$$

$$(K_1)_{\text{at } 500 \text{ K}} = \frac{\ln 2}{2}$$

$$(K_2)_{\text{at } 500 \text{ K}} = \ln 2$$

Now for $C \xrightarrow{K_2} D$

$$\ln \left[\frac{(K_2)_{\text{at } 500 \text{ K}}}{(K_2)_{\text{at } 300 \text{ K}}} \right] = \left(\frac{\ln 2 \times R \times 1500}{4} \right) \times \frac{1}{R} \times \left[\frac{1}{300} - \frac{1}{500} \right]$$

$$(K_2)_{\text{at } 300 \text{ K}} = \frac{\ln 2}{\sqrt{2}} = 0.49$$

$$(K_2)_{\text{at } 300 \text{ K}} = 4.9 \times 10^{-1}$$

Ans is 5.

73. For strong electrolyte Λ_m increases slowly with dilution and can be represented by the equation

$$\Lambda_m = \Lambda_m^\circ - A c^{1/2}$$

Molar conductivity values of the solutions of strong electrolyte AB at 18°C are given below :

$c [\text{mol L}^{-1}]$	0.04	0.09	0.16	0.25
$\Lambda_m [\text{S cm}^2 \text{mol}^{-1}]$	96.1	95.7	95.3	94.9

The value of constant A based on the above data [in $\text{S cm}^2 \text{mol}^{-1}/(\text{mol/L})^{1/2}$] unit is _____.

Ans. (4)

Sol. Using equation : $\Lambda_m = \Lambda_m^\circ - A \sqrt{c}$

$$96.1 = \Lambda_m^\circ - A \sqrt{0.04}$$

$$96.1 = \Lambda_m^\circ - A \times 0.2 \quad \dots\dots\dots(1)$$

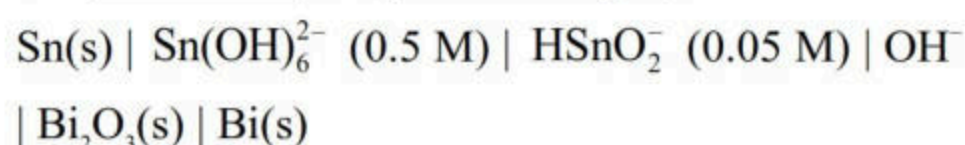
$$95.7 = \Lambda_m^\circ - A \times \sqrt{0.09}$$

$$95.7 = \Lambda_m^\circ - A \times 0.3 \quad \dots\dots\dots(2)$$

From eq. (1) and eq. (2)

$$A = 4$$

74. A volume of x mL of 5 M NaHCO_3 solution was mixed with 10 mL of 2 M H_2CO_3 solution to make an electrolytic buffer. If the same buffer was used in the following electrochemical cell to record a cell potential of 235.3 mV, then the value of x = _____ mL (nearest integer).



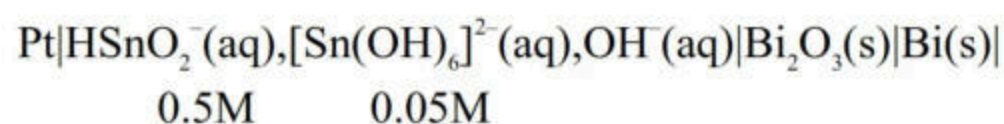
Consider upto one place of decimal for intermediate calculations

$$\left[\begin{array}{l} \text{Given :} \quad E_{\text{HSnO}_2^- \mid \text{Sn(OH)}_6^{2-}}^\circ = -0.9 \text{ V} \\ E_{\text{Bi}_2\text{O}_3 \mid \text{Bi}}^\circ = -0.44 \text{ V} \\ \text{pK}_{\text{a}}(\text{H}_2\text{CO}_3) = 6.11 \\ \frac{2.303 RT}{F} = 0.059 \text{ V} \\ \text{Anti log}(1.29) = 19.5 \end{array} \right]$$

Ans. (78)

Sol. We have considered

$$E_{[\text{Sn}(\text{OH})_6]^{2-}/\text{HSnO}_2^-}^{\circ} = -0.9 \text{ V}$$

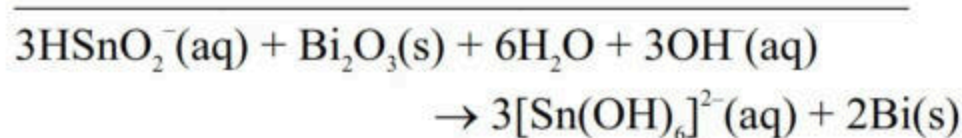
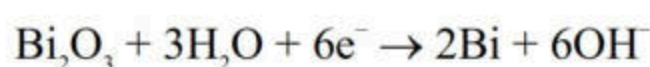


$$E_{\text{cell}}^{\circ} = +0.9 - 0.44 = 0.46 \text{ V}$$

Oxidation Half :



Reduction Half :



$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{0.059}{6} \log \frac{(0.5)^3}{(0.05)^3 \times [\text{OH}^-]^3}$$

$$0.2353 = 0.46 - \frac{0.059}{6} \times 3 \log \left[\frac{10}{[\text{OH}^-]} \right]$$

$$\log \left[\frac{10}{[\text{OH}^-]} \right] = \frac{2 \times 0.2247}{0.059} = 7.6$$

$$1 + \text{pOH} = 7.6$$

$$\text{pOH} = 6.6$$

$$\text{pH} = 14 - 6.6 = 7.4$$

$$\text{pH} = \text{pK}_{a1} + \log \frac{[\text{HCO}_3^-]}{[\text{H}_2\text{CO}_3]}$$

$$7.4 = 6.11 + \log \frac{5x}{20}$$

$$1.29 = \log \frac{x}{4}$$

$$\frac{x}{4} = 19.5$$

$$x = 78$$

Note : In question paper, $E_{\text{HSnO}_2^-/[\text{Sn}(\text{OH})_6]^{2-}}^{\circ} = -0.9 \text{ V}$

data is given, but NTA has given answer by considering $E_{[\text{Sn}(\text{OH})_6]^{2-}/\text{HSnO}_2^-}^{\circ} = -0.9 \text{ V}$ therefore this question should be **BONUS**.

75. The number of isoelectronic species among Sc^{3+} , Cr^{2+} , Mn^{3+} , Co^{3+} and Fe^{3+} is 'n'. If 'n' moles of AgCl is formed during the reaction of complex with formula $\text{CoCl}_3(\text{en})_2\text{NH}_3$ with excess of AgNO_3 solution, then the number of electrons present in the t_{2g} orbital of the complex is _____.

Ans. (6)

Sol.

Sc^{+3}	18
Cr^{+2}	22
Mn^{+3}	22
Co^{+3}	24
Fe^{+3}	23

Cr^{2+} and Mn^{3+} are isoelectronic

$$n = 2$$

Complex is : $[\text{Co}(\text{en})_2\text{NH}_3\text{Cl}]\text{Cl}_2$

