

JEE (Main)-2026

Session-1

28 January 2026 Shift-2

Memory-Based Answers & Solutions (Physics, Chemistry, and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
- (4) Section - A : Attempt all questions.
- (5) Section - B : Attempt all questions.
- (6) Section - A (01 – 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.
- (7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

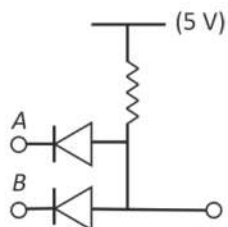
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. For the circuit given below, identify the logic gate.



- (1) AND
(2) OR
(3) NAND
(4) NOR

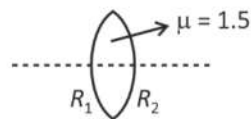
Answer (1)

Sol. Truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Clearly AND Gate.

2. Object is placed at distance 30 cm from lens given below, then distance of image from lens is ($R_1 = 10$ cm, $R_2 = 20$ cm)



- (1) 36 cm
(2) 24 cm
(3) 20 cm
(4) 30 cm

Answer (2)

$$\text{Sol. } \frac{1}{f} = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{3}{40}$$

$$\frac{1}{v} - \frac{1}{-30} = \frac{3}{40}$$

$$\Rightarrow \frac{1}{v} = \frac{5}{120}$$

$$v = 24 \text{ cm}$$

3. The position vector is given as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and if its signs reversed then which of the following physical quantity remains unaffected?

- (1) Acceleration
(2) Velocity
(3) Displacement
(4) Torque

Answer (4)

$$\text{Sol. } \vec{r} \times \vec{F} \equiv -\vec{r} \times -\vec{F} \equiv \vec{\tau}$$

4. If the mass number of nucleus is α , its radius is R_α . And another mass number of β then its radius is R_β ; then

$$\frac{R_\alpha}{R_\beta} = ? \quad [\text{Given } \beta = 8\alpha]$$

- (1) 8 (2) $\frac{1}{2}$
(3) 1 (4) 2

Answer (2)

Sol. We know $R = R_0(A)^{1/3}$

$$\text{So, } R_\alpha = R_0(\alpha)^{1/3}$$

$$R_\beta = R_0(\beta)^{1/3}$$

$$\text{So, } \frac{R_\alpha}{R_\beta} = \left[\frac{\alpha}{\beta} \right]^{1/3} = \left(\frac{1}{8} \right)^{1/3} = \frac{1}{2}$$

5. Which of following physical quantity is not measurable?

- (1) Displacement (2) Voltage
(3) Voltage difference (4) Acceleration

Answer (2)

Sol. Voltage difference or EMF difference, $\Delta\varepsilon = -\int_1^2 \vec{E} \cdot d\vec{l}$ can

be measured. But to define voltage we need a reference point, but as being non-conservative field that will give inconsistent result.

6. Two light sources of 450 nm and 550 nm are used for YDSE with slit distance 2.25 mm and distance between the slits and screen is 1.5 m. Then the distance from central maxima at which minima of both wavelength coincide?

- (1) 1.65 mm (2) 1.20 mm
(3) 1.30 mm (4) 1.40 mm

Answer (1)

Sol. $\frac{\lambda_1}{\lambda_2} = \frac{2m-1}{2n-1} = \frac{9}{11}$

5th maximum of 550 nm coincides will 11th maxima of 450 nm.

$$y = \frac{9 \lambda D}{2 d} = \frac{9}{2} \times \frac{550 \times 10^{-9}}{2.25 \times 10^{-3}} \times \frac{3}{2} = 1.650$$

7. A beam of power 2 μ W is hitting a metal surface beam contains photons of wavelength 662 nm. Find number of photons striking per second.

- (1) 2×10^{14} (2) 6.67×10^{12}
(3) 4×10^{11} (4) 3.2×10^{13}

Answer (2)

Sol. $n = \frac{P}{\text{energy of one photons}}$

$$= \frac{2 \times 10^{-6}}{hc/\lambda}$$

$$= \frac{2 \times 10^{-6} \times 662 \times 10^{-3}}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

$$N = 6.67 \times 10^{12}$$

8. Mean free path of gas particles of diameter 5 Å at temperature and pressure of 41°C and 1.38×10^5 Pa.

- (1) 14.14 nm (2) 20 nm
(3) 28.28 nm (4) 10 nm

Answer (3)

Sol. $\lambda = \frac{1kT}{\sqrt{2}\pi d^2 P}$ and $P = nkT$

$$\lambda = \frac{1.38 \times 10^{-23} \times 3.14}{1.414 \times 3.14 \times 25 \times 10^{-20} \times 1.38 \times 10^5}$$

$$= \frac{\sqrt{2}}{2 \times 25} \times 10^{-21+20-5}$$

9. Find the percentage error in K , where $T = 2\pi\sqrt{\frac{m}{k}}$. Given that 60 oscillations completes in 50 second. Time resolution is 2 second, $m = 10 \text{ g}$ and $\Delta m = \pm 10 \text{ mg}$.

- (1) 8% (2) 9%
(3) 9.1% (4) 8.1%

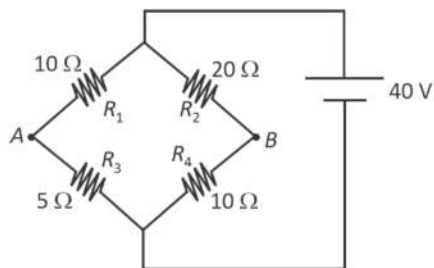
Answer (4)

Sol. $\frac{\Delta T}{T} = \frac{2}{50}$ and $\frac{\Delta m}{m} = 10^{-3}$

$$\Rightarrow \frac{2\Delta T}{T} + \frac{\Delta m}{m} = \frac{\Delta k}{k}$$

$$\Rightarrow \left(\frac{4}{50} + 10^{-3} \right) \times 100 = 8.1\%$$

10. In a balanced wheatstone bridge $R_2 R_3 : R_1 R_4$. Because of heating R_3 increases by 20%. Then potential difference across A & B becomes

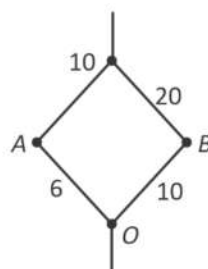


- (1) 1.50 V (2) 2.40 V
(3) 1.67 V (4) 3.60 V

Answer (3)

Sol. $V_A = \frac{40}{16} \times 6$

$$V_B = \frac{40}{3}$$



$$\begin{aligned} \Delta V &= 40 \times \frac{3}{8} - \frac{40}{3} \\ &= 40 \left\{ \frac{3}{8} - \frac{1}{3} \right\} \\ &= \frac{5}{3} \end{aligned}$$

11. Match the two columns and choose the correct option.

	Column-I		Column-II
(a)	Coefficient of viscosity	(p)	$[ML^0T^{-2}]$
(b)	Surface tension	(q)	$[ML^{-1}T^{-2}]$
(c)	Pressure	(r)	$[ML^2T^{-2}]$
(d)	Work	(s)	$[ML^{-1}T^{-1}]$

- (1) (a)-(p), (b)-(q), (c)-(r), (d)-(s)
(2) (a)-(s), (b)-(p), (c)-(q), (d)-(r)
(3) (a)-(q), (b)-(s), (c)-(p), (d)-(r)
(4) (a)-(p), (b)-(q), (c)-(s), (d)-(r)

Answer (2)

Sol. For (a) $F = -\eta A \frac{dv}{dx}$

(b) $T = \frac{F}{L}$

(c) $P = \frac{F}{A}$

(d) $W = \vec{F} \cdot \vec{r}$

12. Given that $v = \sqrt{\frac{Y}{P}}$. Find the maximum % error in v .

Given that $\frac{\Delta Y}{Y} \times 100 = 1\%$ and $\frac{\Delta P}{P} \times 100 = 0.5\%$

- (1) $\frac{3}{2}\%$
 (2) $\frac{3}{4}\%$
 (3) 1%
 (4) $\frac{1}{2}\%$

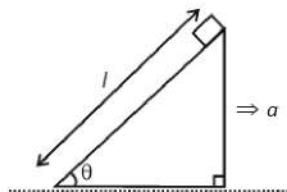
Answer (2)

Sol. $\frac{\Delta v}{v} = \frac{1}{2} \frac{\Delta Y}{Y} + \frac{1}{2} \frac{\Delta P}{P}$

So $\left(\frac{\Delta v}{v} \times 100 \right) = \frac{1}{2} \left(\frac{\Delta Y}{Y} \right) \times 100 + \frac{1}{2} \left(\frac{\Delta P}{P} \times 100 \right)$

$\Rightarrow \left(\frac{\Delta v}{v} \times 100 \right) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}\%$

13. Find the time taken by block to reach the ground when released from a top of a wedge of inclination θ and length l , maintained at a constant acceleration ' a ' to the right as shown. (all contacts case smooth and block doesn't lose contact with wedge).



(1) $t = \sqrt{\frac{l}{g \cos^2 \theta + a \sin \theta}}$

(2) $t = \sqrt{\frac{l}{g \sin \theta + a \cos \theta}}$

(3) $t = \sqrt{\frac{2l}{g \sin \theta + a \cos \theta}}$

(4) $t = \sqrt{\frac{l}{2g \cos \theta + a \sin \theta}}$

Answer (3)

Sol. Applying pseudo force on block and resolving forces along incline we get $A = g \sin \theta + a \cos \theta$.

$\Rightarrow \frac{1}{2} A t^2 = l$

$\Rightarrow t = \sqrt{\frac{2l}{g \sin \theta + a \cos \theta}}$

14.
 15.
 16.
 17.
 18.
 19.
 20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21.
 22.
 23.
 24.
 25.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The plot of $\log_{10}k$ vs $\frac{1}{T}$ gives a straight line. The intercept and slope respectively are

(1) $c = \log A$

$$m = -\frac{E_a}{2.303R}$$

(2) $c = -\frac{E_a}{2.303R}$

$m = \log A$

(3) $c = -\log A$

$$m = -\frac{E_a}{2.303R}$$

(4) $c = \log A$

$$m = \frac{E_a}{2.303R}$$

Answer (1)

Sol. $k = Ae^{-\frac{E_a}{RT}}$

$$\log k = \log A - \frac{E_a}{2.303RT}$$

$y = c + mx$

$c = \text{intercept} = \log A$

$m = \text{slope} = -\frac{E_a}{2.303R}$

2. Consider the following electromagnetic waves :

Wavelength of A = 400 nm

Frequency of B = 10^{16} s^{-1}

Wave number of C = 10^4 cm^{-1}

Order of energies is

(1) $A > B > C$

(2) $B > A > C$

(3) $B > C > A$

(4) $C > A > B$

Answer (2)

Sol. $\lambda_A = 400 \times 10^{-9} \text{ m} = 4 \times 10^{-7} \text{ m}$

$\lambda_B = 3 \times 10^{-8} \text{ m}$

$\lambda_C = 10^{-6} \text{ m}$

$\lambda_C > \lambda_A > \lambda_B$

Energy order will be opposite.

3. Which of the following order is correct.

(1) $\text{HF} > \text{HI} > \text{HBr} > \text{HCl}$ (Boiling point)

(2) $\text{HF} > \text{HI} > \text{HBr} > \text{HCl}$ (Melting point)

(3) $\text{HI} > \text{HF} > \text{HBr} > \text{HCl}$ (Boiling point)

(4) $\text{HI} > \text{HBr} > \text{HF} > \text{HCl}$ (Melting point)

Answer (1)

Sol.	HF	HCl	HBr	HI
B.P.	293	189	206	238
(K)				
MP	190	159	185	222
(K)				

4. Match the isostructural species

	Column-I		Column-II
(a)	XeO_3	(p)	BrF_5
(b)	XeF_2	(q)	NH_3
(c)	XeO_2F_2	(r)	I_3^-
(d)	XeOF_4	(s)	SF_4

(1) $a - q, b - r, c - s, d - p$

(2) $a - p, b - q, c - s, d - p$

(3) $a - q, b - r, c - p, d - s$

(4) $a - p, b - q, c - r, d - s$

Answer (1)**Sol.** XeO_3 and NH_3 are trigonal pyramidal XeF_2 and I_3^- are linear XeO_2F_2 and SF_4 are see-saw XeOF_4 and BrF_5 are square pyramidal

5. Consider a reaction $\text{A} \rightleftharpoons \text{B}$. At 'T' K, the equilibrium concentration of A and B are 0.3 M and 0.315 M. Now, 0.1 mol of A is added to the flask of 1 L, then equilibrium constant and equilibrium concentration of B are

- (1) 1.05, 0.35 M (2) 0.95, 0.37 M
 (3) 1.05, 0.37 M (4) 0.95, 0.35 M

Answer (3)

Sol.

A	$\rightleftharpoons$	B
$t_{\text{eq}} \quad 0.3 \text{ M}$		0.315 M

$$K_{\text{eq}} = \frac{0.315}{0.3}$$

Now 0.1 mole of A is added,

A	$\rightleftharpoons$	B
$t = 0 \quad 0.3 + 0.1 = 0.4$		
$t = t_{\text{eq}} \quad 0.4 - x$		$0.315 + x$

$$1.05 = \frac{0.315 + x}{0.4 - x}$$

$$1.05 \times 0.4 - 1.05x = 0.315 + x$$

$$0.42 - 0.315 = 2.05x$$

$$0.105 = 2.05x$$

$$x = 0.051$$

$$[\text{B}] = 0.366 \text{ M}$$

6. Diamagnetic species among the following complexes is

- (1) $[\text{MnBr}_4]^{2-}$
 (2) $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$
 (3) $[\text{Ni}(\text{CN})_4]^{2-}$
 (4) $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$

Answer (3)**Sol.** $[\text{MnBr}_4]^{2-} \Rightarrow \text{Mn}^{2+} \Rightarrow 3d^5$ $\Rightarrow sp^3$ hybridised $\Rightarrow$ paramagnetic ($n = 5$) $[\text{Cu}(\text{H}_2\text{O})_6]^{2+} \Rightarrow \text{Cu}^{2+} \Rightarrow 3d^9$ $\Rightarrow sp^3d^2$ hybridised $\Rightarrow$ paramagnetic ($n = 1$) $[\text{Ni}(\text{CN})_4]^{2-} \Rightarrow \text{Ni}^{2+} \Rightarrow 3d^8$ $\Rightarrow dsp^2$ hybridised $\Rightarrow$ diamagnetic ($n = 0$) $[\text{Ni}(\text{H}_2\text{O})_6]^{2+} \Rightarrow \text{Ni}^{2+} \Rightarrow 3d^8$ $\Rightarrow sp^3d^2$ hybridised $\Rightarrow$ paramagnetic ($n = 2$)

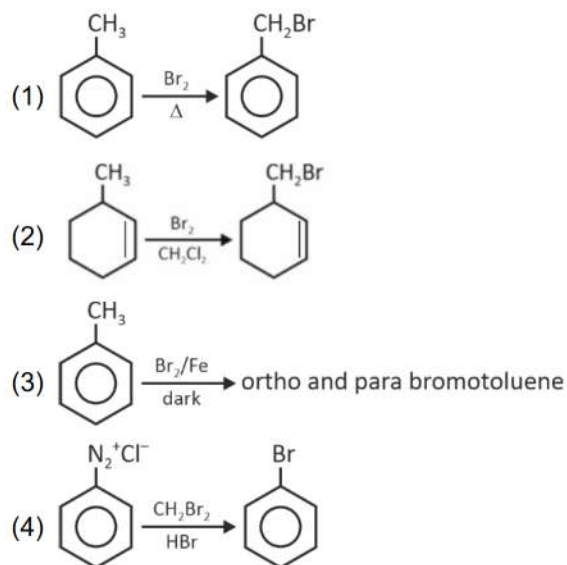
7. Correct statement about $-\text{NO}_2$ group is

- (A) Ring deactivating group in electrophilic substitution
 (B) Ring activating group in electrophilic substitution
 (C) Activating for aromatic nucleophilic substitution in aryl halides
 (D) Deactivating for aromatic nucleophilic substitution in aryl halides.

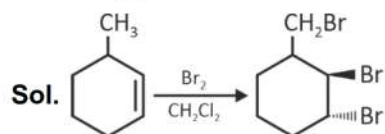
- (1) A, C are correct statement
 (2) B, D are correct
 (3) A, D are correct
 (4) B, C are correct

Answer (1)**Sol.** It shows $-\text{R}$ and $-\text{I}$ effect.

8. The major organic product of which of the following reaction is incorrectly represented?



Answer (2)



9. Consider the following reaction sequence

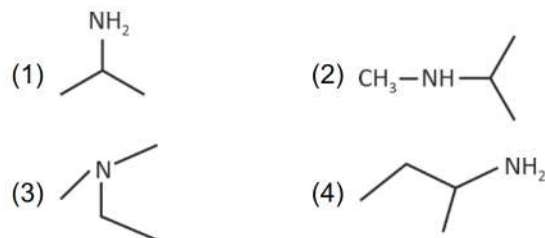


X has %C = 65.75

%H = 15.25

%N = 19

Identify (X)



Answer (4)

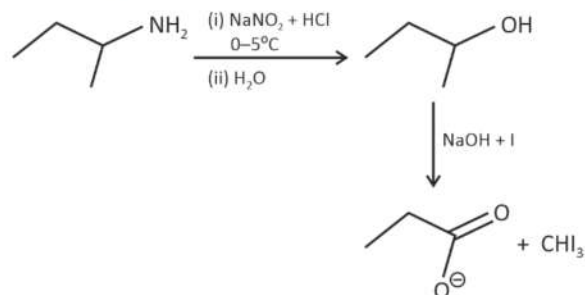
Sol. mole molar ratio

$$C = 65.75 \Rightarrow \frac{65.75}{12} = 5.5 = 4$$

$$H = 15.25 \Rightarrow \frac{15.25}{1} = 15.25 = 11$$

$$N = 19 \Rightarrow \frac{19}{14} = 1.35 = 1$$

$$MF = C_4H_{11}N$$



10. Which of the following compounds on reacting with Heinsberg reagent form an alkali insoluble product.

- (A) Ethanamine
(B) N-methylaniline
(C) N-Ethyl-N-Methylaniline
(D) N-Methylethanamine
(E) N-Phenylaniline
(F) Aniline
(1) A, C, D, E only
(2) B, C, D & E only
(3) B, D, and E only
(4) A, C, F only

Answer (3)

Sol. (A) Ethanamine $\Rightarrow$ $\text{CH}_3\text{CH}_2\text{NH}_2$ (1°)

(B) N-Methylaniline $\Rightarrow$ Ph-NH-CH_3 (2°)

(C) N-Ethyl-N-methylaniline $\Rightarrow$ $\text{Ph-N} \begin{matrix} \text{CH}_3 \\ \text{C}_2\text{H}_5 \end{matrix}$ (3°)

(D) N-Methylethanamine $\Rightarrow \text{CH}_3\text{CH}_2\text{NHCH}_3$ (2°)

(E) N-Phenylaniline $\Rightarrow \text{Ph-NH-Ph}$ (2°)

(F) Aniline $\Rightarrow \text{Ph-NH}_2$ (1°)

2° Amines react with Heinsberg reagent to form an alkali insoluble product.

- 11.
- 12.
- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The sum of valence e^- in element with most and least metallic character among the following is :
Na, P, Cl, S, O and F

Answer (8)

Sol. Element with most metallic character $\Rightarrow \text{Na}$

Element with least metallic character $\Rightarrow \text{F}$

Sum of valence e^- in Na and F = $1 + 7 = 8$

22. In 'S' estimation, 0.314 g of organic compound gave 0.4813 g of barium sulphate. What is % of 'S' in organic compound?

(Report to nearest integer).

Answer (21)

$$\begin{aligned}\text{Sol. \% of S} &= \frac{0.4813}{0.314} \times \frac{32}{233} \times 100 \\ &= 21.05\end{aligned}$$

23. Among Sc^{3+} , Cr^{2+} , Mn^{3+} , Co^{3+} number of isoelectronic species are 'n'.

'n' moles of AgCl is obtained upon reaction with excess of AgNO_3 with 1 mol of $\text{Co(en)}_2\text{NH}_3\text{Cl}_3$. Number of t_{2g} electrons in the complex are

Answer (6)

Sol. $\text{Sc}^{3+} \Rightarrow 3d^0$

$\text{Cr}^{2+} \Rightarrow 3d^4$

$\text{Mn}^{3+} \Rightarrow 3d^4$

$\text{Co}^{3+} \Rightarrow 3d^6$

$n = 2$

2 mol of AgCl is precipitated.

Hence, complex should be $[\text{Co(en)}_2(\text{NH}_3)\text{Cl}]\text{Cl}_2$
complex is inner orbital octahedral complex with hybridisation d^2sp^3

$\text{Co}^{3+} \Rightarrow 3d^6$

$t_{2g}^6 e_g^0$

24. An alpha particle and proton are accelerated in a discharge tube under same potential difference of 200 KeV. The de Broglie wavelength of proton is $x\sqrt{2}$ times of de Broglie wavelength of α -particle.
The value of x is

Answer (2)

$$\text{Sol. } \lambda = \frac{h}{\sqrt{2m(qV)}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \left[\frac{\frac{h}{\sqrt{2 \times 1 \times 1 \times 200 \times 1000}}}{\frac{h}{\sqrt{2 \times 4 \times 2 \times 200 \times 1000}}} \right]$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{8} = 2\sqrt{2}$$

- 25.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The sum of coefficients of x^{499} and x^{500} in the binomial expansion of $(1+x)^{1000} + (1+x)^{999}(x) + x^2(1+x)^{998} + \dots + x^{1000}$ is

- (1) $^{1002}C_{501}$ (2) $^{1002}C_{500}$
(3) $^{1001}C_{500}$ (4) $^{1001}C_{501}$

Answer (2)

Sol. $S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \dots + x^{1000}$

$$\Rightarrow 8 = (1+x)^{1000} \left[\frac{1 - \left(\frac{x}{1+x}\right)^{1001}}{1 - \left(\frac{x}{1+x}\right)} \right]$$

$$= (1+x)^{1001} \left[1 - \left(\frac{x}{1+x}\right)^{1001} \right]$$

$$= (1+x)^{1001} - x^{1001}$$

Coefficient's sum of x^{499} and x^{500}

$$= ^{1001}C_{499} + ^{1001}C_{500} = ^{1002}C_{500}$$

2. If $\sum_{r=1}^{25} \frac{r}{r^4 + r^2 + 1} = \frac{p}{q}$, where p and q are coprime positive integer, then $p+q$ is equal to

- (1) 841 (2) 976
(3) 984 (4) 890

Answer (2)

Sol.
$$\sum_{r=1}^{25} \frac{r}{(r^2+1)^2 - r^2} = \sum_{r=1}^{25} \frac{r}{(r^2+1+r)(r^2+1-r)}$$

$$\begin{aligned} &= \frac{1}{2} \sum_{r=1}^{25} \left(\frac{1}{r^2+1-r} - \frac{1}{r^2+1+r} \right) \\ &= \frac{1}{2} \left[\left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{7} \right) + \dots + \left(\frac{1}{601} - \frac{1}{657} \right) \right] \\ &= \frac{1}{2} \left[1 - \frac{1}{651} \right] \\ &= \frac{1}{2} \left[\frac{650}{651} \right] \\ &= \frac{325}{657} \end{aligned}$$

$$\therefore p+q = 976$$

3. $\frac{6}{3^{26}} + \frac{10}{3^{25}} + \frac{10.2}{3^{24}} + \frac{10.2^2}{3^{23}} + \dots + \frac{10.2^{24}}{3}$ is equal to
- (1) 2^{26} (2) 2^{25}
(3) 3^{26} (4) 3^{25}

Answer (1)

Sol.
$$\frac{6}{3^{26}} + \frac{10.1}{3^{25}} + \frac{10.2}{3^{24}} + \dots + \frac{10.2^{24}}{3}$$

Let $S = 10 \left[\frac{1}{3^{25}} + \frac{2}{3^{24}} + \frac{2^2}{3^{23}} + \dots + \frac{2^{24}}{3} \right]$

$$S = 10 \frac{[6^{25} - 1]}{5 \times 3^{25}}$$

$$= \frac{2}{3^{25}} [6^{25} - 1]$$

Now sum = $\frac{6}{3^{26}} + \frac{2}{3^{25}} [6^{25} - 1]$

$$= \frac{6}{3^{26}} + \frac{6}{3^{26}} [6^{25} - 1]$$

$$= \frac{6^{26}}{3^{26}} = 2^{26}$$

4. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{12(3+[x])}{3+[\sin x]+[\cos x]} dx$, where $[\]$

represents greatest integer function, is equal to

- (1) $3 + 10\pi$ (2) $11\pi + 4$
 (3) $10\pi + 2$ (4) $11\pi + 2$

Answer (4)

Sol. $\int_{-\frac{\pi}{2}}^{-1} \frac{12(3-2)}{3-1+0} dx + \int_{-1}^0 \frac{12(3-1)}{3-1+0} dx +$

$$\int_0^1 \frac{12(3+0)}{3+0+0} dx + \int_1^{\frac{\pi}{2}} \frac{12(3+1)}{3+0+0} dx$$

$$= \int_{-\frac{\pi}{2}}^{-1} 6 dx + \int_{-1}^0 12 dx + \int_0^1 12 dx + \int_1^{\frac{\pi}{2}} 16 dx$$

$$= 6 \left(-1 + \frac{\pi}{2} \right) + 12(1) + 12(1) + 16 \left(\frac{\pi}{2} - 1 \right)$$

$$= -6 + 3\pi + 12 + 12 + 8\pi - 16$$

$$= 2 + 11\pi$$

5. By the principal of inverse trigonometric function, the value of $\tan \left(2\sin^{-1} \left(\frac{2}{\sqrt{13}} \right) - 2\cos^{-1} \left(\frac{3}{\sqrt{10}} \right) \right)$ is equal to

- (1) $\frac{33}{56}$ (2) $\frac{31}{55}$
 (3) $\frac{32}{59}$ (4) $\frac{38}{55}$

Answer (1)

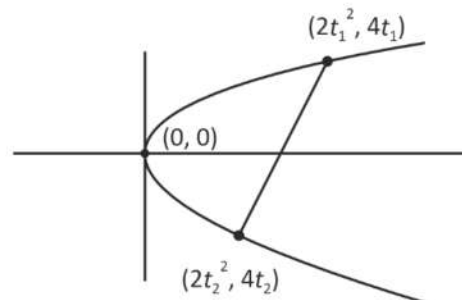
Sol. $\therefore \tan \left(2\sin^{-1} \left(\frac{2}{\sqrt{13}} \right) - 2\cos^{-1} \left(\frac{3}{\sqrt{10}} \right) \right)$
 $= \tan \left(2\tan^{-1} \left(\frac{2}{3} \right) - 2\tan^{-1} \left(\frac{1}{3} \right) \right)$
 $= \tan \left(2\tan^{-1} \left(\frac{\frac{2}{3} - \frac{1}{3}}{1 + \frac{2}{9}} \right) \right)$
 $= \tan \left(2\tan^{-1} \left(\frac{3}{11} \right) \right)$
 $= \tan \left(\tan^{-1} \left(\frac{\frac{6}{11}}{1 - \frac{9}{14}} \right) \right)$
 $= \frac{66}{112} = \frac{33}{56}$

6. Let a triangle ABC such that $A \equiv (0, 0)$ and vertices B and C lie on the parabola $y^2 = 8x$ such that $\left(\frac{7}{3}, \frac{4}{3} \right)$ is the centroid of the $\triangle ABC$ then $(BC)^2$ is equal to

- (1) 90 (2) 120
 (3) 150 (4) 110

Answer (2)

Sol.



$$\Rightarrow \frac{2(t_1^2 + t_2^2)}{3} = \frac{7}{3} \Rightarrow t_1^2 + t_2^2 = \frac{7}{2}$$

$$\frac{4(t_1 + t_2)}{3} = \frac{4}{3} \Rightarrow t_1 + t_2 = 1$$

$$\Rightarrow (t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2$$

$$\text{also, } (t_1 + t_2)^2 = \frac{7}{2} + 2t_1t_2 = 1$$

$$\Rightarrow t_1t_2 = \frac{-5}{4}$$

$$\Rightarrow (t_1 - t_2)^2 = 1 - 4\left(\frac{-5}{4}\right) = 6$$

$$\text{Now, } BC = \sqrt{(2t_1^2 - 2t_2^2)^2 + (4(t_1 + t_2))^2}$$

$$BC = 2|(t_1 - t_2)|\sqrt{(t_1 + t_2)^2 + 4}$$

$$\Rightarrow BC = 2\sqrt{6}(\sqrt{5})$$

$$\Rightarrow BC^2 = 4 \times 6 \times 5 = 120$$

7. Let $A = \{Z \in \mathbb{C} : |Z - 2| \leq 4\}$ and $B = \{Z \in \mathbb{C} : |Z - 2| + |Z + 2| \leq 4\}$ then $\max\{|Z_1 - Z_2| : Z_1 \in A \text{ and } Z_2 \in B\}$ is equal to

- (1) 6 (2) 8
(3) 4 (4) 5

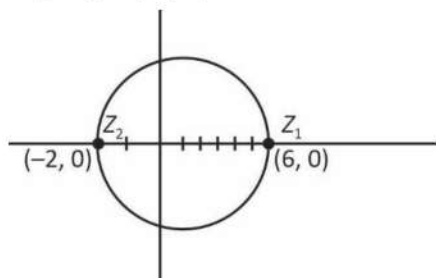
Answer (2)

Sol. $A \Rightarrow (x - 2)^2 + y^2 \leq 16$

$B \Rightarrow$ Line segment

$$\sqrt{(x - 2)^2 + y^2} + \sqrt{(x + 2)^2 + y^2} \leq 4$$

$$\Rightarrow y = 0, x \in (-2, 2)$$



The max $|z - z_2|$ = diameter of circle
= 8

8. If the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{5}{16}$ and $a, 4, \alpha, \beta$ are in increasing A.P. then both the roots of the equation $ax^2 - ax + 2(\alpha - 2\beta) = 0$ lie between

- (1) $(-3, 0)$
(2) $(-2, 3)$
(3) $(0, 3)$
(4) $(-3, 1)$

Answer (2)

Sol. Let $4 - d, 4, 4 + d, 4 + 2d$ are in A.P.

$$\Rightarrow \frac{1}{2} \left(\frac{1}{4 - d} + \frac{1}{4 + 2d} \right) = \frac{5}{10}$$

$$\Rightarrow 5d^2 - 6d - 8 = 0$$

$$\Rightarrow d = 2, \frac{-4}{5}$$

$$\Rightarrow d = 2$$

$$a = 2, \beta = 8, \alpha = 6$$

$$6x^2 - 2x + 2(6 - 16) = 0$$

$$\Rightarrow 3x^2 - x - 10 = 0$$

$$\Rightarrow x = 2, \frac{-5}{3}$$

9. The range of $f(x) = \text{sgn}(\sin x) + \text{sgn}(\cos x) + \text{sgn}(\tan x) + \text{sgn}(\cot x)$, $x \neq \frac{n\pi}{2}, n \in \mathbb{I}$,

$$\text{where } \text{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \\ 0, & t = 0 \end{cases}$$

- (1) $\{4, -4, 2, -2\}$ (2) $\{-2, 0, 4\}$
(3) $\{4, -4, 0, -2\}$ (4) $\{2, -2, 0, 4, -4\}$

Answer (2)

Sol. $f(x) = \text{sgn}(\sin x) + \text{sgn}(\cos x) + \text{sgn}(\tan x) + \text{sgn}(\cot x)$,

$$x \neq \frac{n\pi}{2}, n \in \mathbb{I},$$

The value of

$$f(x) = \begin{cases} 1+1+1+1, & \text{for } x \in \left(0, \frac{\pi}{2}\right) \\ 1-1-1-1, & \text{for } x \in \left(\frac{\pi}{2}, \pi\right) \\ -1-1+1+1, & \text{for } x \in \left(\pi, \frac{3\pi}{2}\right) \\ -1+1-1-1, & \text{for } x \in \left(\frac{3\pi}{2}, 2\pi\right) \end{cases}$$

Hence range of $f(x) = \{-2, 0, 4\}$

10. **Statement I:** $25^{13} + 20^{13} + 8^{13} + 3^{13}$ is divisible by 7.

Statement II: The integral value of $(7+4\sqrt{3})^{25}$ is an odd number

- (1) Neither statements are correct
- (2) Only statement I is correct
- (3) Only statement II is correct
- (4) Both the statements are correct

Answer (4)

Sol. Statement 1 :

$(25^{13} + 3^{13}) + (20^{13} + 8^{13})$ is divisible by 28 as

$(a+b) | a^n + b^n$ for $n \in$ odd natural numbers

or using congruence modular arithmetic,

$$25^{13} + 3^{13} + 20^{13} + 8^{13}$$

$$\equiv (4^{13} + 3^{13} + (-1)^{13} + 1^{13}) \pmod{7}$$

$$\equiv ((-3)^{13} + 3^{13}) \pmod{7}$$

$$\equiv 0 \pmod{7} \quad \Rightarrow 7 \text{ divides the sum}$$

Statement 2

$$\text{Let } I + f_1 = (7+4\sqrt{3})^{25}$$

$$f_2 = (7-4\sqrt{3})^{25} \text{ as } 0 < 7-4\sqrt{3} < 1$$

$$\Rightarrow I + f_1 + f_2 =$$

$$2 \left({}^{25}C_1 (4\sqrt{3})^{24} 7^1 + {}^{25}C_3 (4\sqrt{3})^{22} 7^3 + \dots + {}^{25}C_{25} (4\sqrt{3})^0 7^{25} \right)$$

$$= 2k \text{ for some integer } k$$

$$\Rightarrow \text{Since } f_1 + f_2 \in (0, 2)$$

$$\Rightarrow I + f_1 + f_2 = 2k \Rightarrow I = 2k - 1$$

as only integer in

$$(0, 2) \text{ is } 1 \Rightarrow f_1 + f_2 = 1 \Rightarrow I \text{ is odd.}$$

11. Let $y = y(x)$ be the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \cot x, \quad x \in (0, \pi). \text{ If } y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}, \text{ then}$$

$$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right) \text{ is}$$

- | | |
|------------|-------------|
| (1) 2π | (2) -3π |
| (3) $-\pi$ | (4) π |

Answer (3)

$$\text{Sol. } x \frac{dy}{dx} - y = x^2 \cot x$$

$$\frac{dy}{dx} - \frac{y}{x} = x \cot x$$

$$e^{-\int \frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$$

$$\frac{y}{x} = \int \cot x dx$$

$$\frac{y}{x} = \int \ln |\sin x| + c$$

$$y = x \ln |\sin x| + cx$$

$$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$\Rightarrow c = 1$$

$$\Rightarrow y = x \ln |\sin x| + x$$

$$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$$

$$= 6\left[\frac{\pi}{6}\ln\left(\frac{1}{2}\right) + \frac{\pi}{6}\right] - 8\left[\frac{\pi}{4}\ln\left(\frac{1}{\sqrt{2}}\right) + \frac{\pi}{4}\right]$$

$$\pi - 2\pi = -\pi$$

12. Let $f(x) = \lim_{\theta \rightarrow 0} \frac{\cos \pi x - (x^{2/\theta}) \sin(x-1)}{1 + (x^{2/\theta}) \sin(x-1)}, x \in R$. Then

which of the following is correct.

- (1) f is continuous at $x = 1$ and $f(1) = -1$
- (2) f is discontinuous at $x = -1$ and $f(1) = -1$
- (3) f is continuous at $x = 1$ and $f(1) = 1$
- (4) f is discontinuous at $x = 1$ and $f(1) = 1$

Answer (1)

Sol. $f(x) = \lim_{\theta \rightarrow 0} \frac{\cos \pi x - (x^2)^{1/\theta} \sin(x-1)}{1 + (x^2)^{1/\theta} \sin(x-1)}, x \in R$

$$f(x) = \begin{cases} \frac{-\sin(x-1)}{(x-1)}, & x > 1 \\ -1, & x = 1 \\ \cos(\pi x), & x < 1 \end{cases}$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = -1$$

13. **Statement I:** The function F defined from $R \rightarrow R$

$$F(x) = \frac{x}{1+|x|} \text{ is one-one}$$

Statement II: The function F defined from $R \rightarrow R$

$$F(x) = \frac{x^2 + 4x - 30}{x^2 - 8x + 18} \text{ is many-one}$$

- (1) Statement I is correct but statement II is not correct
- (2) Statement I and statement II both are correct
- (3) Statement I is incorrect but statement II is correct
- (4) Both statement are incorrect

Answer (2)

Sol. Statement 1: $F(x) = \frac{x}{1+|x|}$

$$F(x) = \begin{cases} \frac{x}{1-x} & x < 0 \\ \frac{x}{1+x} & x \geq 0 \end{cases}$$

$$F'(x) = \begin{cases} \frac{1}{(1-x)^2} & x < 0 \\ \frac{1}{(1+x)^2} & x \geq 0 \end{cases}$$

$\Rightarrow F(x)$ is increasing in R and $f(x)$ is continuous $\forall x \in R$

$\Rightarrow F(x)$ is one-one

Statement 2: $F(x) = \frac{x^2 + 4x - 30}{x^2 - 8x + 18}$

$$F'(x) = \frac{(x^2 - 8x + 18)(2x + 4) - (x^2 + 4x - 30)(2x - 8)}{(x^2 - 8x + 18)^2}$$

$$= \frac{(2x^3 - 16x^2 + 36x + 4x^2 - 32x + 72) - (2x^3 + 8x^2 - 60x - 8x^2 - 32x + 240)}{(x^2 - 8x + 18)^2}$$

$$= -\frac{12x^2 + 96x - 72}{(x^2 - 8x + 18)^2}$$

$$= -\frac{12(x^2 - 8x + 6)}{(x^2 - 8x + 18)^2}$$

$F'(x)$ will become 0 at 2 distinct points and also $F(x)$ is continuous.

$\Rightarrow F(x)$ is many-one function.

14. Consider the data:

$x:$	$4k$	$\frac{30}{7}k$	$\frac{32}{7}k$	$\frac{34}{7}k$	$\frac{36}{7}k$	$\frac{38}{7}k$	$\frac{40}{7}k$	$6k$
$p(x):$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

If $E(x) = \frac{263}{15}$, then $P(x < 20)$ is equal to

- (1) $\frac{1}{15}$ (2) $\frac{8}{15}$
 (3) $\frac{4}{15}$ (4) $\frac{14}{15}$

Answer (4)

Sol. $E(x) = \frac{263}{15}$

$$\Rightarrow \sum xP(x) = \frac{263}{15}$$

$$\Rightarrow \frac{526}{105}k = \frac{263}{15}$$

$$\Rightarrow k = \frac{7}{2}$$

$$P(x < 20) = 1 - P(6k)$$

$$= 1 - P(21)$$

$$= 1 - \frac{1}{15}$$

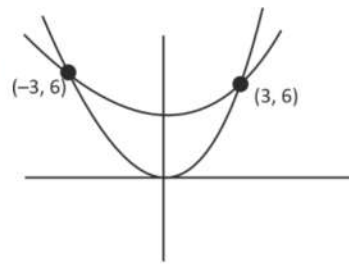
$$= \frac{14}{15}$$

15. Let $P_1 : y = 4x^2$ and $P_2 : x^2 + 27$ be two parabolas. If the area bounded by region enclosed by P_1 and P_2 is 6 times the area bounded by P_1 and $y = \alpha x, \alpha > 0$, then α is equal to

- (1) 8 (2) 12
 (3) 13 (4) 6

Answer (2)

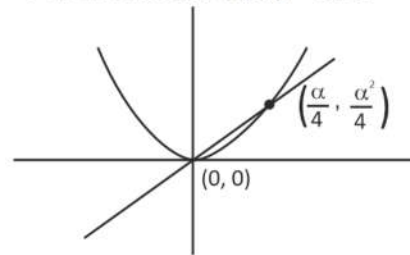
Sol.



$A_1 = \text{Area between } P_1 \text{ and } P_2$

$$\Rightarrow \int_{-3}^3 [(x^2 + 27) - 4x^2] dx = 108$$

$\Rightarrow \text{Area between } P_1 \text{ and } y = \alpha x \text{ is}$



$$\Rightarrow A_2 = \int_0^{\alpha/4} (\alpha x - 4x^2) dx$$

$$= \left. \frac{\alpha x^2}{2} - \frac{4x^3}{3} \right|_0^{\alpha/4}$$

$$= \frac{\alpha^3}{32} - \frac{4}{3} \times \frac{\alpha^3}{64} = \frac{108}{6} = 18$$

$$= \frac{\alpha^3}{32} \left(1 - \frac{2}{3} \right) = 18$$

$$\Rightarrow \alpha^3 = 18 \times 32 \times 3 = 27 \times 64$$

$$\Rightarrow \alpha = 3 \times 4 = 12$$

16. Let Q be the image of the point $P(3, 2, 1)$ in the line

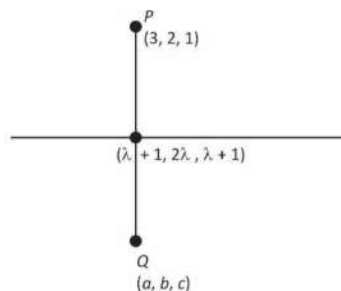
$$\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}, \text{ then the distance of } Q \text{ from the line}$$

$$\frac{x-9}{3} = \frac{y-9}{2} = \frac{3-5}{-2} \text{ is}$$

- (1) 3 (2) 4
 (3) 5 (4) 7

Answer (4)

Sol.



$PQ \perp$ line

$$\Rightarrow (\lambda + 1 - 3)(1) + (2\lambda - 2)(2) + (\lambda + 1 - 1)(1) = 0$$

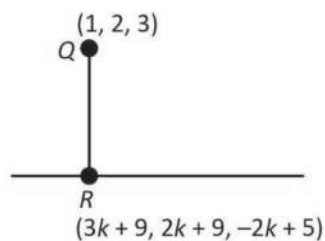
$$\Rightarrow (\lambda - 2) + 4(\lambda - 1) + \lambda = 0$$

$$6\lambda = 6 \Rightarrow \lambda = 1$$

$\Rightarrow$ Image point is $Q(1, 2, 3)$.

The Distance of Point Q from a Second Line

$$\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$$



$$(3k+9-1)(3) + (2k+9-2)(2) + (-2k+5-3)(-2) = 0$$

$$\Rightarrow k = -2$$

$$R \equiv (3k+9, 2k+9, -2k+5), \text{ for } k = -2$$

$$\Rightarrow R \equiv (3, 5, 9)$$

$$QR = \sqrt{2^2 + 3^2 + 6^2} = 7$$

- 17.
- 18.
- 19.
- 20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and B be a 2×2 matrix such that $A^{100} = 1008I + I$, then sum of all elements of B^{100} is

Answer (0)

Sol. $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 3 - 4 & -3 \cdot 4 + 1 \cdot 4 \\ 1 \cdot 3 - 1 & -4 + 1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 \cdot 2 + 1 & -(4 \cdot 2) \\ 1 \cdot 2 & -(2 \cdot 2 - 1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}, A^3 = \begin{bmatrix} 11 & -10 \\ 5 & -9 \end{bmatrix}, \dots$$

$$\therefore A^n = \begin{bmatrix} 2n+1 & -4n \\ n & 1-2n \end{bmatrix}$$

$$\Rightarrow A^{100} = \begin{bmatrix} 201 & -400 \\ 100 & -199 \end{bmatrix} = 100B + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 200 & -400 \\ 100 & -200 \end{bmatrix} = 100B$$

$$\Rightarrow B = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow B^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Sum of elements of $B = 0$

22. Ellipse $\frac{x^2}{144} + \frac{y^2}{169} = 1$ and hyperbola $\frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1$

have same focus and e and L denotes the eccentricity and length of latus rectum of hyperbola then $24(e + L)$ is

Answer (296)

Sol. $\frac{x^2}{144} + \frac{y^2}{169} = 1$

$$e^2 = 1 - \frac{144}{169}$$

$$= \frac{25}{169} \Rightarrow e = \frac{5}{13}$$

$$F_{\text{ellipse}} = \left(0, \pm \frac{5}{13} \times 13 \right)$$

$$= (0, \pm 5)$$

$$\frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1$$

$$e_H^2 = 1 + \frac{16}{\lambda^2}$$

$$F \equiv \left(0, \pm \lambda \sqrt{1 + \frac{16}{\lambda^2}} \right)$$

$$\Rightarrow \lambda \sqrt{1 + \frac{16}{\lambda^2}} = 5$$

$$\lambda^2 + 16 = 25$$

$$\lambda^2 = 9$$

$$\text{Now } e_H = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

$$L(\text{LR}) = \frac{2a^2}{b} = \frac{2 \times 16}{3} = \frac{32}{3}$$

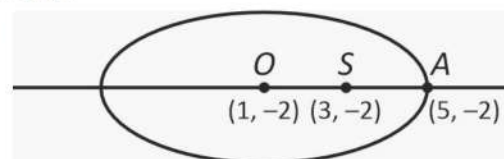
$$24(e + L) = 24 \left(\frac{5}{3} + \frac{32}{3} \right) = 24 \left(\frac{37}{3} \right)$$

$$= 8 \times 37 = 296$$

23. An ellipse has centre at $(1, -2)$ and one of the focus at $(3, -2)$ and one vertex at $(5, -2)$, then the length of its latus rectum is

Answer (6)

Sol.



$$\therefore ae = 2 \text{ and } a = 4$$

$$\therefore e = \frac{1}{2}$$

$$b^2 = a^2(1 - e^2)$$

$$b^2 = 16 \left(1 - \frac{1}{4} \right) = 12$$

$$\therefore \text{Length of lotus rectum} = \frac{2b^2}{a} = \frac{2 \times 12}{4}$$

$$= 6$$

24. $F(x) = \int \frac{dx}{x^{2/3} + 2x^{1/2}}$ be such that $F(0) = -26 + 24\ln 2$. If

$$F(1) = a + b\ln 3, \text{ then } a + b \text{ is equal to}$$

Answer (13)

Sol. $\int \frac{dx}{x^{2/3} + 2x^{1/2}}$

$$\text{Let } x = t^6$$

$$= dx = 6t^5 dt$$

$$= 6 \int \frac{t^5 dt}{t^4 + 2t^3}$$

$$= 6 \int \frac{t^2}{t+2} dt$$

$$= 6 \int \frac{t^2 - 4 + 4}{t+2} dt$$

$$= 6 \int t + 2 \frac{+4}{t+2} dt$$

$$= 6 \left[\frac{t^2}{2} + 2t + 4 \ln|t+2| \right] + c$$

$$F(x) = 6 \left[\frac{x^{1/3}}{2} + 2x^{1/3} + 4 \ln|x^{1/6} + 2| \right] + c$$

$$F(0) = 24 \ln 2 + c = -26 + 24 \ln 2$$

$$c = -26$$

$$F(1) = \left[\frac{1}{2} + 2 + 4 \ln 3 \right] - 26 \equiv a + b \ln 3$$

$$-11 + 24 \ln 3 = a + b \ln 3$$

$$a + b = 24 - 11 = 13$$

25.

