JEE (Main)-2026 Session-1

Question Paper with Solutions

(Mathematics, Physics, & Chemistry)

5th April 2026 Shift -1

Time: 3 hrs. **M.M: 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A: Attempt all questions.

(5) Section - B: Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 - 25) contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

SECTION-A

1. Let a, b ∈ C. Let α, β be the roots of the equation $x^2 + ax + b = 0$. If $\beta-\alpha=\sqrt{11}$ and $\beta^2-\alpha^2=3i\sqrt{11}$, then $(\beta^3-\alpha^3)^2$ is equal to :

(1) 160 (2) 176

(3) 194 (4) 187

Ans. (2)

Sol. $\beta-\alpha=\sqrt{11}$ ...(1)

and $\beta^2-\alpha^2=3i\sqrt{11}$

$\Rightarrow (\beta+\alpha)(\beta-\alpha)=3i\sqrt{11}$

$\Rightarrow \beta+\alpha=3i$ ...(2)

from (1) and (2)

$$\beta=\frac{\sqrt{11}+3i}{2},\quad \alpha=\frac{3i-\sqrt{11}}{2}$$

$$\Rightarrow \alpha\beta=\frac{-9-11}{4} \Rightarrow \alpha\beta=-5$$

Now $(\beta^3-\alpha^3)^2=(\beta-\alpha)^2(\beta^2+\alpha^2+\alpha\beta)^2$

$=(\beta-\alpha)^2((\beta+\alpha)^2-\alpha\beta)^2$

$=(11)(-9+5)^2=176$

2. Let the sum of the first n terms of an A.P. be $3n^2 + 5n$. Then the sum of squares of the first 10 terms of the A.P. is :

(1) 10220 (2) 12860

(3) 15220 (4) 19780

Ans. (3)

Sol. $S_n = 3n^2 + 5n$

$T_n = S_n - S_{n-1}$

$\Rightarrow T_n = (3n^2 + 5n) - (3(n-1)^2) + 5(n-1))$

$\Rightarrow T_n = 6n + 2$

$$\sum_{n=1}^{10} T_n^2 = \sum_{n=1}^{10}(6n+2)^2$$

$$= 36\sum_{n=1}^{10} n^2 + 24\sum_{n=1}^{10} n + \sum_{n=1}^{10} 4$$

$= 15220$

3. Let A be a 3 × 3 matrix such that

$$A^T\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}5\\2\\2\end{bmatrix},\quad A^T\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}3\\1\\1\end{bmatrix},\quad A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}3\\4\\4\end{bmatrix}$$ and

$$A\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}1\\3\\1\end{bmatrix}$$

If det(A) = 1, then det(adj(A^2 + A)) is equal to :

(1) 16 (2) 25

(3) 49 (4) 64

Ans. (4)

Sol. Let $A=\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{pmatrix}$

Now $A=\begin{pmatrix}\alpha_1 & \beta_1 & \gamma_1\\ \alpha_2 & \beta_2 & \gamma_2\\ \alpha_3 & \beta_3 & \gamma_3\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}3\\1\\1\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}\gamma_1\\ \gamma_2\\ \gamma_3\end{pmatrix}=\begin{pmatrix}3\\1\\1\end{pmatrix} \quad ...(1)$$

Now $\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\3\\1\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}\alpha_3\\ \beta_3\\ \gamma_3\end{pmatrix}=\begin{pmatrix}1\\3\\1\end{pmatrix} \quad ...(2)$$

$$A^T\begin{pmatrix}1\\0\\1\end{pmatrix}-A^T\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}5\\2\\2\end{pmatrix}-\begin{pmatrix}3\\1\\1\end{pmatrix}$$

$$\Rightarrow A^T\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}2\\1\\1\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}=\begin{pmatrix}2\\1\\1\end{pmatrix} \qquad \ldots(3)$$

$$\text{Now } A\begin{pmatrix}1\\0\\1\end{pmatrix}-A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}3\\4\\4\end{pmatrix}-\begin{pmatrix}1\\3\\1\end{pmatrix}$$

$$\Rightarrow A\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}2\\1\\3\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\alpha_1\\ \beta_1\\ \gamma_1\end{pmatrix}=\begin{pmatrix}2\\1\\3\end{pmatrix} \qquad \ldots(4)$$

$$A=\begin{pmatrix}2&1&1\\1&\beta_2&3\\3&1&1\end{pmatrix}$$

$|A| = 2(\beta_2 - 3) - 1(1-9) + 1(1-3\beta_2) = 1$

$\Rightarrow 2\beta_2 - 6 + 8 + 1 - 3\beta_2 = 1$

$\Rightarrow \beta_2 = 2$

$$\text{So } A=\begin{pmatrix}2&1&1\\1&2&3\\3&1&1\end{pmatrix}$$

$$A^2=\begin{pmatrix}2&1&1\\1&2&3\\3&1&1\end{pmatrix}\begin{pmatrix}2&1&1\\1&2&3\\3&1&1\end{pmatrix}=\begin{pmatrix}8&5&6\\13&8&10\\10&6&7\end{pmatrix}$$

$$A^2+A=\begin{pmatrix}10&6&7\\14&10&13\\13&7&8\end{pmatrix} \Rightarrow \det(A^2+A)=8$$

$\Rightarrow \text{Det}(\text{adj}(A^2+A)) = 8^2 = 64$

4. Consider the system of linear equations in x, y, z :

$x + 2y + tz = 0,$

$6x + y + 5tz = 0,$

$3x + t^2y + f(t)z = 0,$

Where $f : R \to R$ is a differentiable function. If this system has infinitely many solutions for all $t \in R$, then f

(1) is a constant function

(2) is strictly increasing on R

(3) is strictly decreasing on R

(4) has two critical points

Ans. (2)

Sol. $$D=\begin{vmatrix}1&2&t\\6&1&5t\\3&t^2&f(t)\end{vmatrix}=0$$

$\Rightarrow 1(f(t) - 5t^3) - 2(6f(t) - 15t) + t(6t^2 - 3) = 0$

$$f(t)=\frac{t^3+12t}{11}$$

$$f'(t)=\frac{1}{11}(3t^2+12)>0 \quad \forall\, t \in R$$

$\Rightarrow f(t)$ is strictly increasing on R

5. $\sum_{n=1}^{10}\left(\frac{528}{n(n+1)(n+2)}\right)$ is equal to :

(1) 65 (2) 130

(3) 220 (4) 440

Ans. (2)

Sol. Let $\sum_{n=1}^{10}\frac{528}{n(n+1)(n+2)}=\lambda$

$$\text{Let } T_n=\frac{1}{n(n+1)(n+2)}$$

$$T_n=\frac{1}{2}\left\{\frac{(n+2)-n}{n(n+1)(n+2)}\right\}$$

$$T_n=\frac{1}{2}\left\{\frac{1}{n(n+1)}-\frac{1}{(n+1)(n+2)}\right\}$$

$$T_1=\frac{1}{2}\left\{\frac{1}{1.2}-\frac{1}{2.3}\right\}$$

$$T_2=\frac{1}{2}\left\{\frac{1}{2.3}-\frac{1}{3.4}\right\}$$

$\vdots$

$$T_{10}=\frac{1}{2}\left\{\frac{1}{10.11}-\frac{1}{11.12}\right\}$$

$$S_n=\frac{1}{2}\left[\frac{1}{2}-\frac{1}{132}\right]$$

$$S_n=\frac{1}{2}\left[\frac{66-1}{132}\right]=\frac{65}{264}$$

$$\text{So, } \lambda=\frac{65}{264}\times 528$$

$\lambda = 130$

6. Let tan A, tan B, where $A, B \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, be the roots of the quadratic equation $x^2 - 2x - 5 = 0$. Then $20\sin^2\left(\frac{A+B}{2}\right)$ is equal to :

(1) $10+\sqrt{10}$ (2) $10-2\sqrt{10}$

(3) $10-3\sqrt{10}$ (4) $10-\sqrt{10}$

Ans. (3)

Sol. $x^2 - 2x - 5 = 0$

$\tan A + \tan B = 2; \quad \tan A \tan B = -5$

$\therefore \tan(A+B) = \frac{2}{1-(-5)} = \frac{1}{3}$

$\Rightarrow \cos(A+B) = \frac{3}{\sqrt{10}}$

$\therefore 20\left(\sin^2\left(\frac{A+B}{2}\right)\right) = \frac{10}{2}(1-\cos(A+B))$

$= 10\left(1-\frac{3}{\sqrt{10}}\right) = \left(10-3\sqrt{10}\right)$

7. A letter is known to have arrived by post either from KANPUR or from ANANTPUR. On the the envelope just two consecutive letters AN are visible. The probability, that the letter came from ANANTPUR, is :

(1) $\frac{7}{10}$ (2) $\frac{10}{17}$

(3) $\frac{12}{19}$ (4) $\frac{7}{19}$

Ans. (2)

Sol. $P(I) : \frac{1}{2}\ P(A/I) \to \frac{1}{5}$

$P(II) : \frac{1}{2}\ P(A/II) \to \frac{2}{7}$

$\therefore$ Reqd. Prob $= \frac{\frac{1}{2}\times\frac{2}{7}}{\frac{1}{2}\times\frac{1}{5}+\frac{1}{2}\times\frac{2}{7}} = \frac{\frac{2}{7}\times 5}{\frac{17}{7}} = \frac{10}{17}$

8. The mean deviation about the mean for the data

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

is equal to :

(1) $\frac{40}{13}$ (2) $\frac{42}{13}$

(3) $\frac{44}{13}$ (4) $\frac{46}{13}$

Ans. (3)

Sol. $\text{M.D.} = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{40+42+18+20+24+90}{26}$

$= \frac{234}{26} = 9$

$\text{M.D.} = \frac{8(4)+6(2)+2(0)+2(1)+2(3)+6(6)}{26}$

$= \frac{84}{26} = \frac{44}{13}$

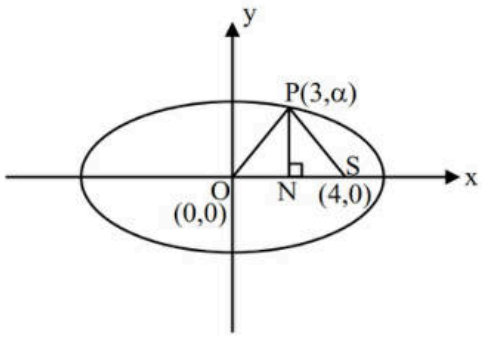
9. Let a focus of the ellipse E : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be S(4, 0) and its eccentricity be $\frac{4}{5}$. If the point P(3, α) lies on E and O is the origin, then the area of ΔPOS is equal to :

(1) 12/5 (2) 14/5

(3) 24/5 (4) 48/5

Ans. (3)

Sol.



$a > b$

Focus $(ae, 0) = (4, 0)$

$\therefore ae = 4$

$a\left(\frac{4}{5}\right) = 4 \Rightarrow a = 5$

$\because b^2 = a^2(1 - e^2)$

$b^2 = 25\left(1 - \frac{16}{25}\right)$

$\therefore E : \frac{x^2}{25} + \frac{y^2}{9} = 1$

$\because$ P lies on the ellipse E

$\therefore \frac{9}{25} + \frac{\alpha^2}{9} = 1$

$\therefore \alpha = \pm\frac{12}{5}$

$\therefore$ Area of $\Delta_{POS} = \frac{1}{2}(OS)(PN)$

$= \frac{1}{2} \times 4 \times \frac{12}{5} = \frac{24}{5}$

10. Let P be a moving point on the circle

$$x^2 + y^2 - 6x - 8y + 21 = 0$$

Then, the maximum distance of P from the vertex of the parabola $x^2 + 6x + y + 13 = 0$ is equal to :

(1) 8 (2) 10

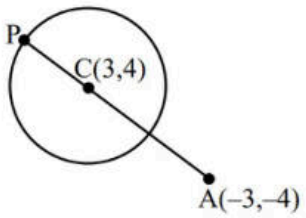
(3) 12 (4) 9

Ans. (3)

Sol. Centre : C(3,4) & r = 2

Parabola $(x + 3)^2 = -(y + 4)$

Vertex is A(–3,–4)



$AP_{max} = AC + r = \sqrt{36 + 64} + 2 = 12$

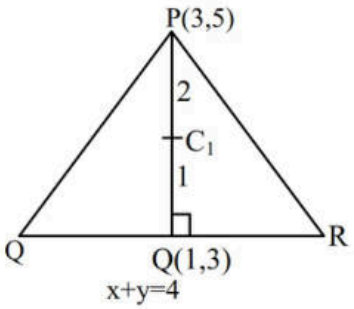
11. In an equilateral triangle PQR, let the vertex P be at (3, 5) and the side QR be along the line $x + y = 4$. If the orthocentre of the triangle PQR is (α, β), then $9(\alpha + \beta)$ is equal to :

(1) 16 (2) 27

(3) 36 (4) 48

Ans. (4)

Sol.



$Q : \frac{x-3}{1} = \frac{y-5}{1} = -\frac{(3+5-4)}{1+1}$

Q(1, 3)

So orthocentre is $\left(\frac{2+3}{3}, \frac{6+5}{3}\right)$

$\left(\frac{5}{3}, \frac{11}{3}\right)$

So $9(\alpha + \beta) = 3(16) = 48$

12. The sum of all the integral values of p such that the equation $3\sin^2 x + 12\cos x - 3 = p$, $x \in R$, has at least one solution, is :

(1) –54 (2) –60

(3) –75 (4) –84

Ans. (3)

Sol. $P = 12\cos x - 3\cos^2 x$

$P = -3(\cos^2 x - 4\cos x)$

$P = -3((\cos x - 2)^2 - 2)$

put $\cos x = -1 \Rightarrow P = -15$

put $\cos x = 1 \Rightarrow P = 9$

$-15 \le P \le 9$

Sum of all integers $= -(10 + 11 + ... + 15) = -75$

13. The square of the distance of the point P(5, 6, 7) from the line $\frac{x-2}{2} = \frac{y-5}{3} = \frac{z-2}{4}$ is equal to :

(1) 3 (2) 5

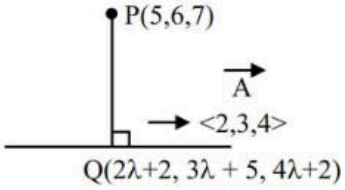
(3) 6 (4) 8

Ans. (3)

Sol. $\overrightarrow{PQ} \cdot \vec{A} = 0$

$\Rightarrow (2\lambda - 3)2 + (3\lambda - 1)3 + (4\lambda - 5)4 = 0$

$\Rightarrow \lambda = 1$



Point Q = (4, 8, 6)

$PQ^2 = 6$

14. Let $\vec{a} = \sqrt{7}\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + 2\hat{k}$. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} + \vec{a} \times \vec{b} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$, then $|3\vec{r}|^2$ is equal to :

(1) 44 (2) 54

(3) 86 (4) 132

Ans. (1)

Sol. $\vec{r} \times \vec{a} - \vec{b} \times \vec{a} = \vec{0}$

$(\vec{r} - \vec{b}) \times \vec{a} = \vec{0}$

$\vec{r} - \vec{b} = \lambda\vec{a}$

$\vec{r} = \vec{b} + \lambda\vec{a}$

$\vec{r}.\vec{a} = 0 \Rightarrow \vec{a}.\vec{b} + \lambda|\vec{a}|^2 = 0$

$\lambda = -\frac{\vec{a}.\vec{b}}{|\vec{a}|^2} = -\frac{(1-2)}{9} = \frac{1}{9}$

$\vec{r} = \vec{b} + \frac{\vec{a}}{9}$

$|3\vec{r}|^2 = 9|\vec{r}|^2 = 9\left(b^2 + \frac{a^2}{81} + \frac{2(\vec{a}.\vec{b})}{9}\right) = 44$

15. The square of the distance of the point of intersection of the lines $\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda(a\hat{i} - \hat{j})$, $a \neq 0$ and $\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu(2\hat{i} + a\hat{k})$ from the origin is :

(1) 5 (2) 10

(3) 17 (4) 26

Ans. (3)

Sol. Equating both the equations

$\left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(a\hat{i} - \hat{j}\right) = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + a\hat{k}\right)$

$1 + a\lambda = 4 + 2\mu$

$1 - \lambda = 0 \Rightarrow \lambda = 1$

$-1 = -1 + a\mu$

$a\mu = 0 \Rightarrow \mu = 0$

$a = 3$

point of intersection $= \hat{i} + \hat{j} - \hat{k} + 3\hat{i} - \hat{j} = 4\hat{i} - \hat{k}$

Ans. 17

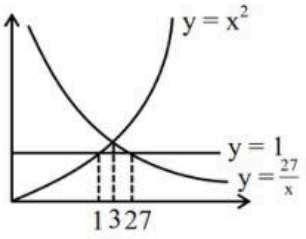
16. The area of the region

$R = \{(x,y): xy \le 27, 1 \le y \le x^2\}$ is equal to :

(1) $78\log_e 3 - \frac{52}{3}$ (2) $54\log_e 3 - \frac{52}{3}$

(3) $54\log_e 3 - \frac{26}{3}$ (4) $54\log_e 3 + \frac{26}{3}$

Ans. (2)



Area $= \int_1^3 (x^2 - 1)dx + \int_3^{27}\left(\frac{27}{x} - 1\right)dx$

$= \left[\frac{x^3}{3} - x\right]_1^3 + [27\ell nx - x]_3^{27}$

$= \frac{26}{3} - 2 + 27\,\ell n\,9 - 24$

$= 27\,\ell n\,9 - \frac{26 \times 2}{3}$

$= 54\,\ell n\,3 - \frac{52}{3}$

17. The product of all possible values of α, for which

$$\lim_{x\to 0}\left(\frac{1-\cos(\alpha x)\cos\left((\alpha+1)x\right)\cos\left((\alpha+2)x\right)}{\sin^2\left((\alpha+1)x\right)}\right)=2,$$

is :

(1) –2 (2) 1

(3) –1 (4) $\frac{5}{4}$

Ans. (3)

Sol. $\lim\limits_{x\to 0}\dfrac{1-\cos\alpha x.\cos((\alpha+1)x)\cos((\alpha+2)x)}{(\alpha+1)^2x^2}$

By using L.H. Rule we get

$\Rightarrow \dfrac{1}{2}(\alpha+1)^2[\alpha^2+(\alpha+1)^2+(\alpha+2)^2]=2$

$\alpha^2+(\alpha+1)^2+(\alpha+2)^2=4(\alpha+1)^2$

$\alpha^2+(\alpha+2)^2=3(\alpha+1)^2$

$\Rightarrow \alpha^2+2\alpha-1=0$

Product = –1

18. The value of the integral $\int_0^\infty \frac{\log_e(x)}{x^2+4}dx$ is:

(1) $\frac{\pi\log_e(2)}{2}$ (2) $\frac{\pi\log_e(2)}{4}$

(3) $1+\pi\log_e(2)$ (4) $2+\pi\log_e(2)$

Ans. (2)

Sol. Put $x=2t \Rightarrow dx=2dt$

$$I=\int_0^\infty \frac{\ell n2t}{4t^2+4}(2dt)=\frac{1}{2}\int_0^\infty \frac{\ell n2+\ell nt}{t^2+1}dt$$

$$=\frac{1}{2}\int_0^\infty \frac{\ell n2}{t^2+1}dt+\frac{1}{2}\int_0^\infty \frac{\ell nt}{t^2+1}dt$$

$$\left[\frac{\ln 2}{2}\tan^{-1}t\right]_0^\infty+I_1$$

$$=\frac{\ell n2}{2}\cdot\frac{\pi}{2}+I_1$$

$$I_2=\frac{1}{2}\int_0^\infty \frac{\ell nt}{t^2+1}dt \qquad t=\frac{1}{u}\Rightarrow dt=-\frac{du}{u^2}$$

$$=\frac{1}{2}\int_\infty^0 \frac{\ell n(1/u)}{\frac{1}{u^2}+1}\left(-\frac{du}{u^2}\right)$$

Add $\Rightarrow I_1=0$

$$I=\frac{\pi\ell n2}{4}$$

19. Let $f: R \to R$ be a differentiable function such that $f\left(\frac{x+y}{3}\right)=\frac{f(x)+f(y)}{3}$ for all x, y $\in$ R and $f'(0) = 3$. Then the minimum value of the function $g(x) = 3 + e^x f(x)$, is

(1) $3\left(\frac{e+1}{e}\right)$ (2) $3\left(\frac{e-1}{e}\right)$

(3) $\frac{3-e}{e}$ (4) 3e

Ans. (2)

Sol. $f\left(\frac{x+y}{3}\right)=\frac{f(x)+f(y)}{3}$, put x = y = 0

$f(0)=\frac{2f(0)}{3}$

$\Rightarrow f(0)=0$.....(1)

$f'\left(\frac{x+y}{3}\right)\cdot\frac{1}{3}=\frac{1}{3}f'(x)$

put x = 0

$f'\left(\frac{y}{3}\right)\frac{1}{3}=\frac{1}{3}\times 3$

$f'\left(\frac{y}{3}\right)=3$

put y = 3x

$f'(x) = 3$

integrate both sides

$f(x) = 3x + c$

from eq. (1)

$f(0) = 0$

$\Rightarrow f(x) = 3x$

Now

$g(x) = 3 + e^x \cdot 3x$

$g'(x) = 3[e^x + x \cdot e^x]$

$= 3e^x(x + 1)$

$g'(x) = 0$, at $x = -1$

$(g(x))_{min} = 3 + e^{-1}(-3)$

$=3\left[1-\frac{1}{e}\right]=\frac{3(e-1)}{e}$

20. The value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\frac{4-\operatorname{cosec}^2 x}{\cos^4 x}\right)dx$ is :

(1) $\frac{11}{\sqrt{3}}$ (2) $\frac{16}{\sqrt{3}}$

(3) $\frac{32}{3\sqrt{3}}$ (4) $\frac{64}{3\sqrt{3}}$

Ans. (3)

Sol. $=\int_{\pi/6}^{\pi/3}\frac{4}{\cos^4 x}dx - \int_{\pi/6}^{\pi/3}\frac{\operatorname{cosec}^2 x}{\cos^4 x}dx$

$=\int_{\pi/6}^{\pi/3}\frac{4}{\cos^4 x}dx - \left[-\frac{\cot x}{\cos^4 x} - \int_{\pi/6}^{\pi/3}(-\cot x)\frac{-4}{\cos^5 x}(-\sin x)\,dx\right]$

$=\frac{\cot x}{\cos^4 x}\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$=\frac{1/\sqrt{3}}{(1/2)^4} - \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^4}$

$=\frac{16}{\sqrt{3}} - \frac{\sqrt{3}.16}{9} = \frac{16}{\sqrt{3}} - \frac{16}{3\sqrt{3}} = \frac{16}{\sqrt{3}}\left(1-\frac{1}{3}\right)$

$\frac{32}{3\sqrt{3}} = \frac{32\sqrt{3}}{9}$

SECTION-B

21. Let A = {1, 2, 3, 4, 5, 6}. The number of one-one functions f : A → A such that f(1) ≥ 3, f(3) ≤ 4 and f(2) + f(3) = 5, is _____.

Ans. (72)

Sol. f(2) + f(3) = 5

C-I : f(2) = 4, f(3) = 1

so, f(1) = 3, 5, 6 → 3 choices

f(4) → 3 choices

f(5) → 2 choices

f(6) → 1 choices

so total function for C-I : 3 × 3 × 2 × 1 = 18

Similarly

C-II : f(2) = 1, f(3) = 4 → 18 function

C-III : f(2) = 3, f(3) = 2 → 18 function

C-IV : f(2) = 2, f(3) = 2 → 18 function

Answer total 72 function

22. Two players A and B play a series of games of badminton. The player, who wins 5 games first, wins the series. Assuming that no game ends in a draw, the number of ways, in which player A wins the series is _____.

Ans. (126)

Sol. Maximum number of games will be 9

A must win 5 gamese so,

Number of ways = ${}^9C_5 = 126$

Answer = 126

23. If the sum of the coefficients of x^7 and x^{14} in the expansion of $\left(\frac{1}{x^3}-x^4\right)^n$, $x \neq 0$, is zero, then the value of n is _____.

Ans. (21)

Sol. $T_{r+1} = {}^nC_r \left(\frac{1}{x^3}\right)^{n-r} \left(-x^4\right)^r$

$T_{r+1} = {}^nC_r\, x^{7r-3n} (-1)^r$

for coeff. of $x^7 \Rightarrow 7r_1 - 3n = 7$

$\Rightarrow r_1 = \frac{7+3n}{7}$

coeff. of $x^7 \Rightarrow {}^nC_{r_1} (-1)^{r_1}$

for coeff. of $x^{14} \Rightarrow 7r_2 - 3n = 14$

$\Rightarrow r_2 = \frac{14+3n}{7} = r_1 + 1$

coeff. of $x^{14} \Rightarrow {}^nC_{r_1+1} (-1)^{r_1+1}$

$\Rightarrow {}^nC_{r_1} (-1)^{r_1} + {}^nC_{r_1+1} (-1)^{r_1+1} = 0$

$\Rightarrow {}^nC_{r_1} = {}^nC_{r_1+1} \Rightarrow r_1 + r_1 + 1 = n$

$\Rightarrow 2r_1 + 1 = n \Rightarrow 2\left(\frac{7+3n}{7}\right) + 1 = n$

Ans. $\Rightarrow \boxed{n = 21}$

24. If $\frac{\pi}{4} + \sum_{p=1}^{11} \tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right) = \alpha$, then $\tan\alpha$ is equal to _____.

Ans. (2048)

Sol. $\alpha = \frac{\pi}{4} + \sum_{p=1}^{11} \tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right)$

$= \frac{\pi}{4} + \sum_{p=1}^{11} \tan^{-1}\left(\frac{2^p - 2^{p-1}}{1+2^p 2^{p-1}}\right)$

$= \frac{\pi}{4} + \sum_{p=1}^{11} \left(\tan^{-1}\left(2^p\right) - \tan^{-1}\left(2^{p-1}\right)\right)$

$= \frac{\pi}{4} + \tan^{-1}\left(2^{11}\right) - \tan^{-1}\left(2^0\right) = 2^{11}$

Ans. 2048

25. Let $y = y(x)$ be the solution of the differential equation

$x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\left(\frac{y}{x}\right) - x\right) dx$, $y(1) = \frac{\pi}{2}$ and let $\alpha = \cos\left(\frac{y\left(e^{12}\right)}{e^{12}}\right)$. Then the number of integral value of p, for which the equation $x^2 + y^2 - 2px + 2py + \alpha + 2 = 0$ represents a circle of radius $r \le 6$, is _____.

Ans. (11)

Sol. $\sin\frac{y}{x}\frac{dy}{dx} = \frac{y}{x}\sin\frac{y}{x} - 1$

put $y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$\sin t\left(t + x\frac{dt}{dx}\right) = t\sin t - 1$

$x\sin t\frac{dt}{dx} + 1 = 0$

$\sin t\, dt + \frac{dx}{x} = 0$

$\Rightarrow -\cos t + \ell nx = C$

$\Rightarrow -\cos\left(\frac{y}{x}\right) + \ell nx = C$

$y(1) = \frac{\pi}{2} \Rightarrow C = 0$

$\Rightarrow \cos\left(\frac{y}{x}\right) = \ell nx$

$\operatorname{cosec}\left(\frac{y(e^{12})}{e^{12}}\right) = 12 \Rightarrow \alpha = 12$

$x^2 + y^2 - 2px + 2py + 14 = 0$

$r = \sqrt{p^2 + p^2 - 14}$

$\Rightarrow r \le 6 \Rightarrow r^2 \le 36$

$2p^2 - 14 \le 36$

$p^2 \le 25$

$p \in [-5, 5]$

number of integral value of $p = 11$

SECTION-A

26. In a Vernier calipers, when both jaws touch each other, zero of the Vernier scale is shifted to the right of zero of the main scale and 7th Vernier division coincides with a main scale reading. If the value of 1 main scale division is 1 mm and there are 10 Vernier scale divisions, then the Vernier caliper has

(1) 0.07 cm negative zero error

(2) 0.7 cm negative zero error

(3) 0.07 cm positive zero error

(4) 0.7 cm positive zero error

Ans. (3)

Sol. Scale reading = main scale + Least count × V.S.R.

$= 0 + \frac{1}{10} \times 7 = 0.7$ mm

$= 0.07$ cm

27. L, C and R represents physical quantities inductance, capacitance and resistance respectively.

The dimensional formula $M\,L^2T^{-4}A^{-2}$ corresponds to _____.

(1) $\frac{R}{\sqrt{LC}}$ (2) $\frac{R}{LC}$

(3) $\frac{C}{\sqrt{LR}}$ (4) $\frac{1}{R}\sqrt{\frac{L}{C}}$

Ans. (1)

Sol. $R = \frac{V}{I} = \frac{w}{qI} = \frac{ML^2T^{-2}}{I^2T} = ML^2T^{-3}I^{-2}$

$U = \frac{1}{2}LI^2 \rightarrow ML^2T^{-2} = LA^2$

$L = ML^2T^{-2}A^{-2}$

$Q = CV$

$C = \frac{IT \times q}{ML^2T^{-2}} = \frac{A^2T^2}{ML^2T^{-2}}$

$C = M^{-1}L^{-2}A^{-2}T^4$

$LC = ML^2T^{-2}A^{-2} \times M^{-1}L^{-1}A^2T^4$

$LC = T^2$

$\frac{R}{\sqrt{LC}} = \frac{ML^2T^{-3}A^{-2}}{T} = (ML^2T^{-4}A^{-2})$

28. When one moves from a point 16 km below the earth's surface to a point 16 km above the earth's surface. The change in g is approximately α %. The value of α is _____.

(Take radius of the earth = 6400 km.)

(1) 0.12 (2) 0.25

(3) 0.50 (4) 0.75

Ans. (2)

Sol. $g_{above} = g - \frac{2hg}{R} = 10 - \frac{2h}{R}$

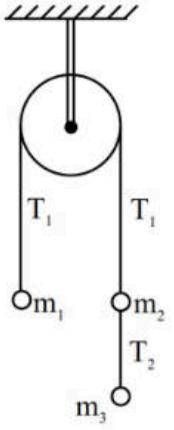
$g_{below} = g - \frac{hg}{R}$

$\Delta g = g_{above} - g_{below} = g - \frac{2hg}{R} - g + \frac{hg}{R}$

$\Delta g = \frac{hg}{R}$

$\frac{\Delta g}{g} = \frac{h}{R} = \frac{16 \times 100}{6400} = 0.25\%$

29. Three masses $m_1 = 4$ kg, $m_2 = 4$kg and $m_3 = 6$ kg are suspended from a fixed smooth frictionless pully as shown in the figure below. The value of T_1 / T_2 is _____. (take g = 10 m/s^2)

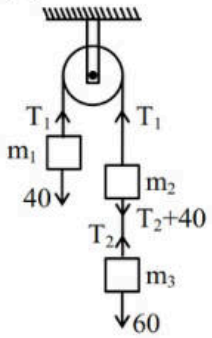


(1) $\frac{5}{3}$ (2) $\frac{2}{3}$

(3) $\frac{3}{5}$ (4) $\frac{2}{5}$

Ans. (1)

Sol.



$60 - T_2 = 6a$

$T_2 + 40 - T_1 = 4a$

$60 = 14a$

$a = \frac{60}{14} = \frac{30}{7}$

$T_1 = 40 + 4 \times \frac{30}{7} = \frac{280+120}{7}$

$T_1 = \frac{400}{7}$

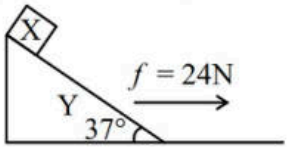
$60 - T_2 = 6\,6 \times \frac{30}{7}$

$60 - \frac{180}{7} = T_2$

$\frac{420-180}{7} = T_2 = \frac{240}{7}$

$\frac{T_1}{T_2} = \frac{5}{3}$

30. A wedge Y with mass of 10 kg and all frictionless surfaces and the inclined surface making 37° with horizontal. A block X with mass 2 kg is placed at the highest point of the wedge as shown in figure is at rest. At t = 0 wedge (Y) is pulled toward right with constant force (f) of 24 N. Taking the block X at rest at t = 0, the time taken by it to slide down 8.8 m on the slope, while Y is on the move, is __ s. (take tan (37°) = 3/4 and g = 10 m/s^2)

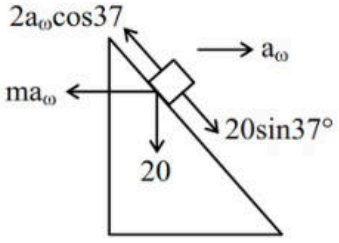


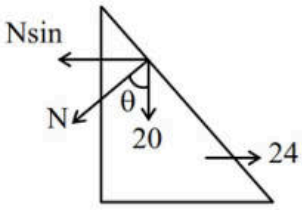
(1) 2 (2) 4

(3) $\sqrt{2}$ (4) $2\sqrt{2}$

Ans. (1)

Sol.





$20\sin37° - 2a_\omega\cos37° = 2a_{rel}$

$20\cos37° - N + 2a_\omega\sin37° = 0$

$24 - N\sin37° = 10 \times a_\omega$

$24 - (20\cos37° + 2a_\omega \sin37°) \times \sin37° = 10a_\omega$

$$24 - \left(16 + 2a_\omega \times \frac{3}{5}\right)\frac{3}{5} = 10a_\omega$$

$$24 - \frac{48}{5} - \frac{18}{25}a_\omega = 10a_\omega$$

$$\frac{72}{5} = \left(10 - \frac{18}{25}\right)a_\omega$$

$$\frac{72}{5} = a_\omega = \frac{180}{116} = \frac{90}{58}$$

$$a_\omega = \frac{45}{29}$$

$$20\sin37° - 2 \times \frac{45}{29} \times \frac{4}{5} \times \cos37° = 2\, a_{rel}$$

$$12 - \frac{72}{29} = 2a_{rel}$$

$$\frac{348 - 72}{29} = 2a_{rel}$$

$$\frac{276}{29 \times 2} = a_{rel}\,;\ \frac{138}{29} = a_{rel}$$

$$t = \sqrt{\frac{2 \times 8.8 \times 29}{138}}$$

$$t = \sqrt{\frac{2 \times 8.8 \times 29}{10 \times 138}}$$

$$t = \sqrt{3.69}$$

$$t \approx 2\,sec$$

After rounding off

31. The Young's modulus of steel wire of radius r and length L is Y. If the radius r and length L of the wire are doubled then the value of Y

(1) increases by two times

(2) reduces by half

(3) remains unchanged

(4) becomes one fourth

Ans. (3)

Sol. Young's modulus is the property of material not depends on dimensions of material.

32. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Statement I : Change in internal energy of a system containing n mole of ideal gas can be written as $\Delta U = nC_v(T_f - T_i) = \frac{nR}{\gamma - 1}(T_f - T_i)$, where $\gamma = \frac{C_P}{C_v}$, T_i = initial temperature, T_f = final temperature.

Statement II : Relation between degree of freedom f and $\gamma\ (= C_p/C_v)$ is $\left(\gamma = 1 + \frac{2}{f}\right)$

Choose the **correct** answer from the options given below

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(3) **A** is true but **R** is false

(4) **A** is false but **R** is ttue

Ans. (2)

Sol. Both statement are correct but between does not explain the statement-1

33. Consider the following statements:

A. Zeroth law of thermodynamics gives concept of temperature

B. First law of thermodynamics gives concept of internal energy

C. In isothermal expansion of ideal gas, $\Delta Q \neq \Delta W$

D. Product of intensive and extensive variables is extensive

E. The ratio of any extensive variable to mass will be an extensive variable

Choose the correct combination of statements from the options given below:

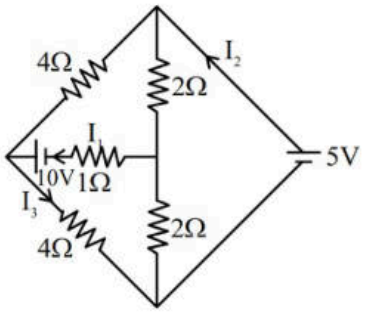
(1) C, D and E Only (2) A, B and C Only

(3) A, B and D Only (4) B, C and D Only

Ans. (3)

Sol. Statement A, B and D are correct

34. Refer to the figure given below. The values of I_1, I_2 and I_3 are _____.



(1) $I_1 = 2.5$ A, $I_2 = 1.875$ A, $I_3 = 1.875$ A

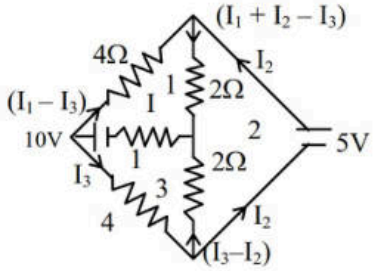
(2) $I_1 = 1.875$ A, $I_2 = 2.5$ A, $I_3 = 1.875$ A

(3) $I_1 = 1.875$ A, $I_2 = 1.875$ A, $I_3 = 2.5$ A

(4) $I_1 = 2.5$ A, $I_2 = 2.5$ A, $I_3 = 1.875$ A

Ans. (1)

Sol.



Loop-1 :

$-4(i_1 - i_3) - 2(i_1 + i_2 - i_3) - i_1 + 10 = 0$

$-7i_1 - 2i_2 + 6i_3 + 10 = 0$ ...(1)

Loop-2 :

$-5 - 2(i_3 - i_2) + 2(i_1 + i_2 - i_3) = 0$

$2i_1 + 4i_2 - 4i_3 - 5 = 0$ ...(2)

Loop-3 :

$-i_1 + 10 - 4i_3 - 2(i_3 - i_2) = 0$

$2i_2 - 6i_3 - i_1 + 10 = 0$ ...(3)

On solving equation- (i) & (ii) & (iii)

$i_1 = 2.5$ A

$i_2 = 1.875$ A

$i_3 = 1.875$ A

35. An electron of mass m is moving in an electric field $\vec{E} = -2E_0\hat{i}$ (E_0 = constant > 0), with an initial velocity $\vec{V} = v_0\hat{i}$ (v_0 = constant > 0). If $\lambda_0 = \frac{h}{4mv_0}$, its de Broglie wavelength at time t is _____. (e = charge of electron)

(1) $\frac{4\lambda_0}{\left[1 - \frac{E_0 e}{2m}\frac{t}{v_0}\right]}$ (2) $\frac{4\lambda_0}{\left[1 + \frac{E_0 e}{2m}\frac{t}{v_0}\right]}$

(3) $\frac{4\lambda_0}{\left[1 + \frac{2E_0 e}{m}\frac{t}{v_0}\right]}$ (4) $\frac{4\lambda_0}{\left[1 - \frac{2E_0 e}{m}\frac{t}{v_0}\right]}$

Ans. (3)

Sol. $V = U + at$

$$V = V_0 + \left(\frac{2eE_0}{m}\right)t$$

$$\because a = \frac{(2E_0)e}{m}$$

$$\lambda = \frac{h}{mV}$$

$$\lambda = \frac{h}{m\left[V_0 + \frac{2eE_0 t}{m}\right]}$$

Put $\lambda_0 = \frac{h}{4mV_0} \Rightarrow h = 4\lambda_0 mV_0$

$$\lambda = \frac{4m\lambda_0 V_0}{m\left[V_0 + \frac{2eE_0}{m}t\right]}$$

$$\lambda = \frac{4\lambda_0}{1 + \frac{2eE_0}{mV_0}t}$$

36. In the hydrogen atom, the electron makes a transition from the higher orbit (i) to a lower orbit (*f*). The ratio of the radius of the orbits in given by $r_i : r_f = 16 : 4$. The wavelength of photon emitted due to this transition is ___ nm. (Given Rydberg constant = 1.0973×10^7 /m)

(1) 121 (2) 242

(3) 486 (4) 974

Ans. (3)

Sol. Given : $\frac{r_i}{r_f}=\frac{16}{4}=\frac{n_1^2}{n_f^2}$

$$\frac{n_i}{n_f}=\frac{4}{2}$$

$$\frac{1}{\lambda}=R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)=1.0973\times10^7\left(\frac{1}{(2)^2}-\frac{1}{(4)^2}\right)$$

$\lambda = 4.86\times10^{-7}\ m \approx 486\ nm$

37. A displacement current of 4.0 A can be set up in the space between two parallel plates of 6 μF capacitor. The rate of change of potential difference across the plates of the capacitor is nearly $\alpha \times 10^6$ V/s. The value of α is _____.

(1) 0.58 (2) 0.67

(3) 0.82 (4) 0.75

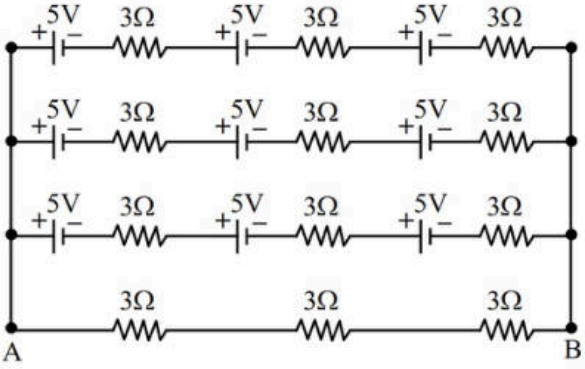
Ans. (2)

Sol. $I_d = C\frac{dV}{dt}$

$4 = (6\times10^{-6})(\alpha\times10^6)$

$\alpha=\frac{2}{3}\approx0.67$

38. Refer to the figure given below, current between terminals A and B is _____A.

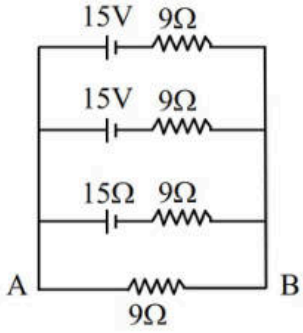


(1) 12.5 (2) 1.25

(3) 7.5 (4) 5

Ans. (2)

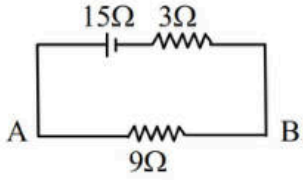
Sol. Solving each branch as series



$$\varepsilon_{eq}=\frac{\frac{15}{9}+\frac{15}{9}+\frac{15}{9}}{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}}=15\text{volt}$$

and $\frac{1}{r_{eq}}=\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$

$r_{eq}=3\Omega$



$I_{AB}=\frac{15}{12}=1.25\ A$

39. In Young's double slit experiment, the frings width of the interferences pattern produced on the screen is 2.4 μm. If the experiment is carried out in another medium having refractive index 1.2, the fringe width will be _____ μm.

(1) 1.2 (2) 2

(3) 2.4 (4) 2.88

Ans. (2)

Sol. Since Fringe width in a medium becomes

$\beta'=\frac{\beta}{\mu}$ $\left(\text{Here } \beta=\frac{\lambda D}{d}\right)$

$\beta'=\frac{\beta}{\mu}=\frac{2.4}{1.2}=2\mu m$

40. A ray of light passing through an equilateral prism is having velocity 2.12×10^8 m/s in the prism material, then the minimum angle of deviation is _____ degrees.

(1) 45 (2) 30

(3) 28 (4) 58

Ans. (2)

Sol. $$\mu = \frac{\sin\left[\frac{\delta_{min} + A}{2}\right]}{\sin\frac{A}{2}}$$

$$\mu = \frac{C}{V} = \frac{3\times10^8}{2.12\times10^8} = \sqrt{2}$$

$$\sqrt{2} = \frac{\sin\left[\frac{\delta_{min} + 60°}{2}\right]}{\sin 30°}$$

$$\sin\left[\frac{\delta_{min} + 60°}{2}\right] = \frac{1}{\sqrt{2}}$$

$$\delta_{min} + 60° = 90° \Rightarrow \delta_{min} = 30°$$

41. Light source having wavelength 331 nm is used to generate photo-electrons whose stopping potential is 0.2 V. The work function of the used metal in the experiment is $\alpha \times 10^{-19}$ J. The value of α is _____ .

$(h = 6.62 \times 10^{-34}\,J\,s, e = 1.6 \times 10^{-19}\,C \text{ and } c = 3 \times 10^8\,m/s)$

(1) 3.68 (2) 4.68

(3) 5.68 (4) 2.68

Ans. (3)

Sol. $$V_s = \frac{1}{e}\left(\frac{hc}{\lambda} - \phi\right)$$

$$\phi = \frac{hc}{\lambda} - eV_s$$

$$= \left(\frac{6.62\times10^{-34}\times3\times10^8}{331\times10^{-9}}\right) - (1.6 \times 10^{-19} \times 0.2)$$

$$= 5.68\times10^{-19}\ J$$

42. A compound microscope is designed with two symmetric biconvex lenses. The objective lens is cut vertically, creating two identical plano-convex lenses. One of them is used in place of original objective lens. To retain same magnification keeping the object distance unchanged, the tube length has to be :-

(1) increased two times

(2) increased $\frac{3}{2}$ times

(3) decreased two times

(4) decreased $\frac{3}{2}$ times

Ans. (1)

Sol. $$M = \frac{L}{F_0}\frac{D}{F_e}$$

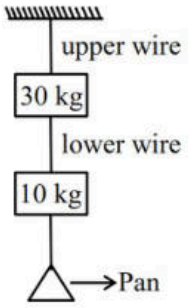
$$\frac{1}{F_0} = (n-1)\left(\frac{2}{R}\right)$$

But when cut

$$\frac{1}{F_0'} = (n-1)\left(\frac{1}{R}\right) = \frac{1}{2F_0}$$

So, $$\frac{L}{F_0} = \frac{L'}{F_0'} = 2L$$

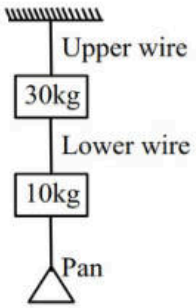
43. Two wires as shown in the figure below, made of steel and have breaking stress of 12×10^8 N/m^2 . Area of cross-section of upper wire is 0.008 cm^2 and of lower wire is 0.004 cm^2. The maximum mass that can be added to pan without breaking any wire is _____ kg. (take g = 10 m/s^2)



(1) 56 (2) 38 (3) 96 (4) 5.6

Ans. (2)

Sol.



$T_1 = mg$

$T_2 = (10 + m)\ g$

$T_3 = (30 + 10 + m)\ g = g\ (40 + m)g$

Stress in upper wire

$$= \frac{(40+m)g}{0.008cm^2}$$

$$= \frac{(40+m)\times 10}{8\times 10^{-7}}$$

Stress in lower wire =

$$= \frac{(10+m)g}{0.004cm^2}$$

$$= \frac{(10+m)\times 10}{4\times 10^{-7}}$$

Upper wire breakes :

$$\frac{(40+m)\times 10}{8\times 10^{-7}} = 12\times 10^8$$

So, maximum mass, without breaking any wire is 38 kg

44. An a.c. source of angular frequency ω is connected across a resistor R and a capacitor C in series. The current is observed as I. Now the frequency of the source is changed to ω/4, (keeping the voltage unchanged) the current is found to be I/3. The ratio of resistance to reactance at frequency ω is

(1) $\sqrt{\frac{6}{7}}$ (2) $\sqrt{\frac{3}{5}}$

(3) $\sqrt{\frac{7}{8}}$ (4) $\sqrt{\frac{3}{4}}$

Ans. (3)

Sol. As we know that current in RLC circuit is given by

$$I = \frac{V}{Z} = \frac{V}{\left[R^2 + (X_L - X_C)^2\right]^{1/2}}$$

$$I = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

As circuit is RC

$$I_0 = \frac{V_0}{(R^2 + X_C^2)^{1/2}}$$

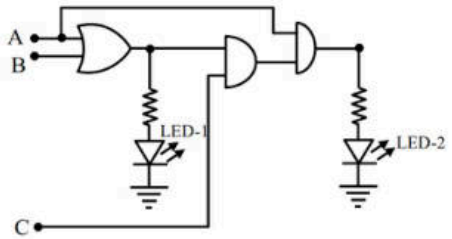
$$\frac{I_0}{3} = \frac{V_0}{[R^2 + (4X_C)^2]^{1/2}}$$

$$R^2 + (X_C)^2 = \frac{(R^2 + 16X_C^2)}{9}$$

$$\frac{8R^2}{9} = \frac{7X_C^2}{9}$$

$$\frac{R}{X_C} = \sqrt{\frac{7}{8}}$$

45. For the given logic circuit, which of the following inputs combination will make both LED-1 and LED-2 to glow ?



(1) A = 0, B = 1, C = 1

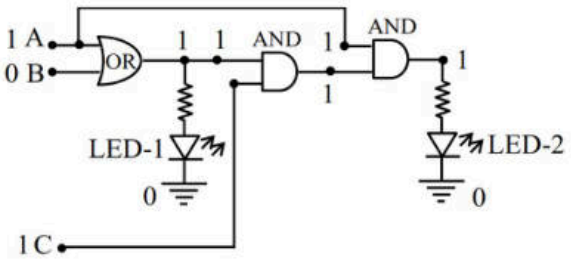
(2) A = 1, B = 0, C = 0

(3) A = 1, B = 0, C = 1

(4) A = 1, B = 1, C = 0

Ans. (3)

Sol.

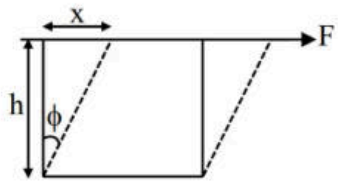


SECTION-B

46. A cube has side length 5 cm and modulus of rigidity 10^5 N/m^2. The displacement produced by a force of 10 N in the upper face of cube is _____ mm.

Ans. (2)

Sol.



$$\eta = \frac{F/A}{\phi} = \frac{Fh}{Ax}$$

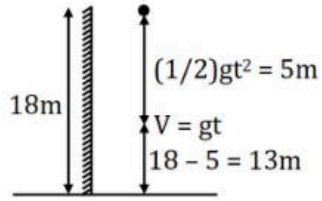
$$x = \frac{Fh}{A\eta} = \frac{10 \times 5 \times 10^{-2}}{25 \times 10^{-4} \times 10^{5}} = \frac{1}{500}\text{m}$$

$\therefore$ x = 2 mm

47. From 18 m height above the ground a ball is dropped from rest. The height above the ground at which the magnitude of velocity equal to the magnitude of acceleration (in the same set of units) due to gravity is _____ m.
(Take g = 10 m/s^2 and neglect the air resistance)

Ans. (13)

Sol.

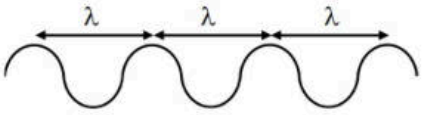


At any time t after it is released velocity V = gt
Acceleration of the ball remains constant = g
$\therefore$ Both will be equal at t = 1 sec
$\therefore$ Position of ball from ground will be (18 – 5) m

48. A transverse wave on a string is described by y = 3sin (36t + 0.018x + π/4). Where x, y are in cm and t in seconds. The least distance between the two successive crests in the wave is _____ cm. (Nearest integer) (π = 3.14)

Ans. (349)

Sol.



Least distance between successive crest = λ

$y = A\sin(\omega t + kx + \phi)$, $\omega = 2\pi f$ & $k = \frac{2\pi}{\lambda}$

& $y = A\sin(36t + 0.018x + \pi/4)$

$\Rightarrow k = 0.018\ \text{cm}^{-1}$ $\Rightarrow \frac{2\pi}{\lambda} = 0.018$

$$\lambda = \frac{2\pi}{0.018} = \frac{2 \times 3.14}{18 \times 10^{-3}}\text{cm} = 348.88\ \text{cm}$$

$\approx$ 349cm (nearest integer)

49. The charged particle moving in a uniform magnetic field of $\left(3\hat{i} + 2\hat{j}\right)$T has an acceleration $\left(4\hat{i} - \frac{x}{2}\hat{j}\right)$m/s^2. The value of x is

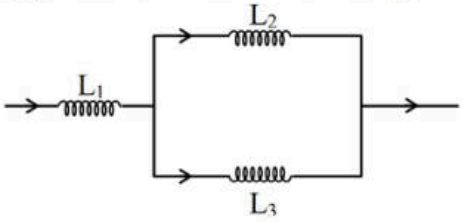
Ans. (12)

Sol. $\vec{F}_B \propto (\vec{V} \times \vec{B})$ $\Rightarrow$ $\vec{F}_B \perp \vec{B}$

$\vec{a} \perp \vec{B} \rightarrow \vec{a} \cdot \vec{B} = 0$

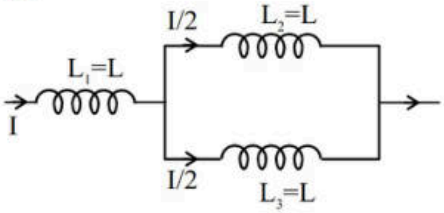
$12 - x = 0$ $\Rightarrow$ $x = 12$

50. In the given circuit below inductance values of L_1, L_2 and L_3 are same. The magnetic energy stored in the entire circuit is (U_t) and that stored in the L_2 inductor is (U_l). (U_t/U_l) is _____.
(Ignore the mutual inductance if any)



Ans. (6)

Sol.



Magnetic energy of inductor is $U = \frac{1}{2}Li^2$

$$\text{So } U_t = \frac{1}{2}LI^2 + \frac{1}{2}L\left(\frac{I}{2}\right)^2 + \frac{1}{2}L\left(\frac{I}{2}\right)^2 = \frac{3}{4}LI^2$$

$$U_\ell = \frac{1}{2}L\left(\frac{I}{2}\right)^2 \Rightarrow \frac{U_t}{U_\ell} = 6$$

SECTION-A

51. How many grams of residue is obtained by heating 2.76 g of silver carbonate ?

(Given : Molar mass of C, O and Ag are 12, 16 and 108 g mol^{-1} respectively)

(1) 1.08 g (2) 2.16 g

(3) 3.24 g (4) 4.32 g

Ans. (2)

Sol. $Ag_2CO_3(s) \xrightarrow{\Delta} 2Ag(s) + CO_2(g) + \frac{1}{2}O_2(g)$

2.76g

$\frac{1}{100}$ mol

Moles of Ag(s) = $\frac{2}{100}$ mol

Mass of residue = 2.16 g

52. Arrange the following atomic orbitals of multi electron atoms in order of increasing energy.

(A) n = 3, ℓ = 2, m = +1 (B) n = 4, ℓ = 0, m = 0

(C) n = 6, ℓ = 1, m = 0 (D) n = 5, ℓ = 1, m = +1

(E) n = 2, ℓ = 1, m = +1

Choose the correct answer from the options given below :

(1) C < D < B < A < E (2) B < A < E < C < D

(3) E < C < D < B < A (4) E < B < A < D < C

Ans. (4)

Sol. (A) n = 3, l = 2, m = +1 ; (n + l) = 5

(B) n = 4, l = 0, m = 0 ; (n + l) = 4

(C) n = 6, l = 1, m = 0 ; (n + l) = 7

(D) n = 5, l = 1, m = +1 ; (n + l) = 6

(E) n = 2, l = 1, m = +1 ; (n + l) = 3

More the value of (n + l), more will be the energy of that particular orbital.

Ans. E < B < A < D < C

53. Identify the **correct** statements from the following :

A. Heisenberg uncertainty principle is applicable to electron.

B. The size of $2p_x$ orbital is less than the size of $3p_x$ orbital.

C. The energy of 2s orbital of H atom is equal to the energy of 2s orbital of Li.

D. The electronic configuration of Cr is [Ar] $3d^5 4s^1$

Choose the correct answer from the options given below :

(1) A, B and C only (2) A, B and D only

(3) B, C, and D only (4) A, C and D only

Ans. (2)

Sol. (A) Heisenberg uncertainty principle is applicable to all the moving microscopic particles.

(B) $2p_x$ has less value of n thus has lesser size than $3p_x$.

(C) $E_{2s}(H) \neq E_{2s}(Li)$

(D) Cr :- [Ar] $3d^5$ $4s^1$

Correct statement are A, B and D.

54. What is the mole fraction of water in 10% by weight (w/w) of aqueous urea solution ?

[Given : Molar mass of H, O, C and N are 1, 16, 12 and 14 g mol^{-1} respectively]

(1) 0.825 (2) 0.032

(3) 0.867 (4) 0.967

Ans. (4)

Sol. Mass ratio = mol ratio × GMM ratio

$\frac{10}{90} = \frac{n}{N} \times \frac{60}{18}$

$\frac{n}{N} = \frac{1}{30}$

$x_{H_2O} = \frac{W}{n+N} = \frac{30}{31} = 0.967$

55. M_3A_2 is a sparingly soluble salt of molar mass y g mol^{-1} and solubility x g L^{-1}. The ratio of the molar concentration of the anion (A^{3-}) to the solubility product of the salt is

(1) $\frac{1}{54}\cdot\frac{y^4}{x^4}$ (2) $\frac{y^5}{108x^4}$

(3) $108\cdot\frac{x^5}{y^5}$ (4) $\frac{1}{108}\frac{y^4}{x^4}$

Ans. (1)

Sol.

$$M_3A_2(s) \rightleftharpoons 3M^{+2}(aq.) + 2A^{-3}(aq.)$$

Molarity	$\frac{x}{y}$	–	–
	–	3 x/y	2 x/y

$K_{sp} = [M^{+2}]^3 [A^{-3}]^2$

$K_{sp} = [3x/y]^3 [2x/y]^2$

$$\text{Ratio of } \frac{[A^{-3}]}{K_{sp}} = \frac{2x/y}{\left[\frac{3x}{y}\right]^3\left[\frac{2x}{y}\right]^2}$$

$$= \frac{1}{54}\frac{y^4}{x^4}$$

56. Arrange the following resultant mixtures in increasing order of their pH values

A. 10 mL 0.2 M $Ca(OH)_2$ + 25 mL 0.1 M HCl

B. 10 mL 0.01 M H_2SO_4 + 10 mL 0.01 M $Ca(OH)_2$

C. 10 mL 0.1 M H_2SO_4 + 10 mL 0.1 M KOH

Choose the correct answer from the options given below :

(1) B < C < A (2) C < A < B

(3) C < B < A (4) A < C < B

Ans. (3)

Sol. (A) $H^+_{(aq)} + OH^-_{(aq)} \rightarrow H_2O(\ell)$

t = 0 2.5 mmol 4 mmol

t = ∞ 0 1.5 mmol

an final solution, $[OH^-] = \frac{1.5 \text{ mmol}}{35 \text{ ml}} = \frac{1.5}{35}$

$$pOH = -\log\frac{1.5}{35}$$

$$pH = 14 + \log\left(\frac{1.5}{35}\right)$$

(B) $H^+_{(aq)} + OH^-_{(aq)} \rightarrow H_2O(\ell)$

t = 0 0.2 mmol 0.2 mmol

final pH = 7

(C) $H^+_{(aq)} + OH^-_{(aq)} \rightarrow H_2O(\ell)$

t = 0 2 mmol 1 mmol

t = ∞ 1 mmol 0

Final pH < 7

So, order of pH values :

(C) < (B) < (A)

57. First order gas phase reaction

$A \rightarrow B + C$

p_i = initial pressure of gas A, p_t = total pressure of the reaction mixture at time t

Expression of rate constant (k) is

(1) $\frac{1}{t}\ln\frac{p_i}{2p_i - p_t}$ (2) $\frac{1}{t}\ln\frac{2p_i}{p_i - p_t}$

(3) $\frac{1}{t}\ln\frac{p_i}{3p_i - 2p_t}$ (4) $\frac{1}{t}\ln\frac{3p_i}{4p_i - p_t}$

Ans. (1)

Sol.

	A(g)	→ B(g)	+ C(g)
t = 0	P_i	–	–
t = t	$P_i - x$	x	x

As given $(P_i - x) + x + x = P_t$

$x = (P_t - P_i)$

$$K = \frac{1}{t}\ell n\frac{P_i}{P_i - x} = \frac{1}{t}\ell n\frac{P_i}{P_i - (P_t - P_i)}$$

$$K = \frac{1}{t}\ell n\frac{P_i}{2P_i - P_t}$$

58. **Statement I :** The correct order of electronegativity of fluorine, oxygen nitrogen is F > O > N.

Statement II : The oxidation state of oxygen in OF_2 is +2 and in Na_2O is –2.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Ans. **(1)**

Sol. Statement-I

$\left.\begin{matrix}(EN)_F = 4.0 \\ (EN)_O = 3.5 \\ (EN)_N = 3.0\end{matrix}\right\}$ on pauling scale

$(EN)_F > (EN)_O > (EN)_N$

Statement-II

Oxidation state of oxygen in OF_2

$OF_2 \Rightarrow x - 1 \times 2 = 0$

$x = +2$

Oxidation state of oxygen in Na_2O

$Na_2O \Rightarrow +1 \times 2 + x = 0$

$x = -2$

Both the statement I and statement II are correct.

59. Correct statements from the following are :

A. Nitrogen in oxidation states from +1 to + 4 disproportionates in acid medium.

B. Nitrogen has the ability of form dπ-pπ multiple bonds with itself and other elements with small size and high electronegativity.

C. N-N single bond is stronger than P-P single bond.

D. Nitrogen has highest density in its group due to small size.

E. The maximum covalency of nitrogen is four since it has only four valence orbitals for bonding.

Choose the correct answer from the options given below :

(1) B, C, and D only (2) C, D, and E only

(3) A, C and E only (4) A and E only

Ans. **(4)**

Sol. (A) In the case of nitrogen, all oxidation states from + 1 to +4 tend to disproportionate in acid solution. For example.

$3HNO_2 \rightarrow HNO_3 + H_2O + 2NO$

(B) Nitrogen form $p_\pi - p_\pi$ bonds with itself and other elements with small size and high electronegativity, $d_\pi - p_\pi$ is not possible with itself as it has no d-orbital.

(C) The single N-N bond is weaker than the single P–P bond because of high intereelecronic repulsion of the non-bonding electrons (lone pair), owing to the small bond length.

(D) Nitrogen has lowest density in its group.

(E) The maximum covalency of nitrogen is four.

Correct answer are (A) and (E)

60. Which of the following is NOT a physical or chemical characteristics of interstitial compounds ?

(1) They have high melting points, higher than those of pure metals

(2) They are very soft ionic in nature

(3) They retain metallic conductivity

(4) They are chemically inert and usually non-stoichiometric

Ans. **(2)**

Sol. The physical and chemical characteristies of interstitial compounds are :

(1) They have high melting points, higher than those of pure metals.

(2) They are very hard, some borides approach diamond in hardness.

(3) They retain metallic conductivity

(4) They are chemically inert and usually non-stoichiometric.

Ans. (2)

61. The correct statements about metal carbonyls are

A. The metal-carbon bonds in metals carbonyls possess both σ and π-character.

B. Due to synergic bonding interactions between metals and CO ligand the metal-carbon bond becomes weak.

C. The metal-carbon σ bond is formed by the donation of lone pair of electrons on the carbonyl carbon into a vacant orbital of metal.

D. The metal-carbon π bond is formed by the donation of electrons from filled d-orbital of metal into vacant π* orbital of CO.

Choose the correct answer from the options given below :

(1) A and B only (2) A, C and D only

(3) B and C only (4) A and D only

Ans. (2)

Sol. (A) The metal-carbon bonds in metal carbonyls possess both σ and π-character.

(B) The metal to ligand bonding creates a synergic effect which strengthens the bond between CO and the metal i.e. metal carbon bond becomes strong.

(C) The metal-carbon σ is formed by the donation of lone pair of electrons on the carbonyl carbon into a vacant orbital of metal.

(D) The metal-carbon π bond is formed by the donation of electrons from filled d-orbial of metal into vacant π* orbital of CO.

Correct answer are A, C, D

62. Given below are two statements :

Statement I : Each electron in e_g orbitals destabilizes the orbitals by $+0.6\ \Delta_0$ and each electron in the t_{2g} orbitals stabilizes the orbitals by $-0.4\ \Delta_0$ in an octahedral field on the basis of crystal field theory.

Statement II : All the d-orbitals of the transition metals have the same energy in their free atomic state but when a complex is formed the ligands destroy the degeneracy of these orbitals on the basis of crystal field theory.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are correct

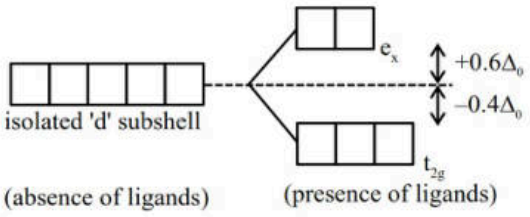
(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

Ans. (1)

Sol. Statement-I



Statement-II

The degeneracy of the d-orbitals is removed due to ligand electron-metal electron repulsions according to crystal field theory.

Both the statement I and II are correct.

63. Given below are two statements :

Statement I : On the basis of inductive effect, the order of stability of alkyl carbanions is $CH_3^- > CH_3 - CH_2^- > (CH_3)_2CH^- > (CH_3)_3C^-$.

Statement II : Allyl and benzyl carbanions are more stabilised by inductive effect and not by resonance effect.

In the light of the above statements choose the correct answer from the options given below

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct and Statement II is incorrect

(4) Statement I is incorrect and Statement II is correct

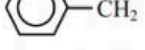
Ans. (3)

Sol. Statement-I

$$\overset{\ominus}{C}H_3 > CH_3 - \overset{\ominus}{C}H_2 > (CH_3)_2\overset{\ominus}{C}H > (CH_3)_3\overset{\ominus}{C}$$

It is correct order of anion stability and it is based on inductive effect.

Statement-II

$CH_2 = CH - \overset{\ominus}{C}H_2$ & 

Allyl anion

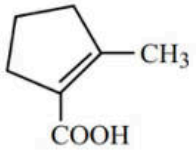
Benzyl carbanion

Both are stabilised by resonance

Statement-I : Correct

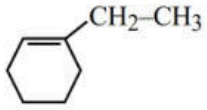
Statement-II : Incorrect

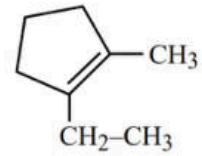
64. "P" is hydrocarbon of molecular formula : C_8H_{14}. On ozonolysis, "P" forms "Q". "Q" on treatment with alkali under reflux condition produces. "R", which on treatment with I_2/NaOH gives a yellow precipitate. Acidification of the solution gives "S". The structure of "S" given below :

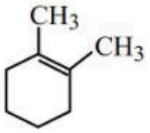


The correct structure of "P" is

(1) 

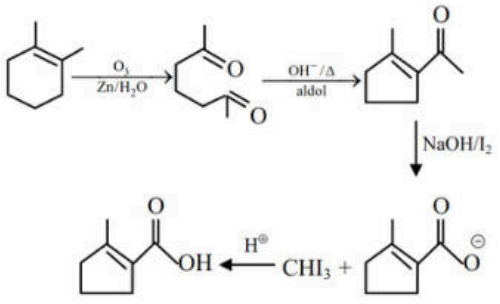
(2) 

(3) 

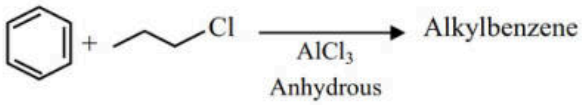
(4) 

Ans. (4)

Sol.



65. For the following Friedel Craft's alkylation reaction, which of the statements are **correct** ?



A. Major product is n-propyl benzene.

B. iso-propyl carbocation intermediate is also generated

C. Multiple substitution is inevitable.

D. Introducing electron-donating substituent on benzene will not produce any alkylbenzene.

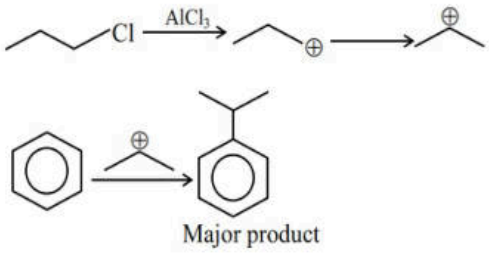
Choose the correct answer from the options given below :

(1) A and D only (2) B and C only

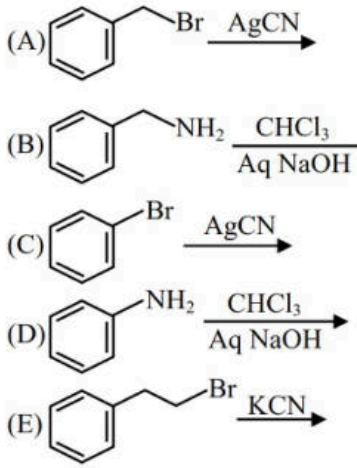
(3) A and C only (4) B and D only

Ans. (2)

Sol.



66. Benzyl isocyanide can be obtained from

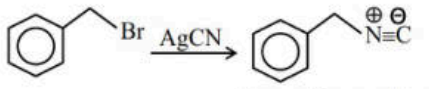


Choose the correct answer from the options given below :

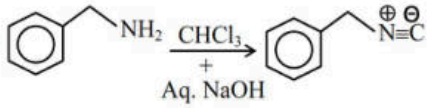
(1) A and B only (2) A and C only

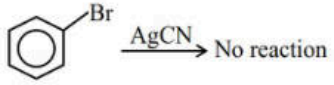
(3) B and D only (4) D and E only

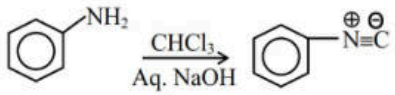
Ans. (1)

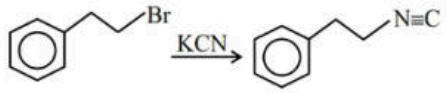
Sol. (A) 

Benzyl Isocyanide

(B) 

(C) 

(D) 

(E) 

(A) & (B) gives correct product.

Ans. (1) (A) & (B) only

67. Consider compounds A, B and C with following structural formulae

$A = CH_3 - CH_2 - CH_2 - CH_2 - CH_2 - OH$

$B = CH_2 = CH - CH_2 - CH_2 - CH_3$

$C = HO - CH_2 - CH_2 - CH(OH) - CH_3$

For the conversion of B from A, reagent (D) required is _____ and structural formula of product (E) obtained when C undergoes same reaction using excess reagent (D) is _____

(1)	D	E
	Conc. H_2SO_4	$CH_2 = CH - CH(OH)/CH_3$

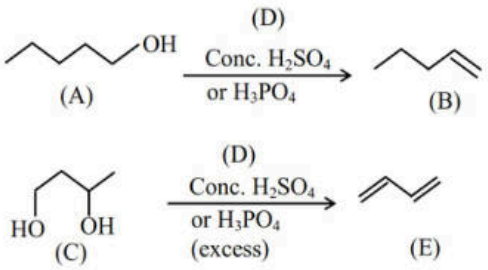
(2)	D	E
	PCC	$HO - CH_2 - CH_2 - CH = CH_2$

(3)	D	E
	PCC	$CH_2 = CH - CH = CH_2$

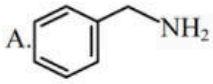
(4)	D	E
	Conc. H_2SO_4 or H_3PO_4	$CH_2 = CH - CH = CH_2$

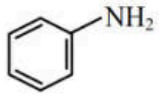
Ans. (4)

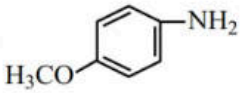
Sol.

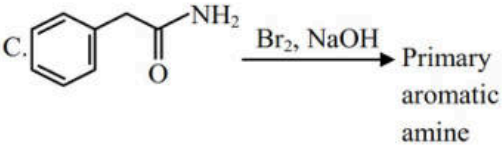


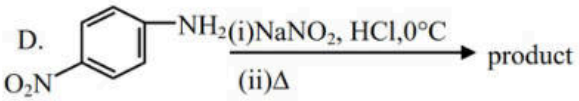
68. Identify the incorrect statements :

A.  is a stronger base than



B.  can be synthesized by Gabriel phthalimide synthesis.

C. 

D. 

will dissolve in NaOH

Choose the correct answer from the options given below :

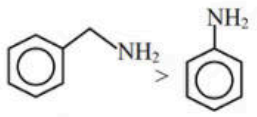
(1) A and D only

(2) A and C only

(3) B and C only

(4) A and B only

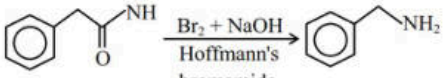
Ans. (3)

Sol. (A) 

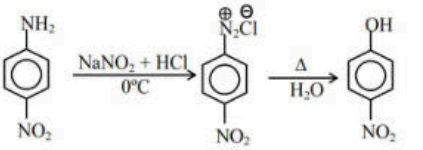
(Basic strength order)

(B) 

Cannot be synthesized by Gabriel pathalamide synthesis

(C) 

(Aliphatic amine)

(D) 

Soluble in NaOH

Only (B) and (C) are incorrect hence Ans. is (3)

69. Identify the correct statements

A. Glucose exist in two anomeric forms.

B. Anomers of glucose differ in configuration at C-1 in cyclic hemiacetal structure.

C. Melting point of α-anomer of glucose is greater than β-anomer.

D. Specific rotation of α-anomer is +19° while for β-anomer is +112°

E. α and β-anomers of glucose are prepared by crystallization of saturated glucose solution at 303 K and 371 K respectively.

Choose the correct answer from the options given below :

(1) A and B only

(2) B and C only

(3) A, B and D only

(4) A, B and E only

Ans. (4)

Sol. It is based on theory

70. **Statement I :** Sodium dichromate and potassium dichromate are classified as primary standard in titrimetric analysis .

Statement II : Phenolphthalein is a weak base, therefore it dissociates in acidic medium.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Ans. (2)

Sol. $Na_2Cr_2O_7$ is hygroscopic in nature and is not used as primary standard.

Phenolphthalein is weak acid.

SECTION-B

71. Consider the following species

$BrF_5, XeF_5^-, BF_4^-, ICl_4^-, XeF_4, SF_4, NH_4^+, ClF_3, XeF_2, ICl_2^-$

Number of species having sp^3d hybridized central atom is _____

Ans. (4)

Sol.

Species	Hybridisation
(1) BrF_5	sp^3d^2
(2) $XeF_5^{\ominus}$	sp^3d^3
(3) $BF_4^{\ominus}$	sp^3
(4) $ICl_4^{\ominus}$	sp^3d^2
(5) XeF_4	sp^3d^2
(6) SF_4	sp^3d
(7) NH_4^+	sp^3
(8) ClF_3	sp^3d
(9) XeF_2	sp^3d
(10) $ICl_2^{\ominus}$	sp^3d

Number of species having sp^3d hybridisation are 4.

72. In an estimation of sulphur by Carius method 0.2g of the substance gave 0.6 g of $BaSO_4$. The percentage of sulphur in the substance is _____%.

(Given molar mass in g mol^{-1} S : 32, $BaSO_4$: 231)

Ans. (42)

Sol. Moles of $BaSO_4$ formed $\xrightarrow{\frac{0.6gm}{231gm}} = 0.0026$

Moles of $BaSO_4$ = Moles of S

Mass of sulphur (W_S) = $\xrightarrow{\frac{0.6}{231}} \times 32$

% of sulphur (S) = $\frac{0.6\times32}{231\times0.2}\times100$

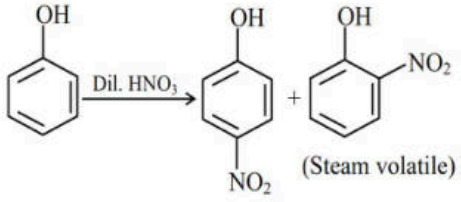
$= \frac{1920}{46.2} = 41.558 \approx 42$

73. One mole of phenol is treated with dilute HNO_3 at 298K to give a mixture of products. The mixture is separated by steam distillation. The seam volatile compound (X) is separated. The increase in percentage of oxygen in (X) with respect to phenol is _____10^{-1}%

(Given molar mass in g mol^{-1} H:1, C:12, N:14, O:16)

Ans. (175)

Sol.



(Steam volatile)

% oxygen in phenol = $\frac{16}{94}\times100 = 17.02\%$

% oxygen in o-Nitrophenol ($C_6H_5NO_3$)

Molecular mass = $C_6H_5NO_3$ = 139 g/m

$= \frac{48}{139} \times 100 = 34.53\%$

% increase = (34.53 – 17.02) = 17.5

$17.5 \times 10^{-1} = 175$ Answer

74. The values of pressure equilibrium constant recorded at different temperatures for the following equilibrium reaction have been given below:

$A(g) \rightleftharpoons B(g) + C(g)$

$\frac{1}{T}(K^{-1})$	$Log_{10}K_p$
0.05	3.5
0.06	2.5
0.07	1.5

The magnitude of $\frac{\Delta H^\circ}{R}$ calculated from the above data is _____.(Nearest integer)

Ans. (230)

Sol. Using formula

$\log K_{P_2} - \log K_{P_1} = \frac{\Delta H}{2.303\,R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$3.5 - 2.5 = \frac{\Delta H}{2.303\,R}(0.06 - 0.05)$

$\frac{\Delta H}{R} = \frac{2.303}{0.01}$

$\frac{\Delta H}{R} = 230.3 \approx 230$

75. If the half life of a first order reaction is 6.93 minutes then the time required for completion of 99% of the reaction will be_____minutes.

(Given : log 2 = 0.3010)

Ans. (46)

Sol. $t_{1/2} = \frac{\ell n2}{K} \Rightarrow K_1 = \frac{0.693}{6.93} \Rightarrow 0.1\ min^{-1}$

$t_{99\%} = \frac{1}{K}\ell n\left[\frac{A_0}{A_t}\right]$

$= \frac{1}{0.1}\ell n\left[\frac{100}{1}\right] \Rightarrow \frac{2\ell n10}{0.1}$

$= \frac{2\times2.303}{0.1}$

$\Rightarrow 46.06\ min \approx 46$