

JEE (Main)-2025 (Online) Session-2
Memory Based Question with & Solutions
(Physics, Chemistry and Mathematics)
2nd April 2025 (Shift-1)

Time: 3 hrs.

M.M.: 300

IMPORTANT INSTRUCTIONS:

- (1) The test is of 3 hours duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
- (4) Section - A : Attempt all questions.
- (5) Section - B : Attempt all questions.
- (6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.
- (7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MEMORY BASED QUESTIONS JEE-MAIN EXAMINATION – APRIL, 2025

(Held On Wednesday 2nd April, 2025)

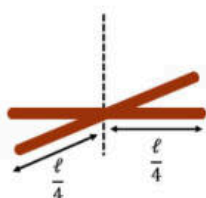
TIME : 9 : 00 AM to 12 : 00 PM

PHYSICS

SECTION-A

1. The moment of inertia of a uniform rod of mass m and length ℓ is α when rotated about an axis passing through centre and perpendicular to the length.

If the rod is broken into equal halves and arranged as shown, then the moment of inertia about the given axis is



- (1) 2α (2) $\frac{\alpha}{2}$
(3) 4α (4) $\frac{\alpha}{4}$

Ans. (4)

Sol. $\alpha = \frac{m\ell^2}{12}$

$$I' = 2I = \frac{2 \times \frac{m}{2} \left(\frac{\ell}{2}\right)^2}{12}$$

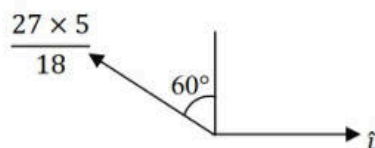
$$I' = \frac{\alpha}{4}$$

2. A river flowing with 9 km/hr in which a man can swim at the rate of 27 Km/hr in still water, if he crosses the river in 30 s heading at an angle 150° with the direction of river flow. Find width of river (in m).

- (1) $\frac{225}{4}$ (2) $\frac{225}{2}$
(3) $\frac{225}{5}$ (4) $\frac{125}{2}$

Ans. (2)

Sol.



$$t = \frac{d}{v_{MR} \cos 60^\circ}$$

$$\Rightarrow 30 = \frac{d}{27 \times \frac{5}{18} \times \frac{1}{2}}$$

$$d = \frac{225}{2}$$

3. $x_1 = \sqrt{7} \sin \omega t$

$$x_2 = 2\sqrt{7} \sin(\omega t + \pi/3)$$

Find magnitude of max. acceleration of particle if it travels according to superposition of two SHM's.

- (1) $7\omega^2$ (2) $5\omega^2$
(3) ω^2 (4) $4\omega^2$

Ans. (1)

Sol. $x_{\text{net}} = 7 \sin(\omega t + \phi)$

$$a_{\text{net}} = -7\omega^2 \sin(\omega t + \phi)$$

$$|a_{\text{max}}| = 7\omega^2$$

4. A rod is acted upon by a tensile force. Find energy density of rod.

$\eta = 0.5$: Poisson's ratio, Transverse strain $= 10^{-3}$,
 $Y = 2 \times 10^{11} \text{ N/m}^2$.

- (1) $2 \times 10^4 \text{ J/m}^3$ (2) $2 \times 10^5 \text{ J/m}^3$
(3) $1 \times 10^3 \text{ J/m}^3$ (4) $4 \times 10^5 \text{ J/m}^3$

Ans. (4)

Sol. $\frac{10^{-3}}{\frac{\Delta \ell}{\ell}} = 0.5 \Rightarrow \frac{\Delta \ell}{\ell} = 2 \times 10^{-3}$

$$\text{Energy Density} = \frac{1}{2} \times (\text{strain})^2 \times Y$$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times 2 \times 10^{11}$$

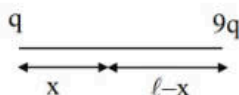
$$= 4 \times 10^5 \text{ J/m}^3$$

5. Two point charges q and $9q$ are placed at distance of ℓ from each other. The electric field is zero at a

- (1) Distance $\frac{\ell}{4}$ from charge $9q$
 (2) Distance $\frac{3\ell}{4}$ from charge q
 (3) Distance $\frac{\ell}{3}$ from charge $9q$
 (4) Distance $\frac{\ell}{4}$ from charge q

Ans. (4)

Sol.



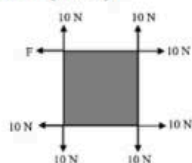
$$\frac{kq}{x^2} = \frac{k9q}{(\ell - x)^2}$$

$$\frac{\ell - x}{x} = 3$$

$$\ell = 4x$$

$$x = \frac{\ell}{4}$$

6. A square shape lamina of mass M kg is at rest. Find value of F (in N).



- (1) 10 N (2) 15 N
 (3) 20 N (4) 30 N

Ans. (1)

Sol. For equilibrium of mass M , $F_{\text{net}} = 0$
 So, $F = 10$ N

7. Find the ratio of $\left(\gamma = \frac{C_p}{C_v}\right)$ for two gases having degree of freedoms $f = 3$ and $f = 5$.

- (1) $\frac{21}{25}$ (2) $\frac{3}{7}$
 (3) $\frac{25}{21}$ (4) $\frac{7}{3}$

Ans. (3)

Sol. $\gamma = 1 + \frac{2}{f}$

$$\gamma_1 = 1 + \frac{2}{3} = \frac{5}{3}$$

$$\gamma_2 = 1 + \frac{2}{5} = \frac{7}{5}$$

$$\frac{\gamma_1}{\gamma_2} = \frac{25}{21}$$

8. The ratio of magnetic field to center of circular coil to magnetic field at distance x from the centre of circular coil $\left(\frac{x}{R} = \frac{3}{4}\right)$

- (1) $\frac{64}{125}$ (2) $\frac{64}{25}$
 (3) $\frac{32}{125}$ (4) $\frac{32}{25}$

Ans. (1)

Sol. $B_{\text{center}} = \frac{\mu_0 i}{2R}$

$$B_{\text{axis}} = \frac{\mu_0 i R^2}{2R^3 \left(1 + \frac{x^2}{R^2}\right)^{3/2}}$$

$$= \frac{\mu_0 i}{2R \left(1 + \frac{9}{16}\right)^{3/2}}$$

$$\frac{B_{\text{center}}}{B_{\text{axis}}} = \frac{64}{125}$$

9. $\left(P + \frac{an^2}{V^2}\right)(V - nb) = RT$. The dimensional formula of $\frac{a}{b^2}$ is equal to

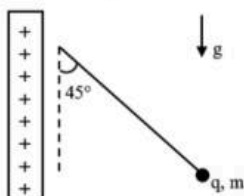
- (1) ML^3T^{-2} (2) $ML^{-1}T^{-2}$
 (3) MLT (4) ML^3T^{-1}

Ans. (2)

Sol. $[a] = \frac{PV^2}{n^2}$, $[b] = \frac{V}{n}$

$$\left[\frac{a}{b^2}\right] = \frac{PV^2}{\frac{n^2}{V^2}} = P$$

10. The figure shows an infinite charge plate having uniform charge density σ and a small charged particle having charge q and mass m suspended by a light insulating thread. Find σ if the charge is in equilibrium.

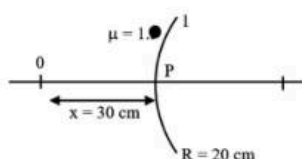


- (1) $\frac{2\epsilon_0 mg}{q}$ (2) $\frac{\epsilon_0 mg}{2q}$
 (3) $\frac{2q}{\epsilon_0 mg}$ (4) $\frac{2q\epsilon_0}{mg}$

Ans. (1)

Sol. $45^\circ = \frac{mg}{\frac{q\sigma}{2\epsilon_0}}$
 $\sigma = \frac{2\epsilon_0 mg}{q}$

11. Find the distance of image from point P

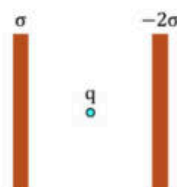


- (1) $\frac{40}{3}$ cm (2) $\frac{80}{3}$ cm
 (3) $\frac{10}{3}$ cm (4) $\frac{20}{3}$ cm

Ans. (1)

Sol. $\frac{1}{v} - \frac{1.5}{-30} = \frac{1-1.5}{20}$
 $v = -\frac{40}{3}$ cm

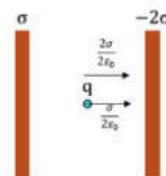
12. A charge q is placed between two infinite non-conducting sheets as shown in figure find force on charge q .



- (1) $\frac{3\sigma q}{\epsilon_0}$ (2) $\frac{2\sigma q}{\epsilon_0}$
 (3) $\frac{3\sigma q}{2\epsilon_0}$ (4) Zero

Ans. (3)

Sol. $E = \frac{3\sigma}{2\epsilon_0}$
 $F = q \left(\frac{3\sigma}{2\epsilon_0} \right)$



13. Which of the following is correct

- (1) Energy of ground state of hydrogen is equal to energy of Li^{2+} in 2nd excited state.
 (2) Energy of H^+ in ground state energy is equal to He^+ 1st excited state energy
 (3) Li^{2+} ground state energy is equal to He^+ 1st excited state energy
 (4) None of these

Ans. (1)

Sol. $E = -13.6 \frac{Z^2}{n^2}$

14. Which of the following is true

- (1) $\chi = \frac{\mu}{\mu_0} - 1$ (2) $\chi = \frac{\mu_0}{\mu} + 1$
 (3) $\chi = \frac{\mu_0}{\mu} - 1$ (4) $\chi = 1 - \frac{\mu_0}{\mu}$

Ans. (1)

Sol. $(1 + \chi) = \frac{\mu}{\mu_0}$
 $\chi = \frac{\mu}{\mu_0} - 1$

15. What is the ratio of radius of n^{th} orbit of H, He^+ , Li^{2+} ?

(Assume Bohr Model is applicable)

- (1) 6 : 3 : 1 (2) 6 : 3 : 2
 (3) 3 : 6 : 2 (4) 4 : 3 : 2

Ans. (2)

Sol. $R = K \frac{n^2}{Z}$

$$R_1 : R_2 : R_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$$

$$= 6 : 3 : 2$$

16. Match the List-I with the List-II.

List-I		List-II	
(i)	Coefficient of viscosity	(a)	$M^0 L^0 T^0$
(ii)	Strain	(b)	$M^{-1} L T^2$
(iii)	Compressibility	(c)	$M L^{-2} T^{-2}$
(iv)	Pressure gradient	(d)	$M L^{-1} T^{-1}$

(1) (i)-(a), (ii)-(c), (iii)-(d), (iv)-(b)

(2) (i)-(d), (ii)-(a), (iii)-(b), (iv)-(c)

(3) (i)-(b), (ii)-(d), (iii)-(c), (iv)-(a)

(4) (i)-(c), (ii)-(b), (iii)-(d), (iv)-(a)

Ans. (2)

Sol. Pressure gradient = $\frac{M L^{-1} T^{-2}}{L} = M L^{-2} T^{-2}$

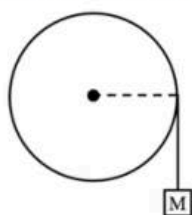
Coefficient of viscosity = $F = \eta A \frac{dv}{dx}$

Dimension = $M L^{-1} T^{-1}$

Compressibility = $\frac{1}{P} = M^{-1} L T^2$

Strain = $M^0 L^0 T^0$

17. The figure shows a disc of mass 5 kg and radius 10 cm having axis fixed and free to rotate about its axis. A 2 kg block is suspended with the help of a string wound around the disc and released from rest. The angular velocity of the disc when the block moves by 0.2 m is (Take $g = 10 \text{ ms}^{-2}$)



(1) $\frac{40}{9} \text{ rad/s}$

(2) $\frac{40}{3} \text{ rad/s}$

(3) $\frac{30}{7} \text{ rad/s}$

(4) $\frac{28}{5} \text{ rad/s}$

Ans. (2)

Sol. $\omega = \frac{V}{R}$

Apply conservation of energy

$$Mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

$$= 2 \times 10 \times 0.2 = \frac{1}{2} \times 2 \times v^2 + \frac{1}{2} \times \frac{5}{2} \times v^2$$

$$v = \frac{4}{3} \text{ m/s}$$

$$\omega = \frac{4}{3 \times 0.1} = \frac{40}{3} \text{ rad/s}$$

18. In a single slit diffraction using light of wavelength λ , the 2nd minima is formed at θ_1 and 3rd maxima is at θ_2 . If $\theta_1 + \theta_2 = 30^\circ$, then the slit width is :-

(1) $\frac{66\lambda}{\pi}$

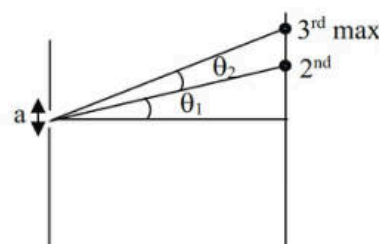
(2) $\frac{22\lambda}{\pi}$

(3) $\frac{33\lambda}{\pi}$

(4) $\frac{11\lambda}{\pi}$

Ans. (3)

Sol.



$$a \sin \theta_n = n\lambda$$

$$a \sin \theta_1 = 2\lambda$$

$$a \theta_1 = 2\lambda$$

$$\theta_1 = \frac{2\lambda}{a}$$

$$a \sin \theta_2 = \frac{7\lambda}{2}$$

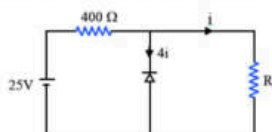
$$\theta_2 = \frac{7\lambda}{2a}$$

$$\frac{2\lambda}{a} + \frac{7\lambda}{2a} = \frac{\pi}{6}$$

$$a = \frac{33\lambda}{\pi}$$

SECTION-B

1. The zener diode maintains 5V across it, find R



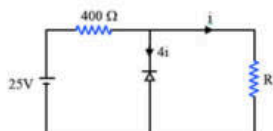
Ans. (500Ω)

Sol. $5i = \frac{400}{20} = 16$

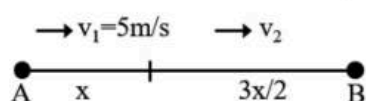
$$i = \frac{1}{100}$$

$$5 = \frac{1}{100}R$$

$$R = 500\Omega$$



2. A particle moves from A to B, such that average velocity is $\langle v \rangle = \frac{50}{7}$ m/s. Find v_2



Ans. (10)

Sol. $\frac{5x}{2} \cdot \frac{7}{50} = \frac{x}{5} + \frac{3x}{2v_2}$

$$v_2 = 10 \text{ m/s}$$

3. A photon of wavelength $4000\text{\AA}$ falls on a metal plate which is placed in a transverse uniform magnetic field of $\frac{5}{8}\mu\text{T}$ as shown in figure. Assuming the electron having maximum kinetic energy is emitted perpendicular to the plate. If the electron hits the plate again at a distance d from the point of ejection, then find the value of 2d (in SI Units). Given the work function of the metal 1.1eV. (Given $\sqrt{10} = 3.3$)

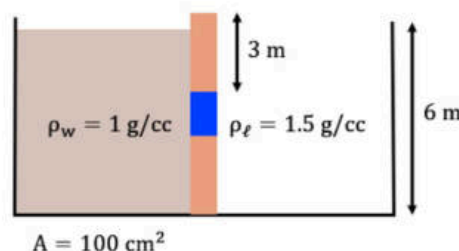


Ans. (8)

Sol. $K = E_p - \phi = 2\text{eV}$

$$R = \frac{mV}{qB} = \frac{\sqrt{2mK}}{qB} \approx 8$$

4. A tank is filled with two liquids separated by a vertical wall as shown in the figure. The left chamber contains water having density 1 g/cc & liquid in the right chamber has a density 1.5 g/cc. If a small window hinged at its bottom is located on the wall at a depth of 3 m from surface. Then find the extra force (in SI unit) needed to be applied on the window if the height of the liquid is 6 m and area of the window is 100 cm^2 .



Ans. (150)

Sol. $F_{\text{net}} = F_2 - F_1$
 $= (\rho_l gh - \rho_w gh)A$
 $= gh(500)100 \times 10^{-4}$
 $= 10 \times 3 \times (500)100 \times 10^{-4}$
 $= 150\text{N}$

CHEMISTRY

SECTION-A

1. Which of the following statement(s) is/are correct for the adiabatic process?
- (A) Molar heat capacity is zero.
 (B) Molar heat capacity is infinite.
 (C) Work done on gas is equal to increase in internal energy.
 (D) The increase in temperature results in decrease in internal energy.

- (1) A and C only (2) B and C only
 (3) A and D only (4) B and D only

Ans. (1)

Sol. Adiabatic Process $\rightarrow q = 0$

$$\Delta U = q + w$$

$$\Delta U = w$$

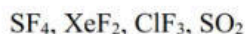
Work done on the gas $\rightarrow$ Compression $\rightarrow$ Heating

$$\Delta T \uparrow \Rightarrow \Delta U \uparrow$$

$$C_m = \frac{q}{n\Delta T} ; q = 0$$

$$C_m = 0$$

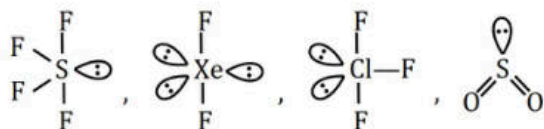
2. Species having non-zero dipole moment and highest number of lone pair on central atom. Hybridisation of central atom is



- (1) dsp^2 (2) sp^3
 (3) sp^3d (4) sp^3d^2

Ans. (3)

Sol.



Hyb. $\Rightarrow$	sp^3d	sp^3d	sp^3d	sp^2
L.P. $\Rightarrow$	1	3	2	1
	Polar	Non Polar	Polar	Polar

3. In an ideal solution 1 mol of liquid A and 3 mole liquid B, total vapour pressure of solution is 500 mm of Hg. Vapour pressure of pure A is 200 mm of Hg and vapour pressure of pure B is x mm of Hg. Find the value of x and which is the least volatile compound

- (1) 1800 A (2) 600 A
 (3) 900 B (4) 500 B

Ans. (2)

Sol. $500 = 200 \times \frac{1}{4} + \frac{3}{4} \times P_B^\circ$

$$2000 = 200 + 3P_B^\circ$$

$$3P_B^\circ = 1800$$

$$P_B^\circ = 600 \text{ mm of Hg}$$

B is more volatile

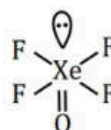
4. Compound AX_4Y , in which
- (i) All elements belongs to p-block.
 (ii) A is monoatomic and non-radioactive.
 (iii) X is most electronegative and Y is less electronegative than X

Find geometry of compound

- (1) Square pyramidal
 (2) Tetrahedral
 (3) Pentagonal bi pyramidal
 (4) Octahedral

Ans. (1)

Sol. On the basis of above information compound should be XeOF_4



Square pyramidal

5. In group 17, which property does not follow regular trend ?

- (1) Electron affinity (2) Ionisation energy
 (3) Covalent radii (4) Ionic radii

Ans. (1)

Sol. Electron affinity order $\Rightarrow \text{Cl} > \text{F} > \text{Br} > \text{I}$

6. Statement I: Atomic radius of Al is less than that of Ga

Statement II: Ionic radius of Al^{3+} is less than Ga^{3+}

Which of the following is correct

- (1) Both Statement are correct.
- (2) Statement I is correct but Statement II is incorrect.
- (3) Statement I is incorrect but Statement II is correct.
- (4) Both Statement are incorrect.

Ans. (3)

Sol. Atomic Radius : Al > Ga
(143pm) (135 pm)
Ionic Radius : Al^{+3} < Ga^{+3}
(53.5 pm) (62 pm)

7. According to Bohr's model of atom _____ has radius of 4th orbital similar to 2nd orbital of hydrogen atom.

- (1) Be^{+3} (2) Li^{+2}
(3) He^{+1} (4) None

Ans. (1)

Sol. $r = 0.529 \times \frac{n^2}{z}$

$$0.529 \times \frac{(4)^2}{z} = 0.529 \times \frac{(2)^2}{1}$$
$$z = \frac{16}{4} = 4$$

8. **Statement-I :** In octahedral complex
 $\Delta_0 > P$ low spin complex are favoured and
 $\Delta_0 < P$ high spin complex are favoured

Statement-II : $\Delta_t < P$ in most of tetrahedral complex

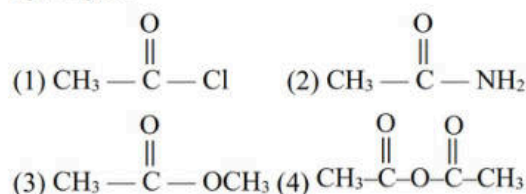
low spin complex are not found.

- (1) Statement I and Statement II both are correct.
- (2) Statement I is correct but Statement II is incorrect.
- (3) Statement I is incorrect but Statement II is correct.
- (4) Both Statements are incorrect.

Ans. (1)

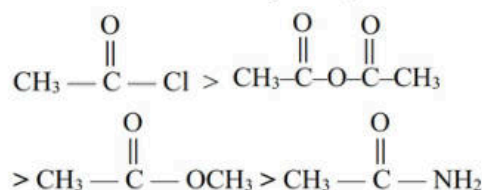
Sol. In octahedral complex
 For low spin complex $\Rightarrow \Delta_0 > P$
 For high spin complex $\Rightarrow \Delta_0 < P$
 In tetrahedral complex $\Rightarrow \Delta_t < P$

9. Which of following molecules shows fastest hydrolysis



Ans. (1)

Sol. Rate of SNAE $\propto$ electrophilicity of acid derivative



10. Which of the following is correct order of basic strength of amines in aqueous medium

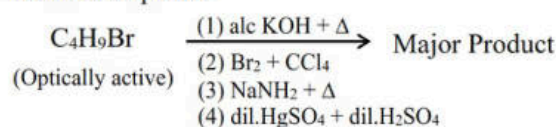
- (1) $\text{CH}_3\text{NH}_2 > (\text{CH}_3)_2\text{NH} > (\text{CH}_3)_3\text{N} > \text{NH}_3$
 (2) $(\text{CH}_3)_2\text{NH} > \text{CH}_3\text{NH}_2 > (\text{CH}_3)_3\text{N} > \text{NH}_3$
 (3) $\text{CH}_3\text{NH}_2 > \text{NH}_3 > (\text{CH}_3)_2\text{NH} > (\text{CH}_3)_3\text{N}$
 (4) $(\text{CH}_3)_3\text{N} > (\text{CH}_3)_2\text{NH} > \text{CH}_3\text{NH}_2 > \text{NH}_3$

Ans. (2)

Sol. Basicity order (factual)

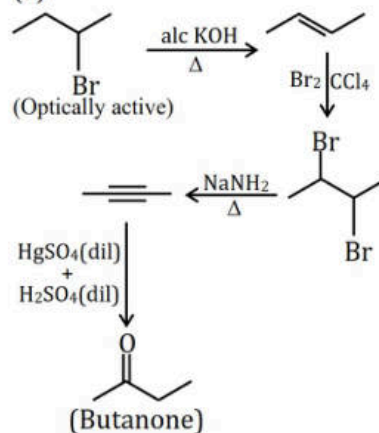
$$(\text{CH}_3)_2\text{NH} > \text{CH}_3\text{NH}_2 > (\text{CH}_3)_3\text{N} > \text{NH}_3$$

11. Which among the following will be correct IUPAC name of the major product in the given reaction sequence

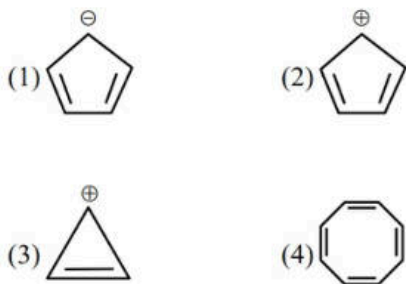


- (1) 2-oxobutane (2) Butanone
(3) 2-Formylbutane (4) Butanal

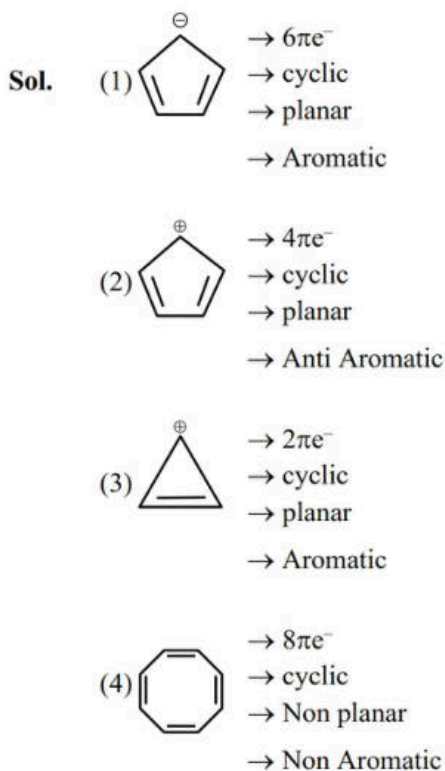
Ans. (2)



12. Out of the following species, which one is anti-aromatic?



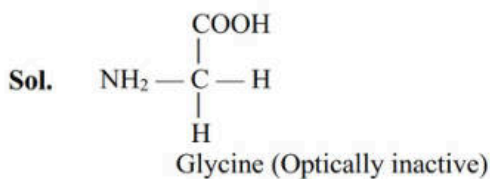
Ans. (2)



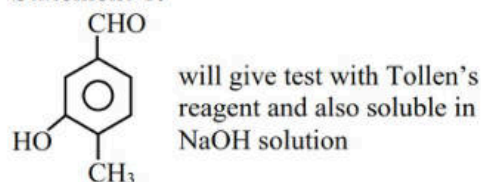
13. Which Amino acids is optically inactive

- (1) Alanine
 (2) Glycine
 (3) Valine
 (4) Aspartic Acid

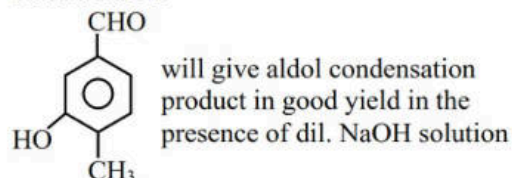
Ans. (2)



14. Statement-1:



Statement-2:



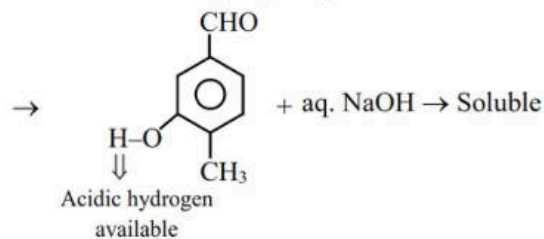
Which among the following is correct for above given statement 1 & 2.

- (1) Statement 1 and Statement 2 both are correct.
 (2) Statement 1 is correct but Statement 2 is incorrect.
 (3) Statement 1 is incorrect but Statement 2 is correct.
 (4) Both Statements are incorrect.

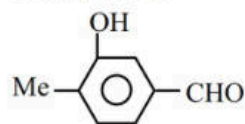
Ans. (2)

Sol. Statement-1:

$\rightarrow$ Aromatic aldehyde give positive tollen's test



Statement-2:



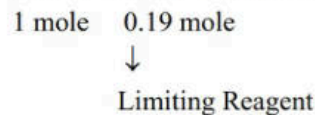
$\rightarrow$ the above compound lacks α -hydrogen
 $\rightarrow$ So, no aldol condensation possible here

SECTION-B

15. 100 g CaCO_3 when reacted with 0.19 mole of HCl then the moles of CaCl_2 formed is $P \times 10^{-3}$ mol. Find P?

Ans. (95)

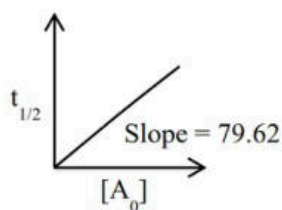
Sol. $\text{CaCO}_3 + 2\text{HCl} \rightarrow \text{CaCl}_2 + \text{H}_2\text{O} + \text{CO}_2$



$$\therefore n_{\text{CaCl}_2} (\text{Formed}) = \frac{1}{2} \times 0.19 = 95 \times 10^{-3}$$

$$\therefore P = 95$$

16. In the following graph between $t_{1/2}$ and initial concentration $[A_0]$. If slope of the graph is $79.62 \text{ M}^{-1} \text{ min.}$ and initial concentration is 2.5 M . Find the concentration of A after 10 min.



Ans. (2)

Sol. $t_{1/2} \propto [A_0]^{1-n}$

$$t_{1/2} \propto [A_0] \quad n = 0$$

$$t_{1/2} = \frac{A_0}{2k}$$

$$m = \frac{1}{2k} = 79.62$$

$$k = \frac{1}{79.62 \times 2}$$

$$[A_0] - [A_t] = kt$$

$$2.5 - [A_t] = \frac{1}{79.62 \times 2} \times 10$$

$$2.5 - \frac{5}{79.62} = [A_t]$$

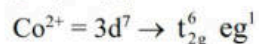
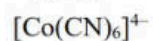
$$[A_t] = 2.43 \text{ M}$$

$$[A_t] \approx 2$$

17. For given element Co, Cr, Fe and Mn an element having highest $E_{M^{+3}/M^{2+}}^\circ$ form a complex $[M(CN)_6]^{4-}$. Find number of electron in e_g set of orbitals.

Ans. (1)

Sol. $E_{Co^{3+}/Co^{2+}}^\circ = 1.97 \text{ V}$



MATHEMATICS

SECTION-A

1. Find the maximum value of n such that $50!$ is divisible by 3^n

Ans. (22)

Sol. Exponent of 3 in

$$50! = \left[\frac{50}{3} \right] + \left[\frac{50}{3^2} \right] + \left[\frac{50}{3^3} \right] + \left[\frac{50}{3^4} \right] + \dots$$

$$= 16 + 5 + 1 + 0 + \dots$$

$$= 22$$

$\Rightarrow$ Maximum value of n is 22.

2. Let $P_n = \alpha^n + \beta^n$, $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$$(1) x^2 + x - 1 = 0 \quad (2) x^2 - 2x + 1 = 0$$

$$(3) x^2 + x - 2 = 0 \quad (4) x^2 - x - 2 = 0$$

Ans. (1)

Sol. $P_1 = 1 \Rightarrow \alpha + \beta = 1$

$$P_8 + P_9 = P_{10} \Rightarrow P_{10} - P_9 - P_8 = 0$$

$\Rightarrow$ Quadratic with roots α, β

$$\Rightarrow x^2 - x - 1 = 0$$

For quadratic with roots $\frac{1}{\alpha}, \frac{1}{\beta}$

$$\text{Put } x = \frac{1}{t} \Rightarrow \frac{1}{t^2} - \frac{1}{t} - 1 = 0$$

$$-t^2 - t + 1 = 0$$

$$\Rightarrow t^2 + t - 1 = 0$$

3. The total number of 10 digits sequences formed by only $\{0, 1, 2\}$ where 1 should be used at least 5 times and 2 should be used exactly three times, is

Ans. (2892)

Sol. Case-1:

$$\left. \begin{array}{l} 1 \text{ used 5 times} \rightarrow 11111 \\ 2 \text{ used 3 times} \rightarrow 222 \\ 0 \text{ used 2 times} \rightarrow 00 \end{array} \right\} 9C_2 \times \frac{2!}{2!} \times \frac{8!}{5!3!} = 2016$$

Case-2:

$$\left. \begin{array}{l} 1 \text{ used 6 times} \rightarrow 111111 \\ 2 \text{ used 3 times} \rightarrow 222 \\ 0 \text{ used 1 times} \rightarrow 0 \end{array} \right\} 9C_1 \times \frac{9!}{6!3!} = 756$$

Case -3 :

$$\left. \begin{array}{l} 1 \text{ used 7 times} \rightarrow 1111111 \\ 2 \text{ used 3 times} \rightarrow 222 \end{array} \right\} \frac{10!}{7!3!} = 120$$

$$\text{Final answer} = (C - I) + (C - II) + (C - III)$$

$$= 2016 + 756 + 120 = 2892.$$

4. Let $\alpha_1, \alpha_2, \alpha_3, \dots$ is an A.P. and

$$\sum_{k=1}^{12} \alpha_{2k-1} = -\frac{72}{5} \alpha_1 \text{ and } \sum_{k=1}^n \alpha_k = 0.$$

Then the value of n is

- (1) 8 (2) 10
(3) 11 (4) 13

Ans. (3)

Sol. $\sum_{k=1}^{12} \alpha_{2k-1} = \frac{-72}{5} \alpha_1$

$$\Rightarrow \alpha_1 + \alpha_3 + \dots + \alpha_{23} = -\frac{72}{5} \alpha_1$$

$$\Rightarrow \frac{12}{2} [2\alpha_1 + 11(2d)] = -\frac{72}{5} \alpha_1$$

$$\Rightarrow 5\alpha_1 + 55d = -6\alpha_1 \Rightarrow \alpha_1 = -5d$$

$$\& \sum_{k=1}^n \alpha_k = 0 \Rightarrow \alpha_1 + \alpha_2 + \dots + \alpha_n = 0$$

$$\Rightarrow \frac{n}{2} [2\alpha_1 + (n-1)d] = 0$$

$$\Rightarrow \frac{n}{2} [-10d + (n-1)d] = 0$$

$$\Rightarrow \frac{nd}{2} [n-11] = 0$$

$$\Rightarrow n = 11, \quad \left\{ \begin{array}{l} \because n \neq 0 \\ \because d \neq 0 \end{array} \right\}$$

5. If $\int_0^e \left[\frac{1}{e^{x-1}} \right] dx = \alpha - \log_e 2$, where $[\cdot]$ is greatest

integer function, then α is equal to

Ans. (2)

Sol. $I = \int_0^e \left[\frac{1}{e^{x-1}} \right] dx$

$$\frac{1}{e^{0-1}} = \frac{1}{e^{-1}} = e$$

$$f(x) = \frac{1}{e^{x-1}}$$

$$f(0) = e \quad e^{1-x} = 2$$

$$f(1) = 1 \quad 1 - x = \ln 2$$

$$f(2) = \frac{1}{e} \quad x = 1 - \ln 2$$

$$\begin{aligned} I &= \int_0^{1-\ln 2} [2] dx + \int_{1-\ln 2}^1 [1] dx \\ &= 2(1 - \ln 2) + 1 - (1 - \ln 2) \\ &= 2 - 2\ln 2 + \ln 2 \\ I &= 2 - \ln 2 = \alpha - \ln 2 \\ \Rightarrow \alpha &= 2 \end{aligned}$$

6. Given the equation of a hyperbola

$$H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ and its directrix is } x = \sqrt{\frac{10}{81}}$$

with a focus at $(\sqrt{10}, 0)$, then find the value of $9(e + \ell^2)$,

where ℓ is length of latus rectum is

- (1) 2697 (2) 2597
(3) 2487 (4) 2587

Ans. (4)

Sol. Given: $\frac{a}{e} = \sqrt{\frac{10}{81}}$ & $ae = \sqrt{10}$

$$\text{So } a^2 = \frac{10}{9} \text{ \& } \frac{ae}{\frac{a}{e}} = \frac{\sqrt{10}}{\sqrt{10}} \times 9 \Rightarrow e^2 = 9$$

$$\Rightarrow e^2 = 1 + \frac{b^2}{a^2} \Rightarrow 9 = 1 + \frac{b^2}{a^2} \Rightarrow b^2 = \frac{80}{9}$$

$$\ell = \frac{2b^2}{a} = \frac{2}{\sqrt{10}} \times \frac{80}{9} \times 3 = \frac{160}{3\sqrt{10}}$$

$$\text{So } 9 \left[3 + \frac{160 \times 160}{9 \times 10} \right] = 2587$$

7. Let the system of equations,

$$3x - y + \beta z = 3, \quad 2x + \alpha y + z = -3$$

and $x + y + 4z = 4$ has infinite solutions,

then $22\beta - 9\alpha$ equals to

- (1) 165 (2) 164
(3) 163 (4) 162

Ans. (2)

Sol. $3x - y + \beta z = 3 \quad \dots(i)$

$$2x + \alpha y + z = -3 \quad \dots(ii)$$

$$x + y + 4z = 4 \quad \dots(iii)$$

From (i) and (iii)

we have,

$$-4y + (\beta - 12)z = -9 \quad \dots(iv)$$

From (ii) & (iii)

We have,

$$(\alpha - 2)y - 7z = -11 \quad \dots(v)$$

From (iv) & (v)

We know that these are coincident lines

$$\Rightarrow \frac{\alpha - 2}{-4} = \frac{-7}{\beta - 12} = \frac{-11}{-9} \quad \dots(vi)$$

$$\Rightarrow 9\alpha - 18 = -44$$

$$\alpha = -\frac{26}{9}$$

$$\text{equation (vi) also } \Rightarrow -63 = 11\beta - 132$$

$$\beta = \frac{69}{11}$$

Now,

$$22\beta - 9\alpha = 138 + 26 = 164$$

8. If a twice differential function f satisfies

$$f''(x) = f(x) \text{ such that } f(0) = \frac{1}{2} = f'(0)$$

Then find $f''\left(\frac{\pi}{3}\right)$

(1) $e^{\frac{\pi}{3}}$ (2) $\frac{e^{\frac{\pi}{3}}}{2}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{e^{\frac{2\pi}{3}}}{2}$

Ans. (2)

Sol. $f'(x)f''(x) = f(x) \cdot f'(x)$

$$\Rightarrow \frac{(f'(x))^2}{2} = \frac{(f(x))^2}{2} + c$$

$$\Rightarrow c = 0$$

$$(f'(x))^2 = (f(x))^2$$

$$\Rightarrow f'(x) = \pm f(x)$$

$$f'(x) = f(x) \Rightarrow \frac{dy}{y} = dx$$

$$\Rightarrow \ln y = x + c$$

$$y = Ae^x \Rightarrow y = \frac{e^x}{2}$$

$$f(x) = \frac{e^x}{2}$$

$$f''(x) = \frac{1}{2}e^x$$

$$f''\left(\frac{\pi}{3}\right) = \frac{e^{\frac{\pi}{3}}}{2}$$

9. Find $\int_0^e \log_e x \, dx$

Ans. (0)

Sol. $\int_0^e \ln x \, dx$

$$\Rightarrow x \ln x - x \Big|_0^e$$

$$\Rightarrow 0 - \lim_{x \rightarrow 0} x \ln x$$

$$\Rightarrow -\lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x}} \quad \left(\frac{\infty}{\infty}\right)$$

$$\Rightarrow -\lim_{x \rightarrow 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}}$$

$$\Rightarrow +\lim_{x \rightarrow 0} x = 0$$

10. Number of solutions in $[-2\pi, 2\pi]$ for equation

$$2\sqrt{2}\cos^2\theta + (2 - \sqrt{6})\cos\theta - \sqrt{3} = 0$$

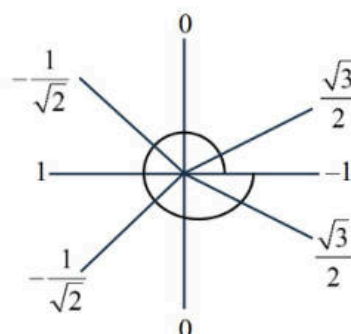
Ans. (8)

Sol. $2\sqrt{2}\cos^2\theta + 2\cos\theta - \sqrt{6}\cos\theta - \sqrt{3} = 0$

$$\Rightarrow 2\cos\theta(\sqrt{2}\cos\theta + 1) - \sqrt{3}(\sqrt{2}\cos\theta + 1) = 0$$

$$\Rightarrow 2\cos\theta - \sqrt{3} \quad \sqrt{2}\cos\theta + 1 = 0$$

$$\therefore \cos\theta = \frac{\sqrt{3}}{2}, \cos\theta = \frac{-1}{\sqrt{2}}$$



No of solution in cycle = 4

$\therefore$ Total no of solution = $2 \times 4 = 8$.

11. Term independent in

$$'x' \left[\frac{x+1}{x^{2/3} + 1 - x^{1/3}} - \frac{x-1}{x - x^{1/2}} \right]^{10}; x > 1 \text{ is}$$

Ans. 5th term

Sol. $\left(x^{\frac{1}{3}}\right)^3 + (1)^3 = \left(x^{\frac{1}{3}} + 1\right)\left(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1\right)$

$$\& x-1 = \left(x^{\frac{1}{2}} + 1\right)\left(x^{\frac{1}{2}} - 1\right)$$

So

$$\left[\left(x^{\frac{1}{3}} + 1\right) - \frac{\left(x^{\frac{1}{2}} - 1\right)\left(x^{\frac{1}{2}} + 1\right)}{x^{\frac{1}{2}}\left(x^{\frac{1}{2}} - 1\right)} \right]^{10} = \left[x^{\frac{1}{3}} - x^{\frac{1}{2}} \right]^{10}$$

$$T_{r+1} = {}^{10}C_r \left(x^{\frac{1}{3}}\right)^{10-r} \left(-x^{\frac{1}{2}}\right)^r = {}^{10}C_r (-1)^r (x)^{\frac{20-5r}{6}}$$

$$20 - 5r = 0 \Rightarrow r = 4$$

So, T_5 , i.e. 5th term.

12. Let $f(x) = 2x^3 + 9x^2a + 12a^2x + 1$.

local minima and local maxima occur at p & q respectively, such that $p^2 = q$.

then the value of $f(3)$ is

Ans. (37)

Sol. $\frac{dy}{dx} = 6x^2 + 18ax + 12a^2 = 0$

$$\Rightarrow x^2 + 3ax + 2a^2 = 0$$

$$(x + a)(x + 2a) = 0 \Rightarrow x = -a - 2a$$

$$a > 0$$

$$a < 0$$



$$a^2 = -2a$$

$$a = -2 \quad \text{Rejected}$$

$$4a^2 = -a$$

$$a = -\frac{1}{4}$$

$$f(x) = 2x^3 + \left(\frac{-1}{4}\right) \times 9x^2 + 12 \times \frac{1}{16}x + 1$$

$$f(3) = 37$$

13. The area denoted by the region

$$S = \{x > 0, y \leq 4, |4 - x^2| < y \leq x^2\} \text{ is}$$

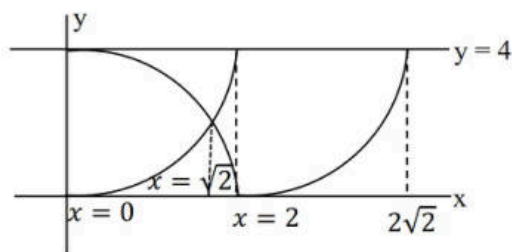
equal to $\frac{80\sqrt{2}}{\alpha} - \beta$ where $\alpha, \beta \in N$ then

$\alpha + \beta$ is equal to

Sol. Ans. (22)

Sol. $x^2 - 4 = 4 \Rightarrow x = 2\sqrt{2}$

$$4 - x^2 = x^2 \Rightarrow x = \sqrt{2}$$



$$\int_{\sqrt{2}}^2 \{x^2 - (4 - x^2)\} dx + (2\sqrt{2} - 2)4 - \int_2^{2\sqrt{2}} (x^2 - 4) dx$$

$$\left[\frac{2x^3}{3} - 4x \right]_{\sqrt{2}}^2 + 8\sqrt{2} - 8 - \left[\frac{x^3}{3} - 4x \right]_2^{2\sqrt{2}}$$

$$= \frac{16}{3} - \frac{4\sqrt{2}}{3} - 8 + 4\sqrt{2} + 8\sqrt{2} - 8 - \frac{16\sqrt{2}}{3} + \frac{8}{3} + 8\sqrt{2} - 8$$

$$= \frac{24}{3} - \frac{20\sqrt{2}}{3} - 24 + 20\sqrt{2} = \frac{40\sqrt{2}}{3} - 16$$

14. If z is a complex number and $k \in R$,

$$\text{such that } |z| = 1, \frac{2 + k^2 z}{k + \bar{z}} = kz,$$

maximum distance from $k + ik^2$ to the circle $|z - (1 + 2i)| = 1$, is

(1) $\sqrt{3} + 1$

(2) 2

(3) 3

(4) $\sqrt{5} + 1$

Ans. (4)

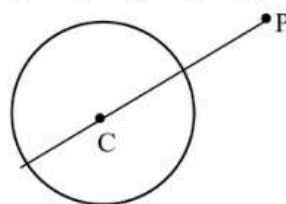
Sol. $2 + k^2 z = k^2 z + k|z|^2$

$$\Rightarrow 2 = k|z|^2$$

$$\Rightarrow 2 = k(1)^2 \Rightarrow k = 2$$

Now distance of $2 + 4i$ to circle

$$(x - 1)^2 + (y - 2)^2 = (1)^2$$



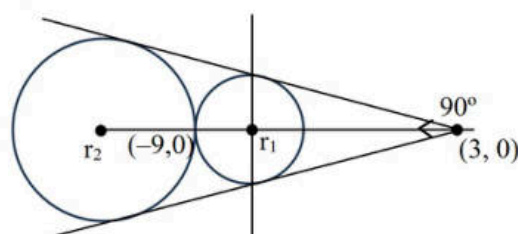
Maximum distance = $CP + r$

$$= \sqrt{(2-1)^2 + (4-2)^2} + 1 = \sqrt{5} + 1$$

15. Two circles are touching the lines $x + y = 3$ and $x - y = 3$ and passing through $(-9, 0)$, then absolute value of difference of their radii is

Ans. (24)

Sol. $r_1 + r_1\sqrt{2} = 12$



$$\sqrt{2}r_2 - r_2 = 12$$

$$r_1(1 + \sqrt{2}) = 12$$

$$r_2(\sqrt{2} - 1) = 12$$

$$|r_1 - r_2| = \left| \frac{12}{1 + \sqrt{2}} - \frac{12}{\sqrt{2} - 1} \right|$$

$$\left| \frac{12(\sqrt{2} - 1) - 12(\sqrt{2} + 1)}{2 - 1} \right| = 24$$

16. If $\lim_{x \rightarrow 0} \frac{(\gamma-1)e^{x^2} + x^2 \sin(\alpha x)}{\sin(2x) - \beta x} = 3$, then

$\alpha + 2\beta + \gamma$ is equal to

- (1) 0 (2) 1
(3) 3 (4) 5

Ans. (2)

Sol.
$$= \lim_{x \rightarrow 0} \frac{(\gamma-1) \left(1 + x^2 + \frac{x^4}{2!} \right) + x^2 \left(\alpha x - \frac{\alpha^3 x^3}{3!} \dots \right)}{2x - \frac{8x^3}{3!} \dots - \beta x}$$

$$= \lim_{x \rightarrow 0} \frac{(\gamma-1) + (\gamma-1)x^2 + \alpha x^3 + \frac{\gamma-1}{2}x^4 \dots}{(2-\beta)x - \frac{8}{3!}x^3 \dots}$$

$$\Rightarrow \gamma = 1 \quad \frac{\alpha}{-8/3!} = 3 \Rightarrow \alpha = -4$$

$$\beta = 2$$

$$\text{Now, } \alpha + 2\beta + \gamma = -4 + 2(2) + 1 = 1$$

17. Let E be an ellipse such that $E: \frac{x^2}{18} + \frac{y^2}{9} = 1$.

Let point P lies on E such that S and S' are foci of ellipse. Then find the sum of

$$\min(PS, PS') + \max(PS, PS')$$

- (1) 18 (2) 36
(3) 9 (4) 27

Ans. (4)

Sol. $PS + PS' = (a - ex_1)(a + ex_1)$

$$\Rightarrow a^2 - e^2 x_1^2$$

$$\text{Max.} = a^2$$

$$\text{min} = a^2 - e^2 a^2$$

$$\Rightarrow a^2(1 - e^2)$$

$$\Rightarrow b^2$$

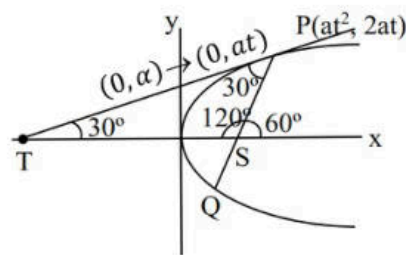
$$\text{Ans. } a^2 + b^2$$

$$\Rightarrow 18 + 9 = 27$$

18. For parabola $y^2 = 4x$, its focal chord PQ making 60° angle with its axis. A circle with PS as diameter (where S is focus), touching the y -axis at $R(0, \alpha)$, then $5\alpha^2$ is equal to

Ans. (15)

Sol.



$$PT: ty = x + at^2$$

$$PS = ST$$

$$m_T = \frac{1}{t} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$t = \sqrt{3}$$

$$\alpha = at$$

$$\Rightarrow \alpha = \sqrt{3}a = \sqrt{3} \quad (a = 1)$$

$$\text{So, } 5\alpha^2 = 15$$

19. If Q and R are two points on line

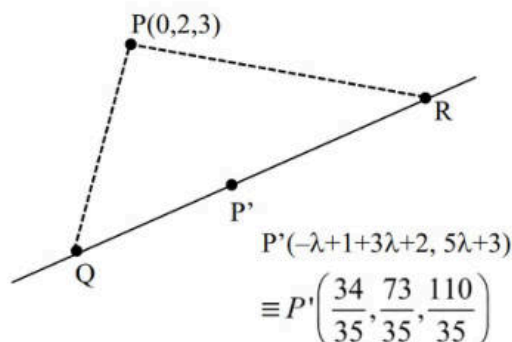
$$L: \frac{x-1}{-1} = \frac{y-2}{3} = \frac{z-3}{5} \text{ such that } QR = 5.$$

If $P(0, 2, 3)$ be any point, then the area of ΔPQR is

- (1) $\sqrt{\frac{85}{14}}$ (2) $\sqrt{\frac{75}{14}}$
(3) $\frac{\sqrt{85}}{14}$ (4) $\frac{\sqrt{75}}{14}$

Ans. (1)

Sol.



$$PP' = -\lambda + 1, 3\lambda + 0, 5\lambda$$

$$-1(-\lambda + 1) + 3(3\lambda) + 5(5\lambda) = 0$$

$$\lambda - 1 + 9\lambda + 25\lambda = 0$$

$$\lambda = \frac{1}{35}$$

$$PP' = \frac{1}{35} \sqrt{34^2 + 3^2 + 5^2}$$

$$= \frac{\sqrt{34}}{\sqrt{35}}$$

$$\text{Area} = \frac{1}{2} \times 5 \times PP'$$

$$= \frac{1}{2} \times 5 \times \frac{\sqrt{34}}{\sqrt{35}} = \frac{\sqrt{85}}{\sqrt{14}}$$