

PHYSICS

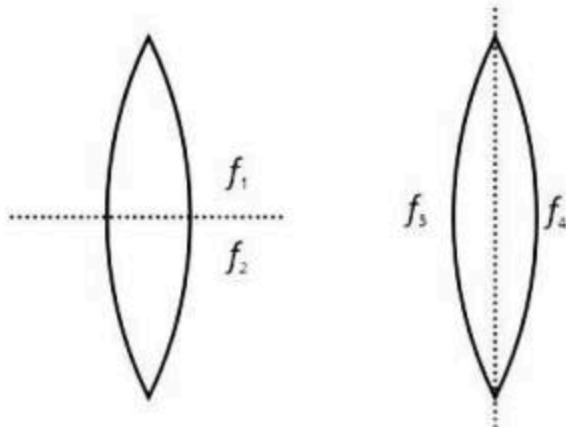
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. An equiconvex lens is cut in two ways as shown. If the focal lengths of the parts are as mentioned in the diagram. Find

$$\frac{f_1}{f_3}$$



(1) 2

(2) 4

(3) $\frac{1}{2}$

(4) $\frac{1}{4}$

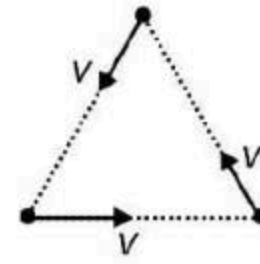
Answer (3)

Sol. $f_1 = f_2 = f$

$$f_3 = f_4 = 2f$$

$$\frac{f_1}{f_3} = \frac{1}{2}$$

2. Three identical particles, each of mass m move under the influence of mutual attractive forces. Initially they are on the vertices of an equilateral triangle of side ' a ' and have equal speed v directed towards the adjacent particle as shown. The net angular momentum about the centre just before collision is



(1) $\frac{3mva}{2}$

(2) $\frac{2}{3}mva$

(3) $\frac{\sqrt{3}}{2}mva$

(4) $\frac{2}{\sqrt{3}}mva$

Answer (3)

Sol. Net angular momentum remains conserved as there is zero net torque about centroid.

$$\Rightarrow L_i = L_f$$

$$L_f = \frac{3amv}{2\sqrt{3}} = \frac{\sqrt{3}}{2}mva$$

3. A solenoid of radius 10 cm carrying current 0.29 A and having total 200 turns. If magnetic field inside solenoid is 2.9×10^{-4} T. Find length of solenoid.

(1) 6π cm

(2) 8π cm

(3) 4.5π cm

(4) 16π cm

Answer (2)

Sol. $B = \mu_0 n_i$

$$2.9 \times 10^{-4} = 4\pi \times 10^{-7} \times \frac{200}{L} \times 0.29$$

$$L = 8\pi \text{ cm}$$

4. Match the physical quantities with their corresponding dimensions

Column-I

Column-II

(A) Young's modulus

(i) $[AL^2]$

(B) Magnetic moment

(ii) $[ML^2T^{-2}A^{-1}]$

(C) Magnetic flux

(iii) $[AL^{-1}]$

(D) Magnetic intensity

(iv) $[ML^{-1}T^{-2}]$

- (1) $A \rightarrow (\text{iv}), B \rightarrow (\text{i}), C \rightarrow (\text{ii}), D \rightarrow (\text{iii})$
 (2) $A \rightarrow (\text{iv}), B \rightarrow (\text{ii}), C \rightarrow (\text{i}), D \rightarrow (\text{iii})$
 (3) $A \rightarrow (\text{iii}), B \rightarrow (\text{i}), C \rightarrow (\text{ii}), D \rightarrow (\text{iv})$
 (4) $A \rightarrow (\text{iii}), B \rightarrow (\text{ii}), C \rightarrow (\text{i}), D \rightarrow (\text{iv})$

Answer (1)

Sol. Young's modulus $\equiv \frac{F}{A} \equiv [ML^{-1}T^{-2}]$

Magnetic moment $\equiv IA \equiv [AL^2]$

Magnetic flux $\equiv \Phi \equiv [ML^2T^{-2}A^{-1}]$

Magnetic intensity $\equiv H = nI \equiv [AL^{-1}]$

5. Two particles of same mass are performing SHM vertically with two different springs of spring constants K_1 and K_2 . If amplitude of both is same. Find ratio of the maximum speed of two particles.

- (1) $\sqrt{\frac{K_1}{K_2}}$ (2) $\sqrt{K_2 K_1}$
 (3) $\sqrt{\frac{K_2}{K_1}}$ (4) $\sqrt{\frac{K_1 + K_2}{K_1 - K_2}}$

Answer (1)

Sol. $V_{\max} = A\omega$

$$\frac{V_1}{V_2} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{K_1}{K_2}}$$

6. A physical quantity Q is given as $Q = \frac{ab^4}{cd}$, if the percentage error is a, b, c and d are 2%, 1%, 2% and 1%, the % error in Q will be
- (1) 5% (2) 15%
 (3) 9% (4) 2%

Answer (3)

Sol. $\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{4\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}$
 $= 2\% + 4\% + 2\% + 1\%$
 $= 9\%$

7. **Assertion** : On increasing the pressure, the volume decrease is more in an isothermal process than in an adiabatic process.

Reason : Adiabatic process is given by PV^γ .

- (1) Assertion is correct and Reason is false
 (2) Assertion is correct and Reason is correct
 (3) Assertion is false and Reason is correct
 (4) Assertion is false and Reason is false

Answer (2)

Sol. Isothermal $PV = \text{Constant}$

$$\frac{dP}{dV} = -\frac{P}{V}$$

$$dP = -\frac{P}{V}(dV)_{\text{isothermal}} \quad \dots(i)$$

Adiabatic $PV^\gamma = \text{Constant}$

$$\frac{dP}{dV} = -\gamma \frac{P}{V}$$

$$dP = -\frac{\gamma P}{V}(dV)_{\text{adiabatic}} \quad \dots(ii)$$

If dP is same in both process

$$(dV)_{\text{isothermal}} = \gamma (dV)_{\text{adiabatic}}$$

8. Two planet A and B are revolving around a massive star such that $r_A = 2r_B$ and $m_A = 4\sqrt{3} m_B$. Find ratio of angular momentum of planet B to planet A .
- (1) $8\sqrt{3}$ (2) $\frac{1}{8\sqrt{3}}$
 (3) $\frac{1}{2\sqrt{3}}$ (4) $\frac{1}{3\sqrt{2}}$

Answer (2)

Sol. $L = mvr$

$$L = m\sqrt{\frac{GM}{r}}r$$

$$L = m\sqrt{GM}r$$

$$\frac{L_B}{L_A} = \frac{1}{4\sqrt{3}} \cdot \frac{1}{2} = \frac{1}{8\sqrt{3}}$$

9. A capacitor $C_1 = 6 \mu\text{F}$, initially charged with a cell of emf 5V is disconnected and connected to another capacitor $C_2 = 12 \mu\text{F}$ which is initially neutral. The charges on C_1 and C_2 after connection are

- (1) $0 \mu\text{C}, 30 \mu\text{C}$ (2) $10 \mu\text{C}, 20 \mu\text{C}$
 (3) $20 \mu\text{C}, 10 \mu\text{C}$ (4) $30 \mu\text{C}, 0 \mu\text{C}$

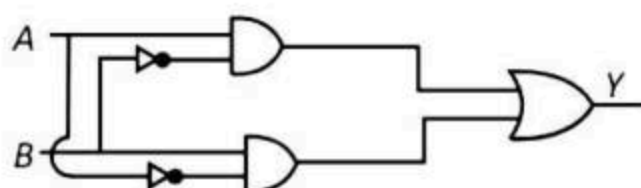
Answer (2)

Sol. Potential difference at equilibrium

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{(6 \mu\text{F})(5 \text{ V})}{(6 \mu\text{F}) + (12 \mu\text{F})} = \frac{5}{3} \text{ V}$$

$$q_1 = C_1 V = (6 \mu\text{F}) \left(\frac{5}{3} \text{ V} \right) = 10 \mu\text{C}$$

10. The truth table for the logical circuit shown below is



(1)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

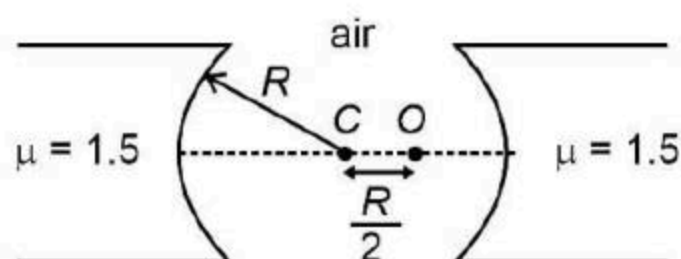
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Answer (2)

Sol. $Y = \overline{A}B + A\overline{B}$

This is a *XOR* gate

11. Figure shows two spherical surfaces of radius R having common centre. If the object is placed at O , find the distance between the first images formed by both the surfaces



- (1) $\frac{4R}{35}$ (2) $\frac{4R}{27}$
 (3) $\frac{4R}{70}$ (4) $\frac{2R}{35}$

Answer (1)

Sol. For right surface

$$\frac{1.5}{v_1} - \frac{1}{-R/2} = \frac{0.5}{-R}$$

$$\frac{1.5}{v_1} = \frac{-2}{R} - \frac{0.5}{R}$$

$$v_1 = \frac{-3R}{5}$$

For left surface

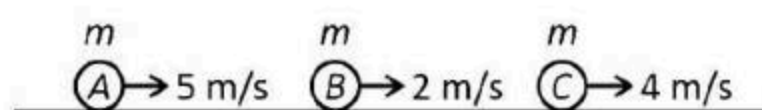
$$\frac{1.5}{v_2} - \frac{1}{-\frac{3R}{2}} = \frac{0.5}{-R}$$

$$v_2 = \frac{-9R}{7}$$

$$d = 2R - \left(\frac{3R}{5} + \frac{9R}{7} \right)$$

$$d = \frac{4R}{35}$$

12. Three particles of same mass are moving as shown. (all collisions are elastic)



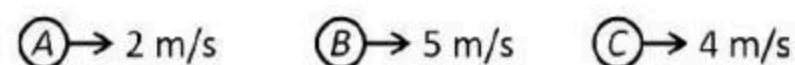
S_1 : After all collisions velocities are 4 m/s, 2 m/s and 5 m/s.

S_2 : Velocities are get interchanged in elastic collision of same mass.

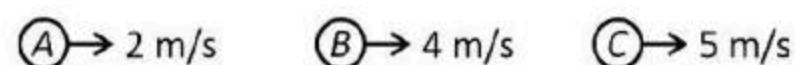
- (1) S_1 : Correct, S_2 : Correct
 (2) S_1 : Incorrect, S_2 : Correct
 (3) S_1 : Incorrect, S_2 : Incorrect
 (4) S_1 : Correct, S_2 : Incorrect

Answer (2)

Sol. After 1st collision



After 2nd collision



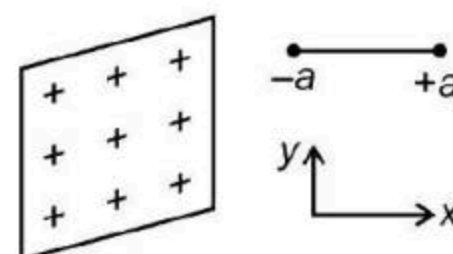
13. An electromagnetic wave propagates in +X-direction. Then, electric field and magnetic field are directed along

- (1) X, Y
 (2) Y, Z
 (3) Z, Y
 (4) Y, X

Answer (2)

Sol. $\hat{C} = \hat{E} \times \hat{B}$

14. A dipole is placed such that its axis is perpendicular to the infinite charged sheet. Select the correct options



- (a) $T_{\text{net}} = 0$, F_{net} is along -ve x-axis
 (b) $T_{\text{net}} = 0$, $U = \text{min}$
 (c) $T_{\text{net}} = 0$, $F_{\text{net}} = 0$
 (d) T_{net} and U both are maximum
 (1) (a), (b), (c) and (d)
 (2) (b) and (c)
 (3) (a) and (c)
 (4) (b) and (d)

Answer (2)

Sol. $T = \vec{p} \times \vec{E} = pE \sin \theta = 0$

$$U = -\vec{p} \cdot \vec{E} = -pE$$

$$\therefore T = 0, U = \text{min}$$

$$F_{\text{net}} = 0$$

15. A cup of coffee take a time 't' to cool from 90°C to 80°C in a surrounding of 20°C. If a similar cup of coffee is cooled from 80°C to 60°C in the same surrounding, it takes a time

- (1) $\frac{13t}{5}$
 (2) $\frac{5t}{13}$
 (3) $\frac{12t}{5}$
 (4) $2t$

Answer (1)

Sol. From Newtons law of cooling

$$-\frac{\theta_2 - \theta_1}{t} = C \left(\frac{\theta_2 + \theta_1}{2} - \theta_s \right)$$

$$\Rightarrow -\left(\frac{80^\circ\text{C} - 90^\circ\text{C}}{t}\right) = C \left(\frac{90^\circ\text{C} + 80^\circ\text{C}}{2} - 20^\circ\text{C} \right)$$

$$\frac{10^\circ\text{C}}{t} = C(65^\circ\text{C})$$

$$C = \frac{2}{13t}$$

Also,

$$-\left(\frac{60^\circ\text{C} - 80^\circ\text{C}}{t'}\right) = C \left(\frac{60^\circ\text{C} + 80^\circ\text{C}}{2} - 20^\circ\text{C} \right)$$

$$\frac{20^\circ\text{C}}{t'} = C(50^\circ\text{C})$$

$$C = \frac{2}{5t'}$$

$$\frac{2}{13t} = \frac{2}{5t'}$$

$$t' = \frac{13t}{5}$$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A converging lens of focal length 24 cm, made of glass ($\mu_{\text{glass}} = 1.5$) is immersed completely in water ($\mu_{\text{water}} = 1.33$). It will now behave like a converging lens of focal length _____ cm.

Answer (96)

$$\text{Sol. } f_{\text{air}} (\mu_{\text{glass}} - 1) = f_{\text{water}} \left(\frac{\mu_{\text{glass}}}{\mu_{\text{water}}} - 1 \right)$$

$$(+24 \text{ cm}) (1.5 - 1) = f_{\text{water}} \left(\frac{1.5}{1.33} - 1 \right)$$

$$24 \times \frac{1}{2} = f \times \frac{1}{8}$$

$$f = 12 \times 8$$

$$f_{\text{water}} = 96 \text{ cm}$$

22. Find the number of spectral lines in H-atom when de-excite from $n = 4$ to ground state

Answer (6)

$$\text{Sol. Number} = 3 \times 2$$

$$= 6$$

23. For a certain mechanical system the rate of accretion $\frac{dm}{dt}$ is proportional to $\sqrt{v}$, where m is mass, t is time and v is velocity, then the power is proportional to $v^{n/2}$ where n is _____.

Answer (5)

$$\text{Sol. } F = \left(\frac{dm}{dt} \right) v = (R \sqrt{v}) v = R v^{3/2}$$

$$P = F v = (R v^{3/2}) v = R v^{5/2}$$

24.

25.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which one of the following forms most stable carbocation ?

- (1) $(\text{Ph})_3\text{C}-\text{Br}$
- (2) $\text{C}_6\text{H}_5\text{CH}_2\text{Br}$
- (3) $\text{C}_6\text{H}_5\text{CH}(\text{Br})\text{CH}_3$
- (4) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{Br}$

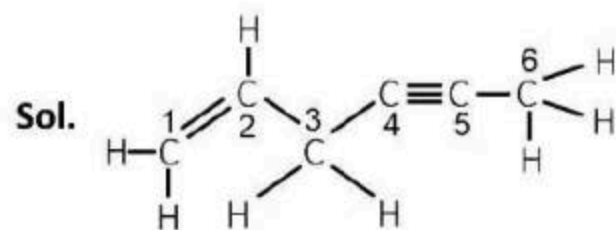
Answer (1)

Sol. $(\text{Ph})_3\text{C}-\text{Br}$ Forms $\text{Ph}-\overset{\oplus}{\text{C}}(\text{Ph})_2$ as the most stable intermediate among the given compounds.

2. Number of σ and π bonds respectively in hex-1-en-4-yne are

- (1) 13, 3
- (2) 14, 3
- (3) 3, 14
- (4) 14, 13

Answer (1)



Hex-1-en-4-yne

$\Rightarrow 13 \sigma$ and 3π bonds

3. Which element in group 15 has the lowest Ionisation Energy

- (1) Bi
- (2) P
- (3) As
- (4) Sb

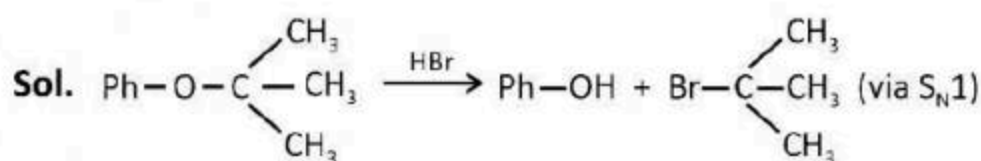
Answer (1)

Sol. $\text{N} > \text{P} > \text{As} > \text{Sb} > \text{Bi}$
 $\text{1402} \quad \text{1012} \quad \text{947} \quad \text{834} \quad \text{703 kJ/mol}$

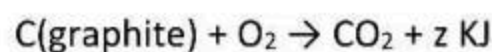
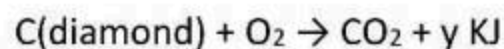
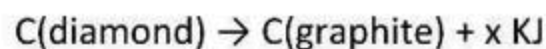
4. Which of the following ether react with HBr to form phenol?

- (1) $\text{Ph}-\text{CH}_2-\text{O}-\text{CH}_2-\text{CH}_3$
- (2) $\text{Ph}-\text{CH}_2-\text{OCH}_3$
- (3) $\text{Ph}-\text{O}-\text{C}(\text{CH}_3)_2$
- (4) $\text{Ph}-\text{CH}_2-\text{O}-\text{CH}_2-\text{Ph}$

Answer (3)



5. Consider the following thermochemical reactions and choose the correct option.



- (1) $x = y + z$
- (2) $x = y - z$
- (3) $x + y = z$
- (4) $x + y = -z$

Answer (2)

- Sol.** (1) $C(\text{diamond}) \rightarrow C(\text{graphite}) \Delta H_1 = -x\text{kJ}$
 (2) $C(\text{diamond}) + O_2(g) \rightarrow CO_2(g) \Delta H_2 = -y\text{kJ}$
 (3) $C(\text{graphite}) + O_2(g) \rightarrow CO_2(g) \Delta H_3 = -z\text{kJ}$

From (1), (2) and (3), we get

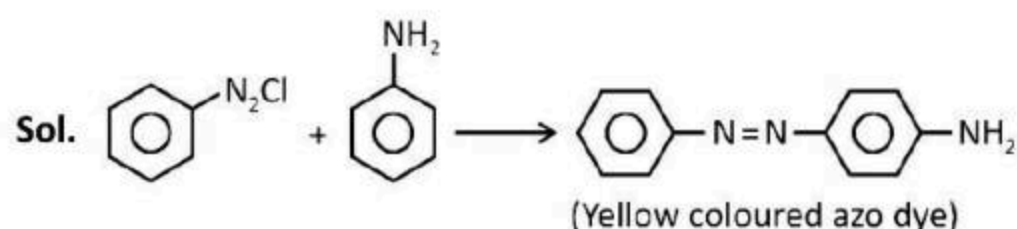
$$\Delta H_1 = \Delta H_2 - \Delta H_3$$

$$-x = -y + z$$

$$x = y - z$$

6. Which of the following will give azo dye test?
 (1) Aniline (2) Anisole
 (3) Benzene (4) Benzaldehyde

Answer (1)



7. Which of the following is an essential amino acid?
 (1) Alanine (2) Glycine
 (3) Valine (4) Aspartic acid

Answer (3)

Sol. Tryptophan, Threonine, Histidine, Valine, Isoleucine, Phenylalanine, Methionine, Arginine, Leucine and Lysine are essential amino acids.

8. A drug becomes ineffective when it decomposes to 50 % its concentration. If 16 mg of said drug becomes 4 mg in 12 months, find the time in which drug becomes ineffective given that decomposition of drug follows first order kinetics.
 (1) 6 months (2) 3 months
 (3) 2 months (4) 12 months

Answer (1)

Sol. Drug $\xrightarrow{1^{\text{st}} \text{ order}}$ Products

Initial mass of drug = 16 mg

Mass of drug after 12 months = 4 mg

$$t_{3/4} = 12 \text{ months}$$

$$2t_{1/2} = 12 \text{ months}$$

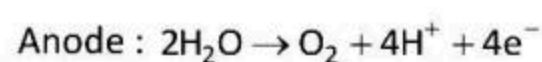
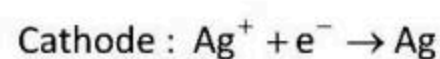
$$t_{1/2} = 6 \text{ months}$$

$\therefore$ Drug becomes ineffective in 6 months.

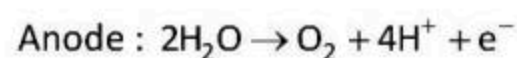
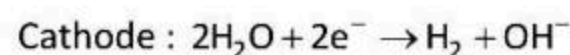
9. Which of the following gives O_2 predominantly on electrolysis among the following?
 A. Aq. $AgNO_3$ (Pt electrodes)
 B. Aq. $AgNO_3$ (Ag electrodes)
 C. Conc. H_2SO_4 (Pt electrodes)
 D. Dilute H_2SO_4 (Pt electrodes)
 (1) A, B only
 (2) B, C only
 (3) A, B, C only
 (4) A, D only

Answer (4)

Sol. Aq. $AgNO_3$ (Pt electrodes)



Dilute H_2SO_4 (Pt electrodes)



SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. 0.41 g of BaSO_4 is obtained from 0.2 g of organic compound in Carius method. What is the percentage of sulphur present in organic compound?

Answer (28)

Sol. Moles of $\text{BaSO}_4 = \frac{0.41}{233} \text{ mol}$

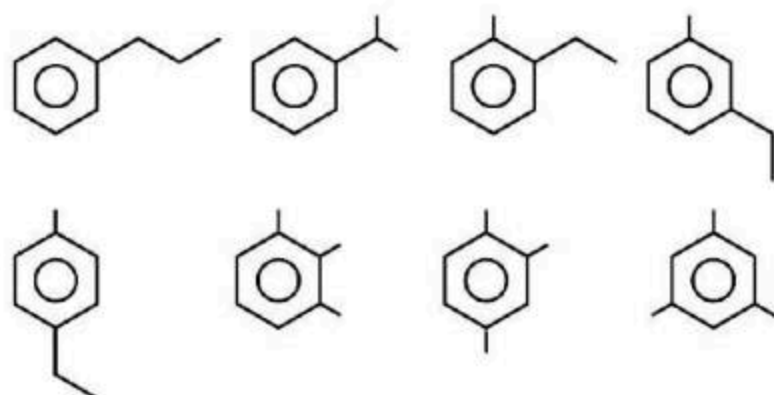
$$\begin{aligned} \text{Mass of sulphur} &= \frac{0.41}{233} \times 32 \text{ g} \\ &= 0.056 \text{ g} \end{aligned}$$

$$\begin{aligned} \% \text{ of sulphur in organic compound} &= \frac{0.056}{0.2} \times 100 \\ &= 28\% \end{aligned}$$

22. The number of benzenoid structural isomers having molecular formula C_9H_{12} which do not give Baeyer's reagent test is ?

Answer (8)

Sol. D.U. = $\frac{18 + 2 - 12}{2} = 4$



Baeyer's Reagent (cold dil. KMnO_4) reacts with alkene and alkynes and not with benzene.

23. How many maximum spectral lines are observed when a sample of hydrogen atoms de-excited from $n = 4$ to $n = 1$?

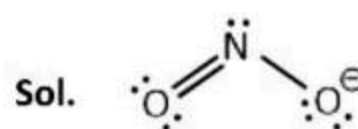
Answer (6)

Sol. Maximum number of spectral lines = $\frac{n(n-1)}{2}$

$$= \frac{4(4-1)}{2} = \frac{12}{2} = 6$$

24. Find number of non-bonding electron in NO_2^- ion is _____.

Answer (12)



$$\begin{aligned} \text{Number of non-bonding electrons will be} \\ &= 4 + 2 + 6 \\ &= 12 \end{aligned}$$

25. Find spin only magnetic moment of yellow coloured complex compound
 $\text{K}_3[\text{Co}(\text{NO}_2)_6]$, $\text{Cu}_2[\text{Fe}(\text{CN})_6]$, $\text{Zn}_2[\text{Fe}(\text{CN})_6]$, $\text{Cu}_3[\text{Fe}(\text{CN})_6]_2$

Answer (0)

Sol. $\text{Cu}_2[\text{Fe}(\text{CN})_6]$ = Chocolate brown ppt

$\text{Zn}_2[\text{Fe}(\text{CN})_6]$ = White ppt

$\text{Cu}_3[\text{Fe}(\text{CN})_6]_2$ = Green ppt

$\text{K}_3[\text{Co}(\text{NO}_2)_6]$ = Yellow ppt

In $\text{K}_3[\text{Co}(\text{NO}_2)_6]$, Co^{3+} with $\text{SFL}(\text{NO}_2^-)$ has electronic configuration $t_{2g}^6 e_g^0$

Number of unpaired $e^- = 0$

So, $\mu = 0$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If the letters of the word "KANPUR" are arranged in dictionary, then the 440th word is

- (1) PRKAUN
- (2) PRKUAN
- (3) PRKNAU
- (4) PRKUNA

Answer (1)

Sol. A K N P R U

A ----- 5!

K ----- 5!

N ----- 5!

PA ----- 4!

PK ----- 4!

PN ----- 4!

PRA ----- 3!

438

PRKANU → 439th

PRKAUN → 440th

2. Let $f(x) = \int_0^x t(t^2 - 3t + 20)dt$, $x \in (1, 3)$ and range of $f(x)$ is (α, β) , then $\alpha + \beta$ is equal to

- (1) $\frac{185}{4}$
- (2) $\frac{185}{2}$
- (3) $\frac{185}{3}$
- (4) $\frac{37}{4}$

Answer (2)

Sol. $f(x) = \frac{x^4}{4} - x^3 + 10x^2$

$$f'(x) = x^3 - 3x^2 + 20x$$

$$= x(x^2 - 3x + 20)$$

in $(1, 3)$ $f'(x)$ is positive

$\therefore f(x)$ is increasing in $(1, 3)$

$$\therefore \alpha = f(1) = \frac{37}{4}$$

$$\beta = f(3) = 83.25 = \frac{333}{4}$$

$$\therefore \alpha + \beta = 92.5 = \frac{185}{2}$$

3. The value of the limit

$$\lim_{x \rightarrow 0} (\operatorname{cosec} x) \left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4} \right) \text{ is}$$

(1) 0

(2) 1

(3) $\frac{1}{2\sqrt{5}}$

(4) $-\frac{1}{2\sqrt{5}}$

Answer (4)

Sol. After rationalization,

$$\lim_{x \rightarrow 0} \frac{1}{\sin x} \left(\frac{(2\cos^2 x + 3\cos x) - (\cos^2 x + \sin x + 4)}{\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}} \right)$$

$$\lim_{x \rightarrow 0} \frac{\cos^2 x + 3\cos x - \sin x - 4}{(\sin x)(\sqrt{5} + \sqrt{5})}$$

$$= \lim_{x \rightarrow 0} \frac{1}{2\sqrt{5}} \frac{\cos^2 x + 3\cos x - \sin x - 4}{\sin x}$$

L'Hopital,

$$\Rightarrow \lim_{x \rightarrow 0} \left(\frac{1}{2\sqrt{5}} \right) \left(\frac{2\cos x(-\sin x) - 3\sin x - \cos x}{\cos x} \right)$$

$$= \frac{1}{2\sqrt{5}} \left[\frac{-1}{1} \right] = -\frac{1}{2\sqrt{5}}$$

4. Let the line L be $\frac{x-1}{1} = \frac{y-4}{3} = \frac{z-7}{5}$ and foot of perpendicular from $(1, -2, -1)$ to L is (α, β, γ) , then $\alpha + \beta + \gamma$ is

- (1) $-\frac{69}{35}$ (2) $\frac{102}{35}$
(3) $\frac{69}{35}$ (4) $-\frac{102}{35}$

Answer (4)

Sol. $\frac{x-1}{1} = \frac{y-4}{3} = \frac{z-7}{5} = \lambda$

Point on line $A(\lambda + 1, 3\lambda + 4, 5\lambda + 7)$

$B(1, -2, -1)$

$$\overrightarrow{AB} \cdot \langle 1, 3, 5 \rangle = 0$$

$$\lambda \cdot 1 + 3(3\lambda + 6) + 5(5\lambda + 8) = 0$$

$$35\lambda + 18 + 40 = 0$$

$$\lambda = -\frac{58}{35}$$

$$(\alpha, \beta, \gamma) = \left(-\frac{23}{35}, -\frac{34}{35}, -\frac{9}{7}\right)$$

$$\alpha + \beta + \gamma = -\frac{102}{35}$$

5. If the exhaustive values of a for which the equation $2x^2 + (a-5)x + 15 = 3a$ has no real roots is (α, β) , then $|4(\alpha + \beta)|$ is equal to

- (1) 56 (2) 52
(3) 54 (4) 18

Answer (1)

Sol. No real roots $\Rightarrow$ discriminant is negative

$$\Rightarrow (a-5)^2 - 4(2)(15-3a) < 0$$

$$\Rightarrow a^2 - 10a + 25 - 120 + 24a < 0$$

$$a^2 + 14a - 95 < 0$$

$$(a+19)(a-5) < 0$$

$$\Rightarrow a \in (-19, 5)$$

$$\alpha = -19$$

$$\alpha = 5$$

$$|4(\alpha + \beta)|$$

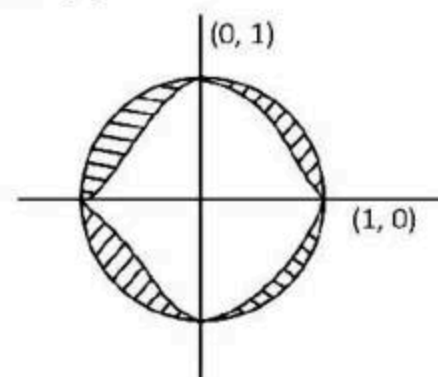
$$= |4(-19)| = 56$$

6. Area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ is $(\pi - \alpha)$ sq. units, then 9α is

- (1) 8 (2) 16
(3) 32 (4) 24

Answer (2)

Sol.



$$\text{Area} = \pi - 4 \int_0^1 (1 - x^2) dx$$

$$= \pi - 4 \left[x - \frac{x^3}{3} \right]_0^1$$

$$= \pi - 4 \times \frac{2}{3} = \left(\pi - \frac{8}{3} \right) \text{ sq unit}$$

$$= \pi - \alpha$$

$$\Rightarrow \pi = \frac{8}{3}$$

$$9\alpha = 16$$

7. If $\log y = x \log \frac{2}{5}$, $x \in \mathbb{N} \cup \{0\}$. Then sum of all values of y equals to

- (1) $\frac{5}{3}$ (2) $\frac{2}{3}$
(3) $\frac{5}{4}$ (4) $\frac{8}{3}$

Answer (1)

Sol. $\log y = x \log \frac{2}{5}$

$$\log y = \log \left(\frac{2}{5} \right)^x$$

$$y = \left(\frac{2}{5}\right)^x$$

$$x \in \mathbb{N} \cup \{0\}$$

$$\Rightarrow y = 1, \frac{2}{5}, \left(\frac{2}{5}\right)^2 \dots \text{which is in G.P.}$$

$$\text{Sum of all values of } \sum y = \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$$

8. There is an arithmetic progression $a_1, a_2, a_3, \dots, a_{2024}$ and $a_1 + (a_5 + a_{10} + a_{15} \dots a_{2020}) + a_{2024} = 2233$. Find the value of $a_1 + a_2 + a_3 + \dots + a_{2024}$.

- (1) 11034 (2) 11132
(3) 10432 (4) 20462

Answer (2)

Sol. $\therefore a_1, a_2, a_3, \dots, a_{2024}$ are in A.P.

$$\text{Then } a_1 + a_{2024} = a_2 + a_{2023} = \dots = a_r + a_{2024-r+1} = l$$

$$\therefore a_1 + (a_5 + a_{10} + \dots + a_{2020}) + a_{2024} = 2023$$

$$\text{or, } (202/1) + l = 2023$$

$$\text{or, } 203l = 2233$$

$$\therefore a_1 + a_2 + \dots + a_{2024} = 1012 \times l$$

$$= 1012 \times \frac{2233}{203}$$

$$= 1012 \times 11$$

$$= 11132$$

9. Two points $(4, 2)$ and $(0, 2)$ lie on the circle whose centre lies on $3x + 2y + 2 = 0$, then length of chord whose mid point is $(1, 2)$, is

- (1) $\sqrt{3}$ (2) $\sqrt{5}$
(3) $2\sqrt{3}$ (4) $2\sqrt{5}$

Answer (3)

Sol. Let the centre be $(-2\alpha, 3\alpha - 1)$

$$\sqrt{(-2\alpha - 4)^2 + (3\alpha - 3)^2} = \sqrt{(-2\alpha - 0)^2 + (3\alpha - 3)^2}$$

$$\Rightarrow (-2\alpha - 4)^2 = (-2\alpha)^2$$

$$\Rightarrow -2\alpha - 4 = -2\alpha$$

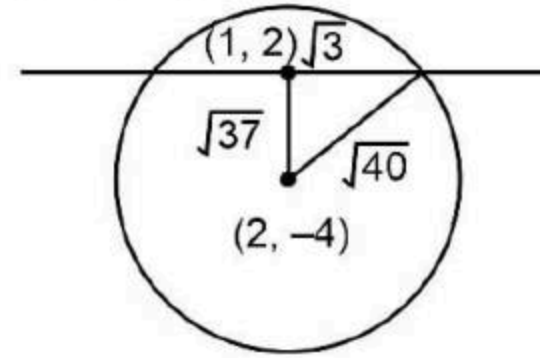
$$\Rightarrow \text{No solution}$$

$$-2\alpha - 4 = -2\alpha$$

$$\Rightarrow \boxed{\alpha = -1}$$

Centre will be $(2, -4)$, radius $\sqrt{4 + 36} = \sqrt{40}$

$$(x - 2)^2 + (y + 4)^2 = 40$$



$$\Rightarrow \text{Length of chord} = 2\sqrt{3}$$

10. If $\lim_{t \rightarrow 0} \left(\int_0^1 (3x + 5)^t dx \right)^{\frac{1}{t}} = \frac{\alpha \left(\frac{8}{5} \right)^{\frac{p}{q}}}{4e}$, then α is

- (1) 32 (2) 16
(3) 8 (4) 64

Answer (1)

Sol. Since, $\int_0^1 (3x + 5)^t dx = \frac{8^{t+1} - 5^{t+1}}{3(t+1)}$

$$\Rightarrow L = \lim_{t \rightarrow 0} \left(\frac{8^{t+1} - 5^{t+1}}{3(t+1)} \right)^{\frac{1}{t}}$$

$$\Rightarrow L = \lim_{t \rightarrow 0} \left(1 + f(t) \right)^{\frac{1}{f(t)} \cdot \frac{f(t)}{t}}$$

$$\text{Where } f(t) = \frac{8^{t+1} - 5^{t+1}}{3(t+1)} - 1 = \frac{8^{t+1} - 5^{t+1} - 3t - 3}{3(t+1)}$$

$$\Rightarrow \text{Since, } \lim_{t \rightarrow 0} f(t) = 0$$

$$L = \lim_{t \rightarrow 0} e^{\frac{f(t)}{t}} = e^{\lim_{t \rightarrow 0} \frac{f(t)}{t}} = e^{\lim_{t \rightarrow 0} f'(t)}$$

$$f'(t) = \frac{8 \cdot 8^t \ln 8 - 5 \cdot 5^t \ln 5 - 3}{3(t+1)^2} - \frac{8^{t+1} - 5^{t+1} - 3t - 3}{3(t+1)^2}$$

$$\lim_{t \rightarrow 0} f'(t) = \frac{8 \ln 8 - 5 \ln 5 - 3}{3} = \frac{1}{3} \ln \left(\frac{8^8}{5^5} \right) - 1$$

$$L = e^{\ln\left(\frac{8^8}{5^5}\right)^{\frac{1}{3}} - 1} = \left(\frac{8^8}{5^5}\right)^{\frac{1}{3}} \cdot e^{-1} = \frac{\left(\frac{8^8}{5^5}\right)^{\frac{1}{3}}}{e}$$

$$= \frac{1}{e} \left(\frac{8}{5}\right)^{\frac{5}{3}} \cdot 8^{\frac{1}{3}} = \frac{32 \left(\frac{8}{5}\right)^{\frac{5}{3}}}{4e}$$

$$\Rightarrow \alpha = 32$$

11. The value of $\int_0^{\frac{\pi}{4}} \left(\sin \left\lfloor 4x - \frac{\pi}{2} \right\rfloor + \sin[2x] \right) dx$ is
(where $[\cdot]$ denotes the greatest integer function)

(1) $\frac{1}{2} + \left(\frac{\pi-2}{4}\right) \sin 1$ (2) $\frac{1}{4} + \left(\frac{\pi-2}{2}\right) \sin 1$

(3) $\frac{1}{2} - \left(\frac{\pi-2}{4}\right) \sin 1$ (4) $\frac{1}{4} - \left(\frac{\pi-2}{2}\right) \sin 1$

Answer (1)

Sol. $\int_0^{\frac{\pi}{4}} \left(\sin \left\lfloor 4x - \frac{\pi}{2} \right\rfloor + \sin[2x] \right) dx$
 $= \int_0^{\frac{\pi}{4}} \sin \left\lfloor 4x - \frac{\pi}{2} \right\rfloor dx + \int_0^{\frac{\pi}{4}} \sin[2x] dx$
 $= \int_0^{\frac{\pi}{8}} \sin \left\lfloor \frac{\pi}{2} - 4x \right\rfloor dx + \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin \left[4x - \frac{\pi}{2} \right] dx + \int_0^{\frac{\pi}{8}} 0 dx$
 $+ \int_{\frac{1}{2}}^{\frac{\pi}{4}} \sin(1) dx$

$$= \int_0^{\frac{\pi}{8}} \cos 4x dx + \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos 4x dx + \sin 1 \cdot \left(\frac{\pi}{4} - \frac{1}{2} \right)$$

$$= \left[\frac{\sin 4x}{4} \right]_0^{\frac{\pi}{8}} - \left[\frac{\sin 4x}{4} \right]_{\frac{\pi}{8}}^{\frac{\pi}{4}} + \frac{(x-2)\sin 1}{4}$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{(\pi-2)\sin 1}{4}$$

$$= \frac{(\pi-2)\sin(1)+2}{4} = \frac{1}{2} + \left(\frac{\pi-2}{4}\right) \sin 1$$

12.
13.
14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $a_{ij} = (\sqrt{2})^{i+j}$, $A = [a_{ij}]_{3 \times 1}$. If sum of third row of A^2 is $\alpha + \beta\sqrt{2}$, then $\alpha + \beta$ is

Answer (224)

Sol. $\begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix} \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix} = \begin{bmatrix} 28 & 28\sqrt{2} & 56 \\ 28\sqrt{2} & 56 & 56\sqrt{2} \\ 56 & 56\sqrt{2} & 112 \end{bmatrix}$

$$56 + 112 + 56\sqrt{2}$$

$$168 + 56\sqrt{2}$$

$$\alpha + \beta\sqrt{2}$$

$$\alpha + \beta = 224$$

22. If 3^{107} is divided by 23, then remainder is

Answer (06)

Sol. Notice that, $3^4 \equiv (12) \pmod{23}$

$$\Rightarrow 3^8 \equiv 144 \equiv 6 \pmod{23}$$

$$3^{11} \equiv 1 \pmod{23}$$

$$(3^{11})^9 \equiv 1 \pmod{23}$$

$$3^{99} \equiv 1 \pmod{23}$$

$$3^8 \cdot 3^{99} \equiv 1 \pmod{23}$$

$$\Rightarrow 3^{107} \equiv 6 \pmod{23}$$

23. If α, β are the values of m where
 $x + y + 2z = 1$
 $x + 2y + 4z = m$
 $x + 4y + 8z = m^2$ have infinitely many solutions.

Then $\sum_{n=1}^{10} (n^\alpha + n^\beta)$ is equal to

Answer (440)

Sol. For infinite solution

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 4 & 8 \end{vmatrix} = 0$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 2 \\ m & 2 & 4 \\ m^2 & 4 & 8 \end{vmatrix} = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & m & 4 \\ 1 & m^2 & 8 \end{vmatrix} = 0 \Rightarrow m^2 + 3m - 2$$

$$\Rightarrow m^2 - 3m + 2 = 0$$

$$m = 2, 1$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & m \\ 1 & 4 & m^2 \end{vmatrix} = 0 \Rightarrow m^2 - 3m + 2 = 0$$

$$\Rightarrow m = 2, 1 \Rightarrow \alpha = 1, \beta = 2$$

$$\sum_{n=1}^{10} (n)^1 + (n)^2 = \frac{10 \times 11}{2} + \frac{10 \times 11 \times 21}{6}$$

$$= 440$$

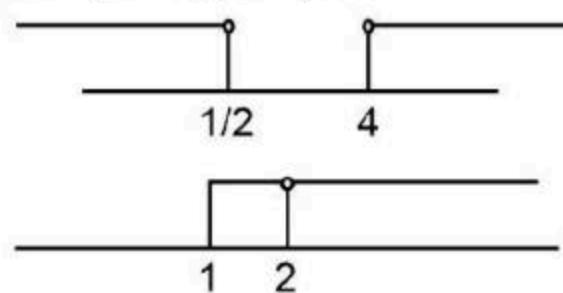
24. If the domain of $\log_{x-1} \left(\frac{2x^2 - 9x + 4}{x^2 - 4x + 5} \right)$ is (α, ∞) and $\log_5(18x - x^2 - 77)$ is (β, γ) , then the value of $\alpha^2 + \beta^2 + \gamma^2$ is

Answer (186)

Sol. $\frac{2x^2 - 9x + 4}{x^2 - 4x + 5} > 0 \quad \dots(i)$

$$x - 1 > 0, x - 1 \neq 1$$

$$\Rightarrow (2x - 1)(x - 4) > 0$$



$$\therefore x \in (4, \infty)$$

$$\therefore \alpha = 4$$

$$\log_5(18x - x^2 - 77)$$

$$\Rightarrow 18x - x^2 - 77 > 0$$

$$\Rightarrow x^2 - 18x + 77 < 0$$

$$\Rightarrow (x - 7)(x - 11) < 0$$

$$x \in (7, 11)$$

$$\therefore \beta = 7, \gamma = 11$$

$$\therefore \alpha^2 + \beta^2 + \gamma^2$$

$$= 16 + 49 + 121$$

$$= 186$$

25. The equation $\alpha x + \beta y = 109$ is chord of ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \text{ having midpoint } \left(\frac{5}{2}, \frac{1}{2} \right), \text{ then } \alpha + \beta \text{ is}$$

Answer (58)

Sol. Chord with given middle point

$$T = S_1$$

$$\frac{5}{18}x + \frac{y}{8} = \frac{25}{36} + \frac{1}{16} = \frac{109}{144}$$

$$40x + 18y = 109$$

$$\equiv \alpha x + \beta y = 109$$

$$\Rightarrow \alpha = 40 \quad \beta = 18$$

$$\alpha + \beta = 58$$

