

JEE (Main)-2025 (Online) Session-2
Question Paper with Solutions
(Mathematics, Physics, And Chemistry)

7 April 2025 Shift – 2

Time: 3 hrs.

M.M : 300

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
- (4) Section - A : Attempt all questions.
- (5) Section - B : Attempt all questions.
- (6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.
- (7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS

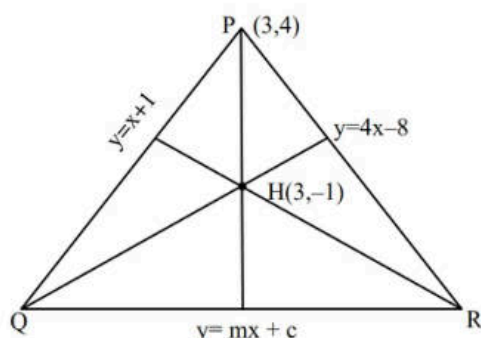
SECTION-A

1. If the orthocentre of the triangle formed by the lines $y = x + 1$, $y = 4x - 8$ and $y = mx + c$ is at $(3, -1)$, then $m - c$ is :

- (1) 0 (2) -2
(3) 4 (4) 2

Ans. (1)

Sol.



Solve line PQ & QR

$$\text{Point Q} \left(\frac{1-c}{m-1}, \frac{1-c}{m-1} + 1 \right)$$

$$m_{2H} = \frac{\frac{1-c}{m-1} + 2}{\frac{1-c}{m-1} - 3} = \frac{1-c+2m-2}{1-c-3m+3} = -\frac{1}{4} \quad \dots(1)$$

$$\therefore m_{PH} = \frac{5}{0} \rightarrow \infty$$

$$\Rightarrow \text{Slope of line QR (m)} = 0$$

Put value of m in equation (1)

$$\frac{1-c-2}{1-c+3} = -\frac{1}{4} \Rightarrow c = 0$$

so $m - c = 0$ Ans.

TEST PAPER WITH SOLUTION

2. Let $\vec{a}$ and $\vec{b}$ be the vectors of the same magnitude such that $\frac{|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}| - |\vec{a} - \vec{b}|} = \sqrt{2} + 1$. Then $\frac{|\vec{a} + \vec{b}|^2}{|\vec{a}|^2}$ is :

- (1) $2 + 4\sqrt{2}$ (2) $1 + \sqrt{2}$
(3) $2 + \sqrt{2}$ (4) $4 + 2\sqrt{2}$

Ans. (3)

Sol. $\frac{|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}| - |\vec{a} - \vec{b}|} = \sqrt{2} + 1$

Apply componendo and dividendo

$$\Rightarrow \frac{2|\vec{a} + \vec{b}|}{2|\vec{a} - \vec{b}|} = \frac{\sqrt{2} + 2}{\sqrt{2}}$$

$$\Rightarrow |\vec{a} + \vec{b}| = (1 + \sqrt{2}) |\vec{a} - \vec{b}|$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = (3 + 2\sqrt{2}) |\vec{a} - \vec{b}|^2$$

$$\Rightarrow 2|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} = (3 + 2\sqrt{2}) (2|\vec{a}|^2 - 2\vec{a} \cdot \vec{b})$$

$$\Rightarrow 2|\vec{a}|^2 (2 + 2\sqrt{2}) = 2\vec{a} \cdot \vec{b} (4 + 2\sqrt{2})$$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} = \frac{2 + 2\sqrt{2}}{4 + 2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Now

$$\frac{|\vec{a} + \vec{b}|^2}{|\vec{a}|^2} = 1 + \frac{|\vec{b}|^2}{|\vec{a}|^2} + \frac{2\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$$

$$= 1 + 1 + 2 \left(\frac{1}{\sqrt{2}} \right) = 2 + \sqrt{2}$$

3. Let

$$A = \{(\alpha, \beta) \in \mathbf{R} \times \mathbf{R} : |\alpha - 1| \leq 4 \text{ and } |\beta - 5| \leq 6\}$$

and

$$B = \{(\alpha, \beta) \in \mathbf{R} \times \mathbf{R} : 16(\alpha - 2)^2 + 9(\beta - 6)^2 \leq 144\}.$$

Then

- (1) $B \subset A$
- (2) $A \cup B = \{(x, y) : -4 \leq x \leq 4, -1 \leq y \leq 11\}$
- (3) neither $A \subset B$ nor $B \subset A$
- (4) $A \subset B$

Ans. (1)

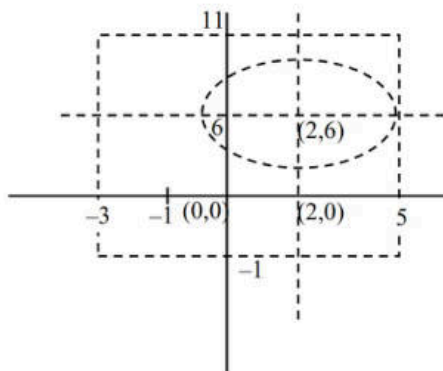
Sol. $A : |x-1| \leq 4 \text{ and } |y-5| \leq 6$

$$\Rightarrow -4 \leq x-1 \leq 4 \Rightarrow -6 \leq y-5 \leq 6$$

$$\Rightarrow -3 \leq x \leq 5 \Rightarrow -1 \leq y \leq 11$$

$$B : 16(x-2)^2 + 9(y-6)^2 \leq 144$$

$$B : \frac{(x-2)^2}{9} + \frac{(y-6)^2}{16} \leq 1$$



From Diagram $B \subset A$

4. If the range of the function $f(x) = \frac{5-x}{x^2-3x+2}$,

$x \neq 1, 2$, is $(-\infty, \alpha] \cup [\beta, \infty)$,

then $\alpha^2 + \beta^2$ is equal to :

- (1) 190
- (2) 192
- (3) 188
- (4) 194

Ans. (4)

Sol. $y = \frac{5-x}{x^2-3x+2}$
 $yx^2 - 3xy + 2y + x - 5 = 0$
 $yz^2 + (-3y+1)x + (2y-5) = 0$

Case I : If $y = 0$ (Accepted)

$$\Rightarrow x = 5$$

Case II : If $y \neq 0$

$$D \geq 0$$

$$(-3y+1)^2 - 4(y)(2y-5) \geq 0$$

$$9y^2 + 1 - 6y - 8y^2 + 20y \geq 0$$

$$y^2 + 14y + 1 \geq 0$$

$$(y+7)^2 - 48 \geq 0$$

$$|y+7| \geq 4\sqrt{3}$$

$$\Rightarrow y+7 \geq 4\sqrt{3} \text{ or } y+7 \leq -4\sqrt{3}$$

$$\Rightarrow y \geq 4\sqrt{3}-7 \text{ or } y \leq -4\sqrt{3}-7$$

From Case I and Case II

$$y \in (-\infty, -4\sqrt{3}-7] \cup [4\sqrt{3}-7, \infty)$$

$$\text{So } \alpha = -4\sqrt{3}-7$$

$$\beta = 4\sqrt{3}-7$$

$$\Rightarrow a^2 + b^2 = (-4\sqrt{3}-7)^2 + (4\sqrt{3}-7)^2$$

$$= 2(48 + 49)$$

$$= 194$$

5. A bag contains 19 unbiased coins and one coin with head on both sides. One coin drawn at random is tossed and head turns up. If the probability that

the drawn coin was unbiased, is $\frac{m}{n}$, $\gcd(m, n) = 1$,

then $n^2 - m^2$ is equal to :

- (1) 80
- (2) 60
- (3) 72
- (4) 64

Ans. (1)

Sol. $P(H) = \frac{19}{20} \times \frac{1}{2} + \frac{1}{20} \times 1$ Head occurs

Selection of unbiased
Head occurs
Selection of biased coin

$$\text{Required probability} = \frac{\frac{19}{20} \times \frac{1}{2}}{\frac{19}{20} \times \frac{1}{2} + \frac{1}{20} \times 1} = \frac{19}{21}$$

$$\therefore \frac{m}{n} = \frac{19}{21}$$

$$\Rightarrow n^2 - m^2 = 441 - 361 = 80$$

6. Let a random variable X take values 0, 1, 2, 3 with $P(X = 0) = P(X = 1) = p$, $P(X = 2) = P(X = 3) = q$ and $E(X^2) = 2E(X)$. Then the value of $8p - 1$ is :

- (1) 0 (2) 2
(3) 1 (4) 3

Ans. (2)

Sol. $2p + 2q = \frac{1}{2}$

$p + q$

$E(X^2) = \sum_{i=0}^3 x_i^2 p(x_i) = 0.p + 1.p + 4.q + 9q$

$= p + 13q$

$E(X) = \sum_{i=0}^3 x_i p(x_i) = 0.p + 1.p + 2q + 3q = p + 5q$

$p + 13q = 2(p + 5q)$

$p = 3q$

So, $q = \frac{1}{8}$ & $p = \frac{3}{8}$

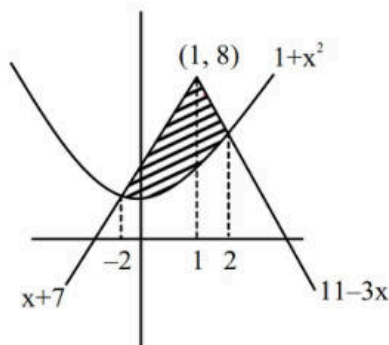
So, $8p - 1 = 2$ Option (2)

7. If the area of the region $\{(x, y) : 1 + x^2 \leq y \leq \min\{x + 7, 11 - 3x\}\}$ is A , then $3A$ is equal to

- (1) 50 (2) 49
(3) 46 (4) 47

Ans. (1)

Sol.



$A = \int_{-2}^1 (x + 7 - x^2 - 1) dx + \int_1^2 (11 + 3x - x^2 - 1) dx$

$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3} \right]_{-2}^1 + \left[10x - \frac{3x^2}{2} - \frac{x^3}{3} \right]_1^2$

$= \frac{50}{3} \Rightarrow 3A = 50$ Option (1)

8. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a polynomial function of degree four having extreme values at $x = 4$ and $x = 5$.

If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 5$, then $f(2)$ is equal to :

- (1) 12 (2) 10
(3) 8 (4) 14

Ans. (2)

Sol. $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 5$

$\lim_{x \rightarrow 0} \frac{ax^4 + bx^3 + cx^2 + dx + e}{x^2} = 5$

$c = 5$ and $d = e = 0$

$f(x) = ax^4 + bx^3 + 5x^2$

$f'(x) = 4ax^3 + 3bx^2 + 10x$

$= x(4ax^2 + 3bx + 10)$

has extremes at 4 and so $f'(4) = 0$ & $f'(5) = 0$

so $a = \frac{1}{8}$ & $b = \frac{-3}{2}$

so $f(2) = \frac{1}{8} \times 2^4 - \frac{3}{2} \times 2^3 + 5 \times 2^2$

$= 2 - 12 + 20 = 10$ Option (2)

9. The number of solutions of the equation

$\cos 2\theta \cos \frac{\theta}{2} + \cos \frac{5\theta}{2} = 2 \cos^3 \frac{5\theta}{2}$

in $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$ is :

- (1) 7 (2) 5
(3) 6 (4) 9

Ans. (1)

Sol. $\cos 2\theta \cos \frac{\theta}{2} + \cos \frac{5\theta}{2} = 2 \cos^3 \frac{5\theta}{2}$

$$\frac{1}{2} \left(2 \cos 2\theta \cos \frac{\theta}{2} \right) + \cos \frac{5\theta}{2}$$

$$= \frac{1}{2} \left(\cos \frac{15\theta}{2} + 3 \cos \frac{5\theta}{2} \right)$$

or solving

$$\cos \frac{3\theta}{2} = \cos \frac{15\theta}{2}$$

$$\cos \frac{15\theta}{2} - \cos \frac{3\theta}{2} = 0$$

$$2 \sin 3\theta \sin \frac{9\theta}{2} = 0$$

$$3\theta = n\pi \text{ or } \frac{9\theta}{2} = m\pi$$

$$\theta = \frac{n\pi}{3} \quad \theta = \frac{2m\pi}{9}$$

$$\theta = \left\{ -\frac{\pi}{2}, \frac{\pi}{3}, 0 \right\}$$

$$\theta = \left\{ -\frac{4\pi}{9}, -\frac{2\pi}{9}, \frac{4\pi}{9}, \frac{2\pi}{9} \right\}$$

Option (1)

10. Let a_n be the n^{th} term of an A. P.

If $S_n = a_1 + a_2 + a_3 + \dots + a_n = 700$, $a_6 = 7$ and

$S_7 = 7$, then a_n is equal to :

(1) 56

(2) 65

(3) 64

(4) 70

Ans. (3)

Sol. $S_n = 700 = \frac{n}{2} [2a + (n-1)d] \dots (i)$

$$a_6 = 7 \Rightarrow a + 5d = 7 \dots (ii)$$

$$S_7 = 7 \Rightarrow \frac{7}{2} (2a + 6d) = 7$$

$$a + 3d = 1 \dots (iii)$$

Solve (ii) and (iii)

$$\frac{n}{2} (-16 + 3n - 3) = 700 \Rightarrow 3n^2 - 19n - 1400 = 0$$

$$(3n + 56)(n - 25) = 0$$

$$\therefore a_{25} = a + 24d = -8 + 24 \times 3$$

$$= -8 + 72$$

$$= 64$$

Ans. $\rightarrow 3$

11. If the locus of $z \in \mathbb{C}$, such that

$$\operatorname{Re} \left(\frac{z-1}{2z+i} \right) + \operatorname{Re} \left(\frac{\bar{z}-1}{2\bar{z}-i} \right) = 2,$$

is a circle of radius r and center (a, b) then $\frac{15ab}{r^2}$ is

equal to :

(1) 24

(2) 12

(3) 18

(4) 16

Ans. (3)

Sol. $\operatorname{Re} \left(\frac{z-1}{2z+i} \right) + \operatorname{Re} \left(\frac{\bar{z}-1}{2\bar{z}-i} \right) = 2$

$$\text{Here, } \frac{z-1}{2z+i} = \left(\frac{\bar{z}-1}{2\bar{z}-i} \right) = 2$$

$$= \operatorname{Re} \left(\frac{z-1}{2z+i} \right) + \operatorname{Re} \left(\frac{\bar{z}-1}{2\bar{z}-i} \right) = 2$$

$$= 2 \operatorname{Re} \left(\frac{z-1}{2z+i} \right) = 2 \Rightarrow \operatorname{Re} \left(\frac{z-1}{2z+i} \right) = 1$$

Let $z = x + iy$

$$\operatorname{Re} \left(\frac{(x-1)+iy}{2x+i(2y+1)} \right) = 1 \Rightarrow \operatorname{Re} \left[\frac{((x-1)+iy)(2x-i(2y+1))}{(2x+i(2y+1))(2x-i(2y+1))} \right] = 1$$

$$\Rightarrow \frac{2x(x-1)+y(2y+1)}{4x^2+(2y+1)^2} = 1$$

$$\Rightarrow 2x^2 - 2x + 2y^2 + y = 4x^2 + 4y^2 + 1 + 4y$$

$$\Rightarrow 2x^2 + 2y^2 + 3y + 2x + 1 = 0$$

$$\Rightarrow x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$$

$$\text{centre} = \left(-\frac{1}{2}, -\frac{3}{4} \right), r = \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \frac{\sqrt{5}}{4}$$

$$a = -\frac{1}{2}, b = -\frac{3}{4}, r^2 = \frac{5}{16}$$

$$15 \frac{ab}{r^2} = 15 \times \left(-\frac{1}{2} \right) \times \left(-\frac{3}{4} \right) \times \frac{16}{5} = 18$$

12. Let the length of a latus rectum of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ be } 10. \text{ If its eccentricity is the}$$

minimum value of the function $f(t) = t^2 + t + \frac{11}{12}$,

$t \in \mathbb{R}$, then $a^2 + b^2$ is equal to :

(1) 125

(2) 126

(3) 120

(4) 115

Ans. (2)

Sol. Length of LR = $\frac{2b^2}{a} = 10 \Rightarrow \boxed{5a = b^2}$ (1)

$$f(t) = t^2 + t + \frac{11}{12}$$

$$\frac{df(t)}{dt} = 2t + 1 = 0 \Rightarrow t = \frac{-1}{2}$$

$$\text{Min value of } f(t) = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + \frac{11}{12}$$

$$= \frac{1}{4} - \frac{1}{2} + \frac{11}{12} = \frac{3-6+11}{12} = \frac{8}{12} = \frac{2}{3} = e$$

$$e^2 = \frac{1-b^2}{a^2} \Rightarrow \frac{4}{9} = \frac{1-b^2}{a^2}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{1-4}{a} = \frac{5}{a} \Rightarrow \boxed{b^2 = \frac{5a^2}{a}} \text{(2)}$$

From (1) & (2)

$$5a = \frac{5a^2}{a} \Rightarrow a = 9, b = \sqrt{45} = 3\sqrt{5}$$

$$\therefore a^2 + b^2 = 81 + 45 = 126$$

13. Let $y = y(x)$ be the solution of the differential equation $(x^2+1)y' - 2xy = (x^4 + 2x^2 + 1) \cos x$,

$$y(0) = 1. \text{ Then } \int_{-3}^3 y(x) dx \text{ is :}$$

(1) 24

(2) 36

(3) 30

(4) 18

Ans. (1)

Sol. $(x^2+1)\frac{dy}{dx} - 2xy = (x^4 + 2x^2 + 1) \cos x$

$$\frac{dy}{dx} - \left(\frac{2x}{x^2+1}\right)y = \frac{(x^2+1)^2 \cos x}{x^2+1} = (x^2+1) \cos x$$

(Linear D.E)

$$P = \frac{-2x}{x^2+1}, Q = (x^2+1) \cos x$$

$$\text{I.F} = e^{\int P dx} = e^{\int \frac{-2x}{x^2+1} dx} = \frac{1}{x^2+1}$$

$$y \cdot \frac{1}{x^2+1} = \int (x^2+1) \cos x \cdot \frac{1}{x^2+1} dx$$

$$\frac{y}{x^2+1} = \sin x + c \Rightarrow y \cos 1 = 1 \Rightarrow c = 1$$

$$y = (x^2+1)(\sin x + 1)$$

$$\int_{-3}^3 y dx = \int_{-3}^3 (x^2+1)(\sin x + 1) dx$$

$$dx = \int_{-3}^3 x^2 \sin x + x^2 \sin x + 1 dx$$

$$\Rightarrow \int_{-3}^3 x^2 \sin x dx + \int_{-3}^3 x^2 dx + \int_{-3}^3 \sin x dx + \int_{-3}^3 1 dx$$

$$= 0 + 18 + 0 + 6 = 24$$

14. If the equation of the line passing through the point $\left(0, -\frac{1}{2}, 0\right)$ and perpendicular to the lines

$$\vec{r} = \lambda(\hat{i} + \hat{a}\hat{j} + \hat{b}\hat{k}) \text{ and}$$

$$\vec{r} = (\hat{i} - \hat{j} - 6\hat{k}) + \mu(-\hat{b}\hat{i} + \hat{a}\hat{j} + 5\hat{k})$$

is $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$, then $a + b + c + d$ is equal to :

(1) 10

(2) 14

(3) 13

(4) 12

Ans. (2)

Sol. Line is $\perp^r$ to 2 line $\Rightarrow$ line will be parallel to

$$(\hat{i} + \hat{a}\hat{j} + \hat{b}\hat{k}) \times (-\hat{b}\hat{i} + \hat{a}\hat{j} + 5\hat{k})$$

Parallel vector along the required line is

$$\hat{i}(5a - ab) - \hat{j}(b^2 + 5) + \hat{k}(a + ab)$$

Dr's of required line $\propto (5a - ab), -(b^2 + 5), (a + ab)$

Also Dr's of required line $\propto -2, d, -4$

$$\therefore \frac{5a - ab}{-2} = \frac{-(b^2 + 5)}{d} = \frac{a + ab}{-4} \text{(1)}$$

Also point $\left(0, -\frac{1}{2}, 0\right)$ will lie on $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$

$$\frac{0-1}{-2} = \frac{-\frac{1}{2}+4}{d} = \frac{0-c}{-4} \Rightarrow d = 7, c = 2$$

$$\text{From (1)} \frac{5a - ab}{-2} = \frac{-b^2 - 5}{7} = \frac{a + ab}{-4}$$

$$\frac{5a - ab}{-2} = \frac{a + ab}{-4}; \frac{-b^2 - 5}{7} = \frac{a + ab}{-4}$$

$$-20a + 4ab = -2a - 2ab \quad | \quad 4b^2 + 20 = 70 + 7ab$$

$$18a = 6ab$$

$$36 + 20 = 70 + 21a$$

$$\boxed{b = 3}$$

$$56 = 28a \Rightarrow \boxed{a = 2}$$

$$a + b + c + d = 2 + 3 + 2 + 7 = 14$$

15. Let p be the number of all triangles that can be formed by joining the vertices of a regular polygon P of n sides and q be the number of all quadrilaterals that can be formed by joining the vertices of P . If $p + q = 126$, then the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{n} = 1$ is :

- (1) $\frac{3}{4}$ (2) $\frac{1}{2}$
 (3) $\frac{\sqrt{7}}{4}$ (4) $\frac{1}{\sqrt{2}}$

Ans. (4)

Sol. Total triangles $\Rightarrow {}^nC_3$

Total quadrilaterals $= {}^nC_4 = q$

$${}^nC_3 + {}^nC_4 = 126 \Rightarrow {}^{n+1}C_4 = 126$$

$$\Rightarrow n+1 = 9 \Rightarrow n = 8$$

$$\frac{x^2}{16} + \frac{y^2}{n} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{8} = 1$$

$$e = \sqrt{1 - \frac{8}{16}} = \sqrt{\frac{8}{16}} = \frac{1}{\sqrt{2}}$$

16. Consider the lines $L_1 : x - 1 = y - 2 = z$ and

$L_2 : x - 2 = y = z - 1$. Let the feet of the perpendiculars from the point $P(5, 1, -3)$ on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A , then $4A^2$ is equal to :

- (1) 139 (2) 147
 (3) 151 (4) 143

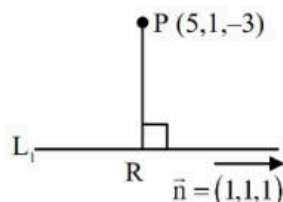
Ans. (2)

Sol. $L_1 : \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-0}{2}$

Let $Q(\lambda + 1, \lambda + 2, \lambda)$

$$\overrightarrow{PQ} = (\lambda - 4, \lambda - 1, \lambda + 3)$$

$$\overrightarrow{PQ} \cdot \vec{m} = 0$$



$$\Rightarrow \lambda - 4 + \lambda + 1, \lambda + 3 = 0$$

$$\Rightarrow 3\lambda = 0$$

$$\lambda = 0$$

$$\Rightarrow \boxed{Q(1, 2, 0)}$$

$$L_2 : \frac{x-2}{1} = \frac{y-0}{1} = \frac{z-1}{2}$$

$$\text{Let } R(\mu + 2, \mu, \mu + 1) \quad \overrightarrow{PR} = (\mu - 3, \mu - 1, \mu + 4)$$

$$\overrightarrow{PR} \cdot \vec{n} = 0$$

$$\mu - 3 + \mu - 1 + \mu + 4 = 0$$

$$3\mu = 0$$

$$\boxed{R(2, 0, 1)}$$

$$\text{Area of } \Delta PQR (A) = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}|$$

$$A = \frac{1}{2} |(-4\hat{i} + \hat{j} + 3\hat{k}) \times (-3\hat{i} + \hat{j} + 4\hat{k})|$$

$$A = \frac{1}{2} |7(\hat{i} + \hat{j} + \hat{k})|$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 1 & 3 \\ -3 & -1 & 4 \end{vmatrix}$$

$$= 7\hat{i} + 7\hat{j} + 7\hat{k}$$

$$4A^2 = 49 \times 3 = 147$$

17. The number of real roots of the equation $x|x-2| + 3|x-3| + 1 = 0$ is :

- (1) 4 (2) 2
 (3) 1 (4) 3

Ans. (3)

Sol. (I) $x < 2$

$$-x^2 + 2x - 3x + 9 + 1 = 0$$

$$\Rightarrow x^2 + x - 10 = 0$$

$$\Rightarrow x = \frac{-1 + \sqrt{41}}{2}, \frac{-1 - \sqrt{41}}{2}$$

(II) $2 \leq x < 3$

$$\Rightarrow x^2 - 2x - 3x + 9 + 1 = 0$$

$$\Rightarrow x^2 - 5x + 10 = 0$$

$$D < 0$$

(III) $x \geq 3$

$$x^2 - 2x + 3x - 9 + 2 = 0$$

$$\Rightarrow x^2 + x - 8 = 0$$

$$x = \frac{-1 + \sqrt{32}}{2}, \frac{-1 - \sqrt{32}}{2}$$

1 real roots

18. Let e_1 and e_2 be the eccentricities of the ellipse $\frac{x^2}{b^2} + \frac{y^2}{25} = 1$ and the hyperbola $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$, respectively. If $b < 5$ and $e_1 e_2 = 1$, then the eccentricity of the ellipse having its axes along the coordinate axes and passing through all four foci (two of the ellipse and two of the hyperbola) is :

- (1) $\frac{4}{5}$ (2) $\frac{3}{5}$
 (3) $\frac{\sqrt{7}}{4}$ (4) $\frac{\sqrt{3}}{2}$

Ans. (2)

Sol. $e_1^2 = 1 - \frac{b^2}{25}$ $e_2^2 = 1 - \frac{b^2}{16}$

$\therefore e_1^2 e_2^2 = 1$

$\left(1 - \frac{b^2}{25}\right) \left(1 - \frac{b^2}{16}\right) = 1$

$\Rightarrow 2 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^2}{400} = 1$

$\Rightarrow \frac{9b^2}{400} = \frac{b^4}{400}$

$b^2 = 9$

$\frac{x^2}{9} + \frac{y^2}{25} = 1$ $\frac{x^2}{16} - \frac{y^2}{9} = 0$

$e_1 = \sqrt{1 - \frac{9}{25}}$ $e_2 = \frac{5}{4}$

$e_1 = \frac{4}{5}$

Foci : - $(0, \pm 4)$ $(\pm 5, 0)$

ellipse passing through all four foci

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

19. Let the system of equations
 $x + 5y - z = 1$
 $4x + 3y - 3z = 7$
 $24x + y + \lambda z = \mu$
 $\lambda, \mu \in \mathbb{R}$, have infinitely many solutions. Then the number of the solutions of this system, If x, y, z are integers and satisfy $7 \leq x + y + z \leq 77$, is
 (1) 3 (2) 6
 (3) 5 (4) 4

Ans. (1)

Sol. For infinitely many solution

$\Delta = 0$

$\begin{vmatrix} 1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow 1(3\lambda + 3) - 5(4\lambda + 72) - 1(4 - 72) = 0$

$\Rightarrow -17\lambda + 3 - 4 \times 72 - 4 = 0$

$\Rightarrow 17\lambda = -289$

$\Rightarrow \boxed{\lambda = -17}$

$\Delta 1 = 0$

$\Rightarrow \begin{vmatrix} 1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17 \end{vmatrix} = 0$

$\Rightarrow 1(-51 + 3) - 5(-119 + 3\mu) - 1(7 - 3\mu) = 0$

$\Rightarrow -48 + 595 - 15\mu - 7 + 3\mu = 0$

$\Rightarrow 12\mu = 540$

$\boxed{\mu = 45}$

$x + 5y - z = 1$

$4x + 3y - 3z = 7$

$24x + y - 17z = 45$

Let $z = 1$

$x + 5y = 1 + \lambda \times 4$

$4x + 3y = 7 + 3\lambda$

$4x + 20y = 4 + 4\lambda$

$\underline{\quad\quad\quad}$
 $-17y = 3 - \lambda$

$y = \frac{\lambda - 3}{17}$, $x = 1 + \lambda - \frac{5\lambda - 15}{17}$

$= \frac{32 - 12\lambda}{17}$

$7 \leq \frac{\lambda - 3}{17} + \frac{32 - 12\lambda}{17} + \lambda \leq 77$

$7 \leq \frac{30\lambda + 29}{17} \leq 77$

$3 \leq \lambda \leq 42$

$\lambda = 3, 20, 37$

20. If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is :

- (1) 760 (2) 755
(3) 750 (4) 757

Ans. (4)

Sol. $ar + ar^3 + ar^5 = 21$, $ar^7 + ar^9 + ar^{11} = 15309$
 $\Rightarrow ar(1 + r^2 + r^4) = 21$, $ar^7(1 + r^2 + r^4) = 15309$

$$\frac{\text{Eq}^n(1)}{\text{Eq}^n(2)} = \frac{21}{15309} \quad \text{--- (2)}$$

$$\Rightarrow \frac{ar}{ar^7} = \frac{21}{15309} \Rightarrow r^6 = 729$$

$$\Rightarrow \frac{a \cdot r^7}{ar} = \frac{15309}{21} \Rightarrow r^6 = 729$$

$$\Rightarrow \frac{a(r^9 - 1)}{r - 1} = \frac{7(19683 - 1)}{2} = \frac{7 \times 19682}{2}$$

$$= \frac{9841}{13} = 757$$

SECTION-B

21. If the function $f(x) = \frac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}$ is continuous at $x = 0$, then $f(0)$ is equal to _____.

Ans. (2)

Sol. $\lim_{x \rightarrow 0} \frac{\frac{\tan(\tan x) - \tan x \tan^3 x}{\tan^3 x} + \frac{\tan x - \sin x}{x^3} + \frac{\sin x - \sin(\sin x)}{\sin^3 x} \cdot \frac{\sin^3 x}{x^3}}{\frac{\tan x - \sin x}{x^3}}$

$$= \frac{\frac{1}{3} + \frac{1}{2} + \frac{1}{6}}{\frac{1}{2}} = 2$$

22. If $\int \left(\frac{1}{x} + \frac{1}{x^3} \right) \left(\sqrt[23]{3x^{-24} + x^{-26}} \right) dx$
- $$= -\frac{\alpha}{3(\alpha+1)} \left(3x^\beta + x^\gamma \right)^{\frac{\alpha+1}{\alpha}} + C, x > 0,$$
- $(\alpha, \beta, \gamma \in \mathbb{Z})$, where C is the constant of integration, then $\alpha + \beta + \gamma$ is equal to _____.

Ans. (19)

Sol. $\int \left(\frac{1}{x^2} + \frac{1}{x^4} \right) \left(\frac{3}{x} + \frac{1}{x^3} \right)^{\frac{1}{23}} dx$

using $t = \frac{3}{x} + \frac{1}{x^3} \Rightarrow dt = -3 \left(\frac{1}{x^2} + \frac{1}{x^4} \right) dx$

$$\int \frac{t^{1/23} dt}{-3} = \frac{t^{24/23}}{\left(\frac{24}{23} \right)(-3)} + C$$

$$\Rightarrow \alpha = 23, \beta = -1, \gamma = -3$$

$$\alpha + \beta + \gamma = 19$$

23. For $t > -1$, let α_t and β_t be the roots of the equation
- $$\left((t+2)^{\frac{1}{7}} - 1 \right) x^2 + \left((t+2)^{\frac{1}{6}} - 1 \right) x + \left((t+2)^{\frac{1}{21}} - 1 \right) = 0.$$

If $\lim_{t \rightarrow -1^+} \alpha_t = a$ and $\lim_{t \rightarrow -1^+} \beta_t = b$, then $72(a+b)^2$ is equal to _____.

Ans. (98)

Sol. $a + b = \lim_{t \rightarrow -1^+} (\alpha + \beta) = \lim_{t \rightarrow -1^+} -\frac{(t+2)^{\frac{1}{6}} - 1}{(t+2)^{\frac{1}{7}} - 1}$

$$\text{let } t + 2 = y$$

$$a + b = \lim_{y \rightarrow 1^+} \frac{y^{1/6} - 1}{y^{1/7} - 1} = \frac{7}{6}$$

$$72(a+b)^2 = 72 \cdot \frac{49}{36} = 98$$

24. Let the lengths of the transverse and conjugate axes of a hyperbola in standard form be $2a$ and $2b$, respectively, and one focus and the corresponding directrix of this hyperbola be $(-5, 0)$ and $5x + 9 = 0$, respectively. If the product of the focal distances of a point $(\alpha, 2\sqrt{5})$ on the hyperbola is p , then $4p$ is equal to _____.

Ans. (189)

Sol. $PF_1 \cdot PF_2 = (ea - a)(ea + a)$

$$P = e^2 \alpha^2 - a^2 = \frac{25}{9} \cdot 9 \cdot \frac{9}{4} - 9 = \frac{189}{4}$$

$$\left. \begin{aligned} ae = 5 \\ \frac{a}{e} = 9/5 \end{aligned} \right\} \begin{aligned} a = 3 \\ e = 5/3 \end{aligned}$$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\frac{\alpha^2}{9} - \frac{4(5)}{16} = 1 \Rightarrow \alpha^2 = 9 \left(\frac{36}{16} \right)$$

25. The sum of the series

$$2 \times 1 \times {}^{20}C_4 - 3 \times 2 \times {}^{20}C_5 + 4 \times 3 \times {}^{20}C_6 - 5 \times 4 \times {}^{20}C_7 + \dots + 18 \times 17 \times {}^{20}C_{20}, \text{ is equal to}$$

Ans. (34)

Sol. $(1-x)^{20} = {}^{20}C_0 - {}^{20}C_1 x + {}^{20}C_2 x^2 - \dots + {}^{20}C_{20} x^{20}$

$$\frac{(1-x)^{20}}{x^2} = \frac{{}^{20}C_0}{x^2} - \frac{{}^{20}C_1}{x} + {}^{20}C_2 - {}^{20}C_3 x + {}^{20}C_4 x^2 - \dots$$

Diff twice and put $x = 1$

$$= 6 - {}^{20}C_1(2) + A$$

$$A = 40 - 6 = 34$$

PHYSICS

SECTION-A

26. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : The outer body of an air craft is made of metal which protects persons sitting inside from lightning-strikes.

Reason (R) : The electric field inside the cavity enclosed by a conductor is zero.

In the light of the above statements, chose the **most appropriate answer** from the options given below :

- (1) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (2) (A) is correct but (R) is not correct
- (3) Both (A) and (R) are correct but (R) is not correct explanation of (A)
- (4) (A) is not correct but (R) is correct

Ans. (1)

Sol. Electric field of outside charge is zero inside conductor

27. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : The density of the copper (${}^{64}_{29}\text{Cu}$) nucleus is greater than that of the carbon (${}^{12}_6\text{C}$) nucleus.

Reason (R) : The nucleus of mass number A has a radius proportional to $A^{1/3}$.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

- (1) (A) is correct but (R) is not correct
- (2) (A) is not correct but (R) is correct
- (3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (4) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Ans. (2)

TEST PAPER WITH SOLUTION

Sol.
$$\rho = \frac{M}{V} = \frac{m_n \times A}{\frac{4}{3}\pi R^3} = \frac{m_n \times A}{\frac{4}{3}\pi A R_0^3}$$

So ρ is almost is constant

$$R = R_0 A^{1/3}$$

$$R \propto A^{1/3}$$

28. The unit of $\sqrt{\frac{2I}{\epsilon_0 c}}$ is :

(I = intensity of an electromagnetic wave, c : speed of light)

- (1) Vm
- (2) NC
- (3) Nm
- (4) NC^{-1}

Ans. (4)

Sol.
$$I = \frac{1}{2} \epsilon_0 E_0^2 \times C$$

$$E_0 = \sqrt{\frac{2I}{\epsilon_0 C}}$$

E_0 : electric field

N/C

29. The dimension of $\sqrt{\frac{\mu_0}{\epsilon_0}}$ is equal to that of :

(μ_0 = Vacuum permeability and ϵ_0 = Vacuum permittivity)

- (1) Voltage
- (2) Capacitance
- (3) Inductance
- (4) Resistance

Ans. (4)

Sol. $L = \frac{\mu_0 NA}{\ell}$

$$C = \frac{A \epsilon_0}{d}$$

$$\frac{L}{C} \propto \frac{\mu_0}{\epsilon_0}$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} \propto \sqrt{\frac{L}{C}}$$

$$\frac{L}{C} = \frac{\tau R}{(\tau / R)} = R^2$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = R$$

- 30.** A photo-emissive substance is illuminated with a radiation of wavelength λ_i so that it releases electrons with de-Broglie wavelength λ_e . The longest wavelength of radiation that can emit photoelectron is λ_0 . Expression for de-Broglie wavelength is given by :

(m : mass of the electron, h : Planck's constant and c : speed of light)

$$(1) \lambda_e = \sqrt{\frac{h}{2mc \left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0} \right)}}$$

$$(2) \lambda_e = \sqrt{\frac{h\lambda_0}{2mc}}$$

$$(3) \lambda_e = \frac{h}{\sqrt{2mc \left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0} \right)}}$$

$$(4) \lambda_e = \sqrt{\frac{h\lambda_i}{2mc}}$$

Ans. (1)

Sol. $K.E = E - W$

$$\lambda_e = \frac{h}{\sqrt{2mK.E}}, E = \frac{hc}{\lambda_i}, W = \frac{hc}{\lambda_0}$$

$$\frac{h^2}{2m\lambda_e^2} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_0}$$

$$\lambda_e = \sqrt{\frac{h}{2mc \left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0} \right)}}$$

- 31.** Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : The radius vector from the Sun to a planet sweeps out equal areas in equal intervals of time and thus areal velocity of planet is constant.

Reason (R) : For a central force field the angular momentum is a constant.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

- (1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**
- (2) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**
- (3) **(A)** is correct but **(R)** is not correct
- (4) **(A)** is not correct but **(R)** is correct

Ans. (1)

Sol. $\frac{dA}{dt} = \frac{L}{2m}$

Due to central force torque is zero & angular momentum is constant.

- 32.** The helium and argon are put in the flask at the same room temperature (300 K). The ratio of average kinetic energies (per molecule) of helium and argon is :

(Give : Molar mass of helium = 4 g/mol, Molar mass of argon = 40 g/mol)

- (1) 1 : 10
- (2) 10 : 1
- (3) 1 : $\sqrt{10}$
- (4) 1 : 1

Ans. (4)

Sol. $K.E = \frac{f}{2} KT$

For He and Ar $f = 3$

$$\frac{K.E_{He}}{K.E_{Ar}} = \frac{1}{1}$$

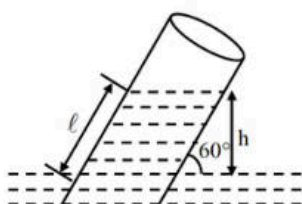
- 33.** A capillary tube of radius 0.1 mm is partly dipped in water (surface tension 70 dyn/cm and glass water contact angle $\approx 0^\circ$) with 30° inclined with vertical. The length of water risen in the capillary is _____ cm.

(Take $g = 9.8 \text{ m/s}^2$)

- (1) $\frac{82}{5}$ (2) $\frac{57}{2}$ (3) $\frac{71}{5}$ (4) $\frac{68}{5}$

Ans. (1)

Sol.



$$h = \frac{2T \cos \theta}{\rho g r} = \frac{2 \times 70 \times 1}{1 \times 980 \times 10^{-2}}$$

$$h = \frac{100}{7} \text{ cm}$$

$$\sin 60^\circ = \frac{h}{l}$$

$$l = \frac{h \times 2}{\sqrt{3}}$$

$$l = \frac{100}{7} \times \frac{2}{\sqrt{3}}$$

$$= \frac{200}{7 \times \sqrt{3}}$$

$$= 16.49 \text{ cm}$$

- 34.** A mirror is used to produce an image with magnification of $\frac{1}{4}$. If the distance between object and its image is 40 cm, then the focal length of the mirror is _____.

- (1) 10 cm (2) 12.7 cm
(3) 10.7 cm (4) 15 cm

Ans. (3)

Sol. $m = -\frac{v}{u} = -\left(\frac{v}{-u}\right) = \frac{v}{u}$

$$\frac{1}{4} = \frac{v}{u} \Rightarrow u = 4v$$

$$v + u = 40$$

$$5v = 40$$

$$v = 8 \text{ cm}$$

$$u = 32 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{8} + \frac{1}{32} = \frac{1}{f}$$

$$\frac{4+1}{32} = \frac{1}{f}$$

$$f = \frac{32}{5} = 10.7 \text{ cm}$$

- 35.** A dipole with two electric charges of $2 \mu\text{C}$ magnitude each, with separation distance $0.5 \mu\text{m}$, is placed between the plates of a capacitor such that its axis is parallel to an electric field established between the plates when a potential difference of 5 V is applied. Separation between the plates is 0.5 mm. If the dipole is rotated by 30° from the axis, it tends to realign in the direction due to a torque. The value of torque is :

- (1) $5 \times 10^{-9} \text{ Nm}$ (2) $5 \times 10^{-3} \text{ Nm}$
(3) $2.5 \times 10^{-12} \text{ Nm}$ (4) $2.5 \times 10^{-9} \text{ Nm}$

Ans. (1)

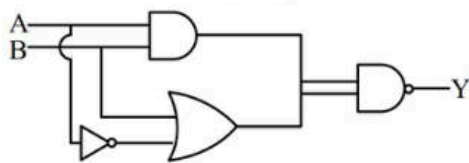
Sol. $E = \frac{V}{d} = \frac{5}{5 \times 10^{-4}} = 10^4 \text{ V/m}$

$$\tau = PE \sin \theta$$

$$\text{Where } P = qa = 2 \times 10^{-6} \times 5 \times 10^{-7} = 1 \times 10^{-12} \text{ C-m}$$

$$\tau = 1 \times 10^{-12} \times 10^4 \times \frac{1}{2} = 5 \times 10^{-9} \text{ N-m}$$

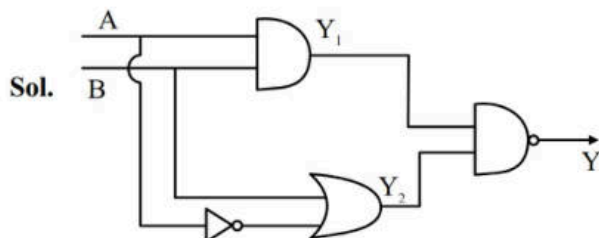
36. Consider the following logic circuit.



The output is $Y = 0$ when :

- (1) $A = 1$ and $B = 1$ (2) $A = 0$ and $B = 1$
 (3) $A = 1$ and $B = 0$ (4) $A = 0$ and $B = 0$

Ans. (1)



Sol.

$$Y_1 = A \cdot B, Y_2 = \bar{A} + B$$

$$Y = \overline{Y_1 \cdot Y_2} = \bar{Y}_1 + \bar{Y}_2$$

$$Y = \overline{A \cdot B} + \overline{\bar{A} + B}$$

$$Y = \bar{A} + \bar{B} + A \cdot \bar{B}$$

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

37. Match List-I with List-II.

List-I		List-II	
(A)	Mass density	(I)	$[ML^2T^{-3}]$
(B)	Impulse	(II)	$[MLT^{-1}]$
(C)	Power	(III)	$[ML^2T^0]$
(D)	Moment of inertia	(IV)	$[ML^{-2}T^0]$

Choose the **correct** answer from the options given below :

- (1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
 (2) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
 (3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
 (4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Ans. (3)

Sol. (A) Mass density = $\frac{M}{V} = M^1L^{-3}$... (iv)

(B) Impulse = $M \times u = M^1L^1T^{-1}$... (ii)

(C) Power = $F \cdot V = M^1L^2T^{-3}$... (i)

(D) Moment of inertia = $Mr^2 = M^1L^2$... (iii)

38. The equation of a wave travelling on a string is $y = \sin[20\pi x + 10\pi t]$, where x and t are distance and time in SI units. The minimum distance between two points having the same oscillating speed is :

- (1) 5.0 cm (2) 20 cm
 (3) 10 cm (4) 2.5 cm

Ans. (1)

- Sol. Minimum distance between 2 points having same speed is $\frac{\lambda}{2}$.

$$\lambda = \frac{2\pi}{k} = \frac{1}{10} \text{ m} = 10 \text{ cm}$$

$$\text{Distance} = \frac{\lambda}{2} = 5 \text{ cm}$$

39. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A) : Refractive index of glass is higher than that of air.

Reason (R) : Optical density of a medium is directly proportionate to its mass density which results in a proportionate refractive index.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

- (1) (A) is not correct but (R) is correct
 (2) Both (A) and (R) are correct and (R) is the correct explanation of (A)
 (3) (A) is correct but (R) is not correct
 (4) Both (A) and (R) are correct but (R) is **not** the correct explanation of (A)

Ans. (3)

- Sol. Refractive index has no relation with mass density because both have different meaning. Hence reason is incorrect.

So (A) is correct but (R) is not correct.

40. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason(R)**.

Assertion (A) : Magnetic monopoles do not exist.

Reason (R) : Magnetic field lines are continuous and form closed loops.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

- (1) Both (A) and (R) are correct but (R) is **not** the correct explanation of (A)
- (2) (A) is correct but (R) is not correct
- (3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (4) (A) is not correct but (R) is correct

Ans. (3)

Sol. Both statements are correct and reason is also the correct explanation of assertion.

41. Which one of the following forces cannot be expressed in terms of potential energy?

- (1) Coulomb's force
- (2) Gravitational force
- (3) Frictional force
- (4) Restoring force

Ans. (3)

Sol. Potential energy is defined for conservative force only. It is not defined for non-conservative force i.e. frictional force.

42. Match **List-I** with **List-II**.

List-I		List-II	
(A)	Isothermal	(I)	ΔW (work done) = 0
(B)	Adiabatic	(II)	ΔQ (supplied heat) = 0
(C)	Isobaric	(III)	ΔU (change in internal energy) $\neq 0$
(D)	Isochoric	(IV)	$\Delta U = 0$

Choose the **correct** answer from the options given below :

- (1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
- (2) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
- (3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
- (4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Ans. (3)

Sol. (A) Isothermal $\rightarrow \Delta T = 0 \rightarrow \Delta U = 0$ (IV)

(B) Adiabatic $\rightarrow \Delta Q = 0$ (II)

(C) Isobaric $\rightarrow \Delta P = 0 \rightarrow \Delta U \neq 0$ (III)

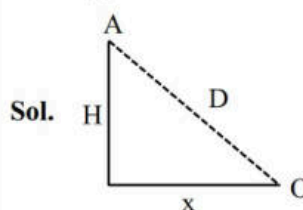
(D) Isochoric $\rightarrow \Delta V = 0 \rightarrow \Delta W = 0$ (I)

43. A helicopter flying horizontally with a speed of 360 km/h at an altitude of 2 km, drops an object at an instant. The object hits the ground at a point O, 20 s after it is dropped. Displacement of 'O' from the position of helicopter where the object was released is :

(use acceleration due to gravity $g = 10 \text{ m/s}^2$ and neglect air resistance)

- (1) $2\sqrt{5}$ km
- (2) 4 km
- (3) 7.2 km
- (4) $2\sqrt{2}$ km

Ans. (4)



Sol.

$$u = 360 \times \frac{5}{18} = 100 \text{ m/s}$$

$$x = u \times t = 2 \times 10^3 \text{ m}$$

$$t = \sqrt{\frac{2H}{g}} \Rightarrow H = \frac{t^2 g}{2}$$

$$H = \frac{400 \times 10}{2}$$

$$H = 2000 \text{ m}$$

$$D = \sqrt{x^2 + H^2}$$

$$D = 2\sqrt{2} \text{ km}$$

44. An object with mass 500 g moves along x-axis with speed $v = 4\sqrt{x}$ m/s. The force acting on the object is :

- (1) 8 N
- (2) 5 N
- (3) 6 N
- (4) 4 N

Ans. (4)

Sol. $F = M \times a$

$$v = 4\sqrt{x}$$

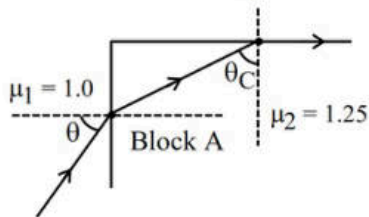
$$v^2 = 16x$$

$$2v \frac{dv}{dx} = 16$$

$$\frac{v dv}{dx} = \frac{16}{2} = 8$$

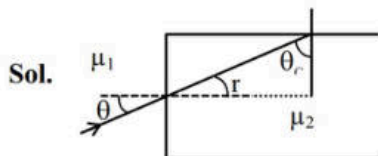
$$F = 0.5 \times 8 = 4 \text{ N}$$

45. A transparent block A having refractive index $\mu = 1.25$ is surrounded by another medium of refractive index $\mu = 1.0$ as shown in figure. A light ray is incident on the flat face of the block with incident angle θ as shown in figure. What is the maximum value of θ for which light suffers total internal reflection at the top surface of the block ?



- (1) $\tan^{-1}(4/3)$ (2) $\tan^{-1}(3/4)$
 (3) $\sin^{-1}(3/4)$ (4) $\cos^{-1}(3/4)$

Ans. (3)



Sol.

$$r + \theta_c = 90^\circ$$

$$\mu_1 \sin \theta = \mu_2 \sin r$$

$$\sin \theta = \frac{\mu_2}{\mu_1} \sin(90 - \theta_c)$$

$$\sin \theta = \frac{\mu_2}{\mu_1} \cos \theta_c$$

$$\sin \theta_c = \frac{\mu_1}{\mu_2}$$

$$\sin \theta = \frac{\mu_2}{\mu_1} \sqrt{1 - \frac{\mu_1^2}{\mu_2^2}}$$

$$\sin \theta = \sqrt{\frac{\mu_2^2 - \mu_1^2}{\mu_1^2}} = \sqrt{\frac{25}{16} - 1}$$

$$\sin \theta = \frac{3}{4}$$

$$\theta = \sin^{-1}\left(\frac{3}{4}\right)$$

SECTION-B

46. A parallel plate capacitor has charge $5 \times 10^{-6} \text{ C}$. A dielectric slab is inserted between the plates and almost fills the space between the plates. If the induced charge on one face of the slab is $4 \times 10^{-6} \text{ C}$ then the dielectric constant of the slab is _____.

Ans. (5)

Sol. $Q_{in} = Q \left(1 - \frac{1}{K}\right)$

$$4 \times 10^{-6} = 5 \times 10^{-6} \left(1 - \frac{1}{K}\right)$$

$$1 - \frac{1}{K} = \frac{4}{5}$$

$$K = 5$$

47. An inductor of reactance 100Ω , a capacitor of reactance 50Ω , and a resistor of resistance 50Ω are connected in series with an AC source of 10 V , 50 Hz . Average power dissipated by the circuit is _____ W.

Ans. (1)

Sol. $P = V_{rms} I_{rms} \cos \phi$

$$P = V_{rms} \times \frac{V_{rms}}{Z} \times \frac{R}{Z}$$

$$P = V_{rms}^2 \times \frac{R}{Z^2}$$

$$Z = \sqrt{R^2 + (x_L - x_C)^2}$$

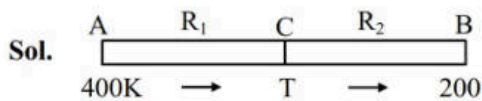
$$Z = 50\sqrt{2} \Omega$$

$$P = 100 \times \frac{50}{2500 \times 2} = 1 \text{ W}$$

48. Two cylindrical rods A and B made of different materials, are joined in a straight line. The ratio of lengths, radii and thermal conductivities of these rods are :

$\frac{L_A}{L_B} = \frac{1}{2}, \frac{r_A}{r_B} = 2$ and $\frac{K_A}{K_B} = \frac{1}{2}$. The free ends of rods A and B are maintained at 400 K, 200 K, respectively. The temperature of rods interface is _____ K, when equilibrium is established.

Ans. (360)



$$R_1 = \frac{\ell_1}{K_1 A_1}, R_2 = \frac{\ell_2}{K_2 A_2}$$

$$\frac{dQ}{dt} = \frac{\Delta T}{R}$$

$$\left(\frac{dQ}{dt} \right)_1 = \left(\frac{dQ}{dt} \right)_2$$

$$\frac{400 - T}{R_1} = \frac{T - 200}{R_2}$$

$$\frac{400 - T}{T - 200} = \frac{R_1}{R_2} = \left(\frac{\ell_1}{\ell_2} \right) \left(\frac{r_2}{r_1} \right)^2 \times \frac{K_2}{K_1}$$

$$= \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times 2$$

$$= \left(\frac{1}{4} \right)$$

$$\frac{400 - T}{T - 200} = \frac{1}{4}$$

$$1600 - 4T = T - 200$$

$$5T = 1800$$

$$T = 360 \text{ K}$$

49. The electric field in a region is given by $\vec{E} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3 \text{ N/C}$. The flux of the field through a rectangular surface parallel to x-z plane is $6.0 \text{ Nm}^2\text{C}^{-1}$. The area of the surface is _____ cm^2 .

Ans. (15)

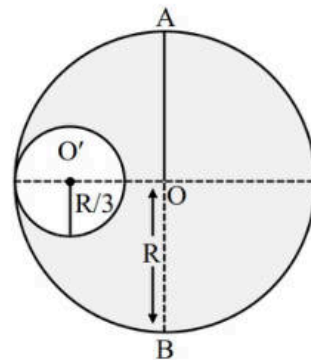
Sol. $\phi = \vec{E} \cdot \vec{A} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3 \cdot A\hat{j}$

$$6 = 4 \times 10^3 A$$

$$A = 1.5 \times 10^{-3} \text{ m}^2$$

$$= 15 \text{ cm}^2$$

50. M and R be the mass and radius of a disc. A small disc of radius $R/3$ is removed from the bigger disc as shown in figure. The moment of inertia of remaining part of bigger disc about an axis AB passing through the centre O and perpendicular to the plane of disc is $\frac{4}{x} MR^2$. The value of x is _____.



Ans. (9)

Sol. Without cavity $I_1 = \frac{MR^2}{2}$

$$\text{Mass of removed disc} = \frac{M}{\pi R^2} \times \left(\frac{R}{3} \right)^2 \pi$$

$$= \left(\frac{M}{9} \right)$$

$$\text{M.I. of removed disc } I_2 = \frac{\frac{M}{9} \left(\frac{R}{3} \right)^2}{2} + \frac{M}{9} \times \left(\frac{2R}{3} \right)^2$$

$$= \frac{MR^2}{18}$$

$$I = I_1 - I_2 = \frac{MR^2}{2} - \frac{MR^2}{18} = \frac{4MR^2}{9}$$

$$(n = 9)$$

CHEMISTRY

SECTION-A

51. Given below are two statements :

Statement (I) : On hydrolysis, oligo peptides give rise to fewer number of α -amino acids while proteins give rise to a large number of β -amino acids.

Statement (II) : Natural proteins are denatured by acids which convert the water soluble form of fibrous proteins to their water insoluble form.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

- (1) Both **statement I** and **statement II** are correct
- (2) **Statement I** is incorrect but **Statement II** is correct
- (3) Both **statement I** and **statement II** are incorrect
- (4) **Statement I** is correct but **Statement II** is incorrect

Ans. (3)

Sol. (i) Protein does not give β -amino acid on hydrolysis

(ii) Fibrous protein are not water soluble

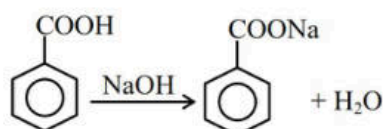
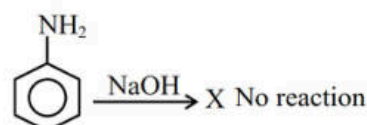
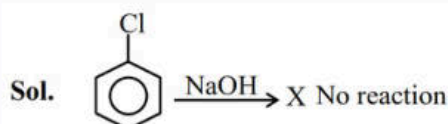
So both statements are wrong

52. Mixture of 1 g each of chlorobenzene, aniline and benzoic acid is dissolved in 50 mL ethyl acetate and placed in a separating funnel, 5 M NaOH (30 mL) was added in the same funnel. The funnel was shaken vigorously and then kept aside. The ethyl acetate layer in the funnel contains :

- (1) benzoic acid
- (2) benzoic acid and aniline
- (3) benzoic acid and chlorobenzene
- (4) chlorobenzene and aniline

Ans. (4)

TEST PAPER WITH SOLUTION

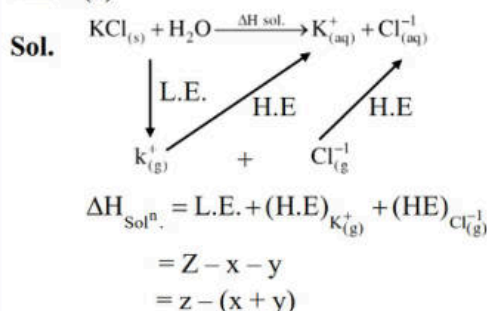


Organic layer in funnel are mixture of chlorobenzene and aniline

53. The hydration energies of K^+ and Cl^- are $-x$ and $-y$ kJ/mol respectively. If lattice energy of KCl is $-z$ kJ/mol, then the heat of solution of KCl is :

- (1) $+x - y - z$
- (2) $x + y + z$
- (3) $z - (x + y)$
- (4) $-z - (x + y)$

Ans. (3)



54. $\text{A(g)} \rightarrow \text{B(g)} + \text{C(g)}$ is a first order reaction.

Time	T	∞
P_{system}	P_t	P_{∞}

The reaction was started with reactant A only. Which of the following expression is correct for rate constant k ?

- (1) $k = \frac{1}{t} \ln \frac{2(p_{\infty} - P_t)}{P_t}$
- (2) $k = \frac{1}{t} \ln \frac{P_{\infty}}{P_t}$
- (3) $k = \frac{1}{t} \ln \frac{P_{\infty}}{2(p_{\infty} - P_t)}$
- (4) $k = \frac{1}{t} \ln \frac{P_{\infty}}{(p_{\infty} - P_t)}$

Ans. (3)

Sol.



$$t = 0 \quad P^0 \quad 0 \quad 0$$

$$t = t \quad P^0 - x \quad x \quad x$$

$$t = \infty \quad 0 \quad P^0 \quad P^0$$

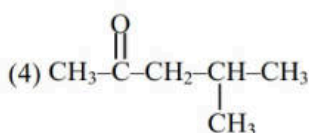
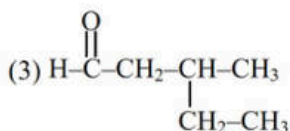
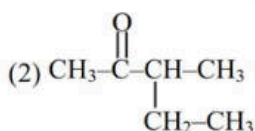
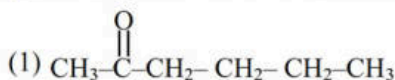
$$P_t = P^0 + x \Rightarrow x = P_t - P^0 = P_t - \frac{P_\infty}{2}$$

$$P_\infty = 2P^0 \Rightarrow P^0 = \frac{P_\infty}{2}$$

$$k = \frac{1}{t} \ln \frac{P^0}{P^0 - x}$$

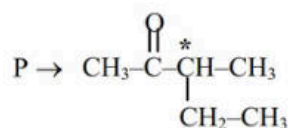
$$k = \frac{1}{t} \ln \frac{P_\infty}{2(P_\infty - P_t)}$$

55. "P" is an optically active compound with molecular formula $C_6H_{12}O$. When "P" is treated with 2,4-dinitrophenylhydrazine, it gives a positive test. However, in presence of Tollens reagent, "P" gives a negative test. Predict the structure of "P".



Ans. (2)

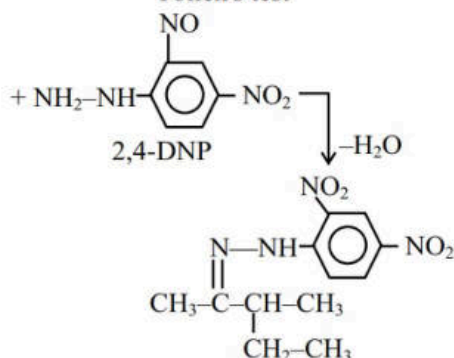
Sol.



Chiral

Does not give

Tollens test



Gives test with

2,4-DNP

56. Choose the incorrect trend in the atomic radii (r) of the elements :

(1) $r_{Br} < r_K$

(2) $r_{Mg} < r_{Al}$

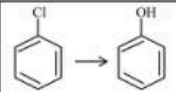
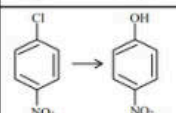
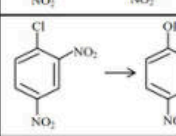
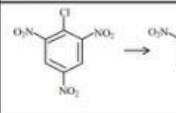
(3) $r_{Rb} < r_{Cs}$

(4) $r_{Al} < r_{Cs}$

Ans. (2)

Sol. In a period from left to right atomic size decreases.

57. Match List-I with List-II

List-I Conversion		List-II Reagents, Conditions used	
(A)		(I)	Warm, H_2O
(B)		(II)	(a) NaOH, 368 K ; (b) H_3O^+
(C)		(III)	(a) NaOH, 443 K; (b) H_3O^+
(D)		(IV)	(a) NaOH, 623 K, 300 atm ; (b) H_3O^+

Choose the correct answer from the options given below :

(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Ans. (3)

Sol. Aromatic halide give nucleophilic substitution reaction at high temperature or in presence of $-I/-M$ group rate of reaction high even at low temperature.

A-IV

B-III

C-II

D-I

58. The correct statement amongst the following is :

(1) The term 'standard state' implies that the temperature is 0°C

(2) The standard state of pure gas is the pure gas at a pressure of 1 bar and temperature 273 K

(3) $\Delta_f H_{298}^\circ$ is zero for $\text{O}(\text{g})$

(4) $\Delta_f H_{500}^\circ$ is zero for $\text{O}_2(\text{g})$

Ans. (4)

Sol. For standard state $\Rightarrow$ pressure = 1 bar and temperature is specified only

$$\Rightarrow (\Delta_f H^\circ)_{\text{O}_{2(\text{g})}} = 0$$

59. Liquid A and B form an ideal solution. The vapour pressure of pure liquids A and B are 350 and 750 mm Hg respectively at the same temperature. If x_A and x_B are the mole fraction of A and B in solution while y_A and y_B are the mole fraction of A and B in vapour phase then :

$$(1) \frac{x_A}{x_B} < \frac{y_A}{y_B}$$

$$(2) \frac{x_A}{x_B} = \frac{y_A}{y_B}$$

$$(3) \frac{x_A}{x_B} > \frac{y_A}{y_B}$$

$$(4) (x_A - y_A) < (x_B - y_B)$$

Ans. (3)

Sol. $P_A^\circ < P_B^\circ$

$$\frac{P_A^\circ}{P_B^\circ} < 1$$

$$\frac{y_A}{y_B} = \frac{P_A^\circ x_A}{P_B^\circ x_B}$$

$$\frac{y_A}{y_B} < 1$$

$$\frac{x_A}{x_B} < 1$$

$$\frac{y_A}{y_B} < \frac{x_A}{x_B}$$

60. 'X' is the number of acidic oxides among VO_2 , V_2O_3 , CrO_3 , V_2O_5 and Mn_2O_7 . The primary valency of cobalt in $[\text{Co}(\text{H}_2\text{NCH}_2\text{CH}_2\text{NH}_2)_3]_2(\text{SO}_4)_3$ is Y.

The value of $X + Y$ is :

(1) 5

(2) 4

(3) 2

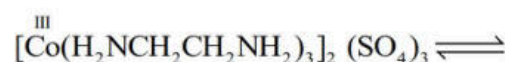
(4) 3

Ans. (1)

Sol. $\text{CrO}_3 = \text{Acidic}$

$\text{Mn}_2\text{O}_7 = \text{Acidic}$

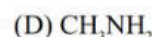
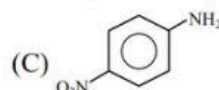
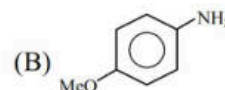
$\therefore x = 2$



$\therefore$ Primary valency = 3

$$\therefore x + y = 5$$

61. The descending order of basicity of following amines is :



Choose the **correct** answer from the options given below :

(1) $\text{B} > \text{E} > \text{D} > \text{A} > \text{C}$

(2) $\text{E} > \text{D} > \text{B} > \text{A} > \text{C}$

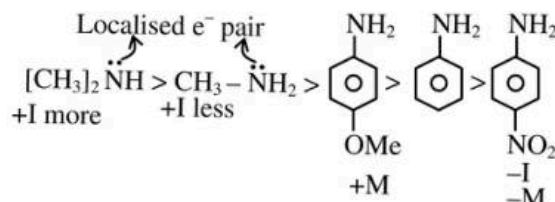
(3) $\text{E} > \text{D} > \text{A} > \text{B} > \text{C}$

(4) $\text{E} > \text{A} > \text{D} > \text{C} > \text{B}$

Ans. (2)

Sol. [2]

$\text{E} > \text{D} > \text{B} > \text{A} > \text{C}$



62. Match List-I with List-II

List-I Complex		List-II Primary valency and Secondary valency		
(A)	$[\text{Co(en)}_2\text{Cl}_2]\text{Cl}$	(I)	3	6
(B)	$[\text{Pt}(\text{NH}_3)_2\text{Cl}(\text{NO}_2)]$	(II)	3	4
(C)	$\text{Hg}[\text{Co}(\text{SCN})_4]$	(III)	2	6
(D)	$[\text{Mg}(\text{EDTA})]^{2-}$	(IV)	2	4

Choose the **correct** answer from the options given below :

- (1) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
 (2) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)
 (3) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
 (4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Ans. (2)

Sol. Primary valency = Oxidation state

Secondary valency = Co-ordination number

	Complex	Primary valency	Secondary
(A)	$[\text{Co(en)}_2\text{Cl}_2]\text{Cl}$	3	6
(B)	$[\text{Pt}(\text{NH}_3)_2\text{Cl}(\text{NO}_2)]$	2	4
(C)	$\text{Hg}[\text{Co}(\text{SCN})_4]$	3	4
(D)	$[\text{Mg}(\text{EDTA})]^{2-}$	2	6

63. Match List-I with List-II

List-I		List-II	
(A)	Solution of chloroform and acetone	(I)	Minimum boiling azeotrope
(B)	Solution of ethanol and water	(II)	Dimerizes
(C)	Solution of benzene and toluene	(III)	Maximum boiling azeotrope
(D)	Solution of acetic acid in benzene	(IV)	$\Delta V_{\text{mix}} = 0$

Choose the **correct** answer from the options given below :

- (1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
 (2) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
 (3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
 (4) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

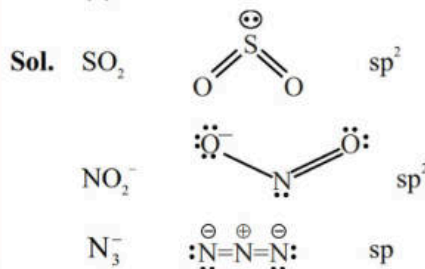
Ans. (1)

- Sol. (A) Solution of chloroform and acetone shows -ve deviation, so maximum boiling azeotrope.
 (B) Solution of ethanol & water shows +ve deviation. So minimum boiling azeotrope.
 (C) Solution of benzene and toluene form ideal solution. $\Delta V_{\text{mix}} = 0$.
 (D) Acetic acid in benzene form dimer.

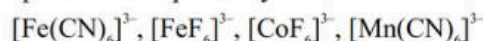
64. In SO_2 , NO_2^- and N_3^- the hybridizations at the central atom are respectively :

- (1) sp^2 , sp^2 and sp (2) sp^2 , sp and sp
 (3) sp^2 , sp^2 and sp^2 (4) sp , sp^2 and sp

Ans. (1)



65. The number of unpaired electrons responsible for the paramagnetic nature of the following complex species are respectively :



- (1) 1, 5, 4, 2 (2) 1, 5, 5, 2
 (3) 1, 1, 4, 2 (4) 1, 4, 4, 2

Ans. (1)

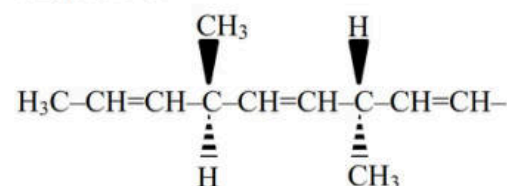
Sol. $[\text{Fe}(\text{CN})_6]^{3-}$ Fe^{3+} $3d^5$ $t_{2g}^{2,2,1}$ $e_g^{0,0}$ unpaired $e^- = 1$

$[\text{FeF}_6]^{3-}$ Fe^{3+} $3d^5$ $t_{2g}^{1,1,1}$ $e_g^{1,1}$ unpaired $e^- = 5$

$[\text{CoF}_6]^{3-}$ Co^{3+} $3d^6$ $t_{2g}^{2,1,1}$ e_g^1 unpaired $e^- = 4$

$[\text{Mn}(\text{CN})_6]^{3-}$ Mn^{3+} $3d^4$ $t_{2g}^{2,1,1}$ $e_g^{0,0}$ unpaired $e^- = 2$

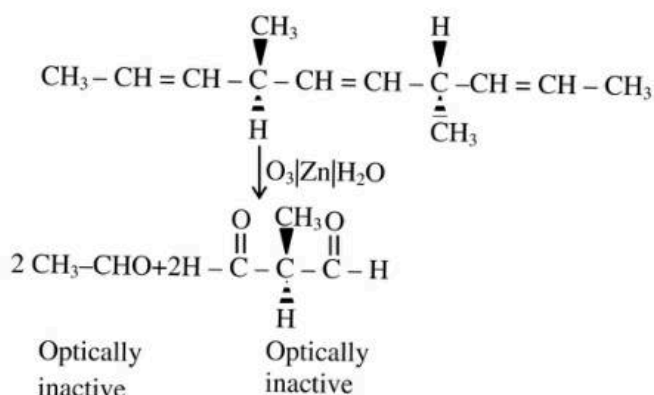
66. The number of optically active products obtained from the complete ozonolysis of the given compound is :



- (1) 2 (2) 0
 (3) 1 (4) 4

Ans. (2)

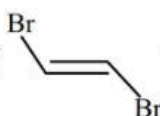
Sol.



67. Given below are two statements :

Statement (I) :  is more polar than



Statement (II) : Boiling point of  is

lower than  but it is more polar than

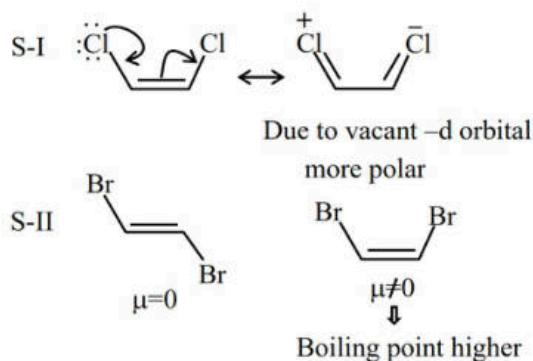


In the light of the above statements, choose the **most appropriate answer** from the options given below :

- (1) **Statement I** is correct but **statement II** is incorrect
- (2) **Statement I** is incorrect but **statement II** is correct
- (3) Both **statement I** and **statement II** are incorrect
- (4) Both **statement I** and **statement II** are correct

Ans. (1)

Sol.



68. The extra stability of half-filled subshell is due to
- (A) Symmetrical distribution of electrons
 - (B) Smaller coulombic repulsion energy
 - (C) The presence of electrons with the same spin in non-degenerate orbitals
 - (D) Larger exchange energy
 - (E) Relatively smaller shielding of electrons by one another

Identify the **correct** statements

- (1) (B), (D) and (E) only
- (2) (A), (B), (D) and (E) only
- (3) (B), (C) and (D) only
- (4) (A), (B) and (D) only

Ans. (2)

Sol. Extra stability of half filled is due to :

- (i) Symmetrical distribution of electrons
- (ii) Large exchange energy
- (iii) Smaller coulombic repulsion
- (iv) Smaller shielding of electrons by one another

69. The correct statements from the following are :

- (A) Ti^{3+} is a powerful oxidising agent
- (B) Al^{3+} does not get reduced easily
- (C) Both Al^{3+} and Ti^{3+} are very stable in solution
- (D) Ti^+ is more stable than Ti^{3+}
- (E) Al^{3+} and Ti^+ are highly stable

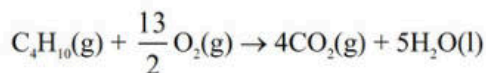
Choose the **correct** answer from the options given below :

- (1) (A), (B), (C), (D) and (E)
- (2) (A), (B), (D) and (E) only
- (3) (B), (D) and (E) only
- (4) (A), (C) and (D) only

Ans. (2)

$$\%N = 18\%$$

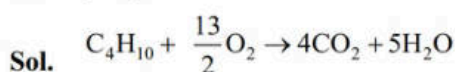
73. Butane reacts with oxygen to produce carbon dioxide and water following the equation given below



If 174.0 kg of butane is mixed with 320.0 kg of O_2 , the volume of water formed in litres is _____. (Nearest integer)

[Given : (a) Molar mass of C, H, O are 12, 1, 16 g mol⁻¹ respectively, (b) Density of water = 1 g mL⁻¹]

Ans. (138)



$$3 \times 10^3 \quad 10 \times 10^3$$

$$\text{Moles of H}_2\text{O formed} = n_{\text{H}_2\text{O}} = 5 \times \frac{2}{13} \times 10 \times 10^3$$

$$\text{Then } w_{\text{H}_2\text{O}} = \frac{10^5}{13} \times 18$$

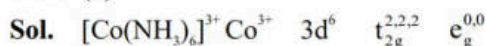
$$= 1.3846 \times 10^5 \text{ g}$$

Volume of H_2O will be = 138.46 litre.

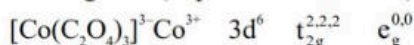
Ans. 138

74. The number of paramagnetic metal complex species among $[\text{Co}(\text{NH}_3)_6]^{3+}$, $[\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}$, $[\text{MnCl}_6]^{3-}$, $[\text{Mn}(\text{CN})_6]^{3-}$, $[\text{CoF}_6]^{3-}$, $[\text{Fe}(\text{CN})_6]^{3-}$ and $[\text{FeF}_6]^{3-}$ with same number of unpaired electrons is _____.

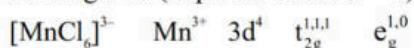
Ans. (2)



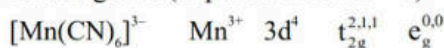
Diamagnetic (unpaired electron = 0)



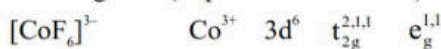
Diamagnetic (unpaired electron = 0)



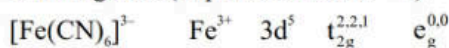
Paramagnetic (unpaired electron = 4)



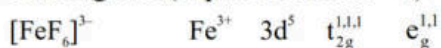
Paramagnetic (unpaired electron = 2)



Paramagnetic (unpaired electron = 4)

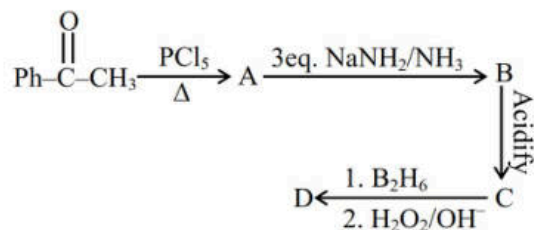


Paramagnetic (unpaired electron = 1)



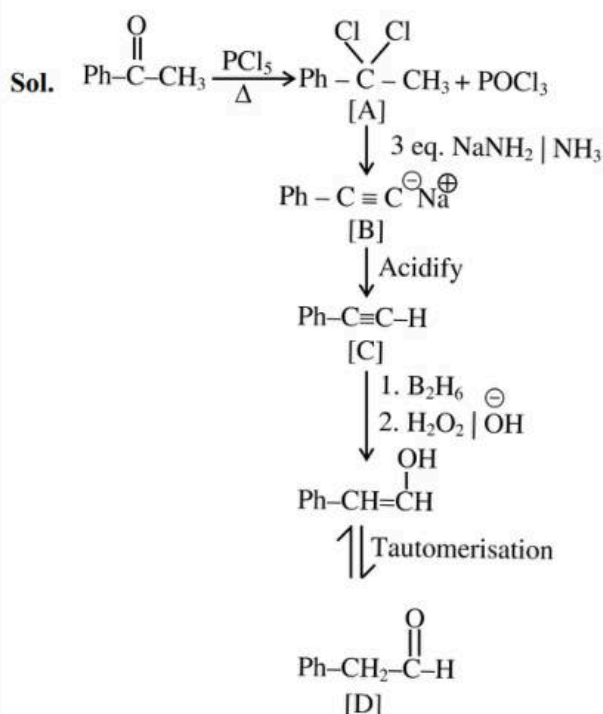
Paramagnetic (unpaired electron = 5)

75. Identify the structure of the final product (D) in the following sequence of the reactions :



Total number of sp^2 hybridised carbon atoms in product D is.

Ans. (7)



$\Rightarrow$ Number of sp^2 C-atoms in product D = 7

NTA Ans. = 7 ALLEN Ans. = 7