

# FINAL JEE-MAIN EXAMINATION – APRIL, 2024

(Held On Monday 08<sup>th</sup> April, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

## MATHEMATICS

### SECTION-A

1. The value of  $k \in \mathbb{N}$  for which the integral

$$I_n = \int_0^1 (1-x^k)^n dx, \quad n \in \mathbb{N}, \text{ satisfies } 147 I_{20} = 148 I_{21}$$

is :

(1) 10 (2) 8

(3) 14 (4) 7

**Ans. (4)**

**Sol.**  $I_n = \int_0^1 (1-x^k)^n \cdot 1 \, dx$

$$I_n = (1-x^k)^n \cdot x - nk \int_0^1 (1-x^k)^{n-1} \cdot x^{k-1} \cdot dx$$

$$I_n = nk \int_0^1 [(1-x^k)^n - (1-x^k)^{n-1}] dx$$

$$I_n = nk I_n - nk I_{n-1}$$

$$\frac{I_n}{I_{n-1}} = \frac{nk}{nk+1}$$

$$\frac{I_{21}}{I_{20}} = \frac{21k}{1+21k}$$

$$= \frac{147}{148} \Rightarrow k = 7$$

2. The sum of all the solutions of the equation  $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$  is :

(1)  $1 + \log_6(8)$  (2)  $\log_8(6)$

(3)  $1 + \log_8(6)$  (4)  $\log_8(4)$

**Ans. (3)**

**Sol.**  $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$

Put  $8^x = t$

$$t^2 - 16 + 48 = 0$$

$$\Rightarrow t = 4 \text{ or } t = 12$$

$$\Rightarrow 8^x = 4 \quad 8^x = 12$$

$$\Rightarrow x = \log_8 4 \quad x = \log_8 12$$

$$\text{sum of solution} = \log_8 4 + \log_8 12$$

$$= \log_8 48 = \log_8(6 \cdot 8)$$

$$= 1 + \log_8 6$$

## TEST PAPER WITH SOLUTION

3. Let the circles  $C_1 : (x - \alpha)^2 + (y - \beta)^2 = r_1^2$  and

$$C_2 : (x - 8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2 \text{ touch each other}$$

externally at the point  $(6, 6)$ . If the point  $(6, 6)$  divides the line segment joining the centres of the circles  $C_1$  and  $C_2$  internally in the ratio  $2 : 1$ , then

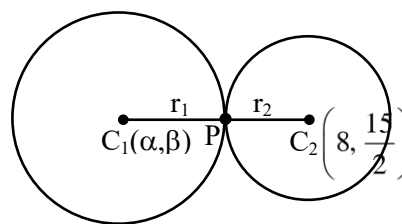
$$(\alpha + \beta) + 4(r_1^2 + r_2^2) \text{ equals}$$

(1) 110 (2) 130

(3) 125 (4) 145

**Ans. (2)**

**Sol.**



$$\begin{array}{c} 2 : 1 \\ \bullet \quad \bullet \quad \bullet \\ (\alpha, \beta) \quad C_1 \quad P \quad C_2 \quad \left(8, \frac{15}{2}\right) \\ \quad \quad \quad (6, 6) \end{array}$$

$$\therefore \frac{16 + \alpha}{3} = 6 \text{ and } \frac{15 + \beta}{3} = 6$$

$$\Rightarrow (\alpha, \beta) \equiv (2, 3)$$

$$\text{Also, } C_1 C_2 = r_1 + r_2$$

$$\Rightarrow \sqrt{(2-8)^2 + \left(3 - \frac{15}{2}\right)^2} = 2r_1 + r_2$$

$$\Rightarrow r_2 = \frac{5}{2} \Rightarrow r_1 = 2r_2 = 5$$

$$\therefore (\alpha + \beta) + 4(r_1^2 + r_2^2)$$

$$= 5 + 4\left(\frac{25}{4} + 25\right) = 130$$

4. Let  $P(x, y, z)$  be a point in the first octant, whose projection in the  $xy$ -plane is the point  $Q$ . Let  $OP = \gamma$ ; the angle between  $OQ$  and the positive  $x$ -axis be  $\theta$ ; and the angle between  $OP$  and the positive  $z$ -axis be  $\phi$ , where  $O$  is the origin. Then the distance of  $P$  from the  $x$ -axis is :

$$(1) \gamma\sqrt{1 - \sin^2 \phi \cos^2 \theta} \quad (2) \gamma\sqrt{1 + \cos^2 \theta \sin^2 \phi}$$

$$(3) \gamma\sqrt{1 - \sin^2 \theta \cos^2 \phi} \quad (4) \gamma\sqrt{1 + \cos^2 \phi \sin^2 \theta}$$

**Ans. (1)**

**Sol.**  $P(x, y, z), Q(x, y, O); x^2 + y^2 + z^2 = \gamma^2$

$$\overrightarrow{OQ} = x\hat{i} + y\hat{j}$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\Rightarrow \sin^2 \phi = \frac{x^2 + y^2}{x^2 + y^2 + z^2}$$

$$\text{distance of } P \text{ from } x\text{-axis} = \sqrt{y^2 + z^2}$$

$$\Rightarrow \sqrt{\gamma^2 - x^2} \Rightarrow \gamma \sqrt{1 - \frac{x^2}{\gamma^2}}$$

$$= \gamma \sqrt{1 - \cos^2 \theta \sin^2 \phi}$$

5. The number of critical points of the function  $f(x) = (x - 2)^{2/3} (2x + 1)$  is :

$$(1) 2 \quad (2) 0$$

$$(3) 1 \quad (4) 3$$

**Ans. (1)**

**Sol.**  $f(x) = (x - 2)^{2/3} (2x + 1)$

$$f'(x) = \frac{2}{3}(x - 2)^{-1/3} (2x + 1) + (x - 2)^{2/3} (2)$$

$$f'(x) = 2 \times \frac{(2x + 1) + (x - 2)}{3(x - 2)^{1/3}}$$

$$\frac{3x - 1}{(x - 2)^{1/3}} = 0$$

$$\text{Critical points } x = \frac{1}{3} \text{ and } x = 2$$

6. Let  $f(x)$  be a positive function such that the area bounded by  $y = f(x), y = 0$  from  $x = 0$  to  $x = a > 0$  is  $e^{-a} + 4a^2 + a - 1$ . Then the differential equation, whose general solution is  $y = c_1 f(x) + c_2$ , where  $c_1$  and  $c_2$  are arbitrary constants, is :

$$(1) (8e^x - 1) \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$$

$$(2) (8e^x + 1) \frac{d^2 y}{dx^2} - \frac{dy}{dx} = 0$$

$$(3) (8e^x + 1) \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$$

$$(4) (8e^x - 1) \frac{d^2 y}{dx^2} - \frac{dy}{dx} = 0$$

**Ans. (3)**

**Sol.**  $\int_0^a f(x) dx = e^{-a} + 4a^2 + a - 1$

$$f(a) = -e^{-a} + 8a + 1$$

$$f(x) = -e^{-x} + 8x + 1$$

$$\text{Now } y = C_1 f(x) + C_2$$

$$\frac{dy}{dx} = C_1 f'(x) = C_1 (e^{-x} + 8) \quad \dots (1)$$

$$\frac{d^2 y}{dx^2} = -C_1 e^{-x} \Rightarrow -e^x \frac{d^2 y}{dx^2}$$

Put in equation (1)

$$\frac{dy}{dx} = -e^x \frac{d^2 y}{dx^2} (e^{-x} + 8)$$

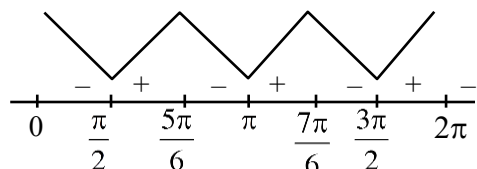
$$(8e^x + 1) \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$$

7. Let  $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$ . The number of points of local maxima of  $f$  in interval  $(0, 2\pi)$  is:

- (1) 1 (2) 2  
(3) 3 (4) 4

**Ans. (2)**

**Sol.**  $f(x) = 4\cos^3(x) + 3\sqrt{3}\cos^2(x) - 10$ ;  $x \in (0, 2\pi)$   
 $\Rightarrow f'(x) = 12\cos^2 x [-\sin(x)] + 3\sqrt{3}(2\cos(x))[-\sin(x)]$   
 $\Rightarrow f'(x) = -6\sin(x)\cos(x)[2\cos(x) + \sqrt{3}]$



local maxima at  $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

8. Let  $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$ . If  $A^3 = 4A^2 - A - 21I$ , where

$I$  is the identity matrix of order  $3 \times 3$ , then  $2a + 3b$  is equal to :

- (1) -10 (2) -13  
(3) -9 (4) -12

**Ans. (2)**

**Sol.**  $A^3 - 4A^2 + A + 21I = 0$   
 $\text{tr}(A) = 4 + 5 + 6 \Rightarrow b = -1$   
 $|A| = -21$   
 $-16 + a = -21 \Rightarrow a = -5$   
 $2a + 3b = -13$

9. If the shortest distance between the lines

$$L_1 : \vec{r} = (2 + \lambda)\hat{i} + (1 - 3\lambda)\hat{j} + (3 + 4\lambda)\hat{k}, \lambda \in \mathbb{R}$$

$$L_2 : \vec{r} = 2(1 + \mu)\hat{i} + 3(1 + \mu)\hat{j} + (5 + \mu)\hat{k}, \mu \in \mathbb{R}$$

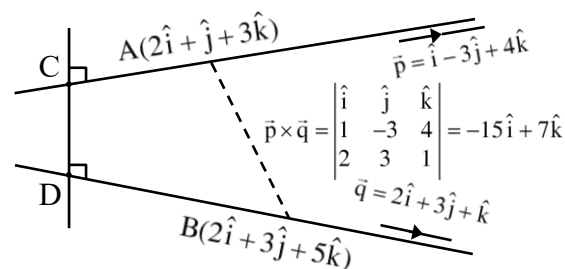
is  $\frac{m}{\sqrt{n}}$ , where  $\gcd(m, n) = 1$ , then the value of

$m + n$  equals.

- (1) 384 (2) 387  
(3) 377 (4) 390

**Ans. (2)**

**Sol.**



$$\begin{aligned} \text{Shortest distance (CD)} &= \frac{|\overline{AB} \cdot \vec{p} \times \vec{q}|}{|\vec{p} \times \vec{q}|} \\ &= \frac{|(0\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (-15\hat{i} + 7\hat{j} + 9\hat{k})|}{\sqrt{355}} \\ &= \frac{0 + 14 + 18}{\sqrt{355}} = \frac{32}{\sqrt{355}} \end{aligned}$$

$$\therefore m + n = 32 + 355 = 387$$

10. Let the sum of two positive integers be 24. If the probability, that their product is not less than

$\frac{3}{4}$  times their greatest positive product, is  $\frac{m}{n}$ ,

where  $\gcd(m, n) = 1$ , then  $n - m$  equals :

- (1) 9 (2) 11  
(3) 8 (4) 10

**Ans. (4)**

**Sol.**  $x + y = 24, x, y \in \mathbb{N}$

$$AM > GM \Rightarrow xy \leq 144$$

$$xy \geq 108$$

Favorable pairs of  $(x, y)$  are

$(13, 11), (12, 12), (14, 10), (15, 9), (16, 8),$   
 $(17, 7), (18, 6), (6, 18), (7, 17), (8, 16), (9, 15),$   
 $(10, 14), (11, 13)$

i.e. 13 cases

Total choices for  $x + y = 24$  is 23

$$\text{Probability} = \frac{13}{23} = \frac{m}{n}$$

$$n - m = 10$$

11. If  $\sin x = -\frac{3}{5}$ , where  $\pi < x < \frac{3\pi}{2}$ ,

then  $80(\tan^2 x - \cos x)$  is equal to :

(1) 109 (2) 108

(3) 18 (4) 19

Ans. (1)

Sol.  $\sin x = -\frac{3}{5}$ ,  $\pi < x < \frac{3\pi}{2}$

$$\tan x = \frac{3}{4} \quad \cos x = -\frac{4}{5}$$

$$80(\tan^2 x - \cos x)$$

$$= 80\left(\frac{9}{16} + \frac{4}{5}\right) = 45 + 64 = 109$$

12. Let  $I(x) = \int \frac{6}{\sin^2 x (1 - \cot x)^2} dx$ . If  $I(0) = 3$ , then

$I\left(\frac{\pi}{12}\right)$  is equal to :

(1)  $\sqrt{3}$  (2)  $3\sqrt{3}$

(3)  $6\sqrt{3}$  (4)  $2\sqrt{3}$

Ans. (2)

Sol.  $I(x) = \int \frac{6 dx}{\sin^2 x (1 - \cot x)^2} = \int \frac{6 \operatorname{cosec}^2 x dx}{(1 - \cot x)^2}$

Put  $1 - \cot x = t$

$\operatorname{cosec}^2 x dx = dt$

$$I = \int \frac{6 dt}{t^2} = \frac{-6}{t} + c$$

$$I(x) = \frac{-6}{1 - \cot x} + c, c = 3$$

$$I(x) = 3 - \frac{6}{1 - \cot x}, I\left(\frac{\pi}{12}\right) = 3 - \frac{6}{1 - (2 + \sqrt{3})}$$

$$I\left(\frac{\pi}{12}\right) = 3 + \frac{6}{\sqrt{3} + 1} = 3 + \frac{6(\sqrt{3} - 1)}{2} = 3\sqrt{3} + 2$$

13. The equations of two sides AB and AC of a triangle ABC are  $4x + y = 14$  and  $3x - 2y = 5$ ,

respectively. The point  $\left(2, -\frac{4}{3}\right)$  divides the third

side BC internally in the ratio 2 : 1. The equation

of the side BC is :

(1)  $x - 6y - 10 = 0$

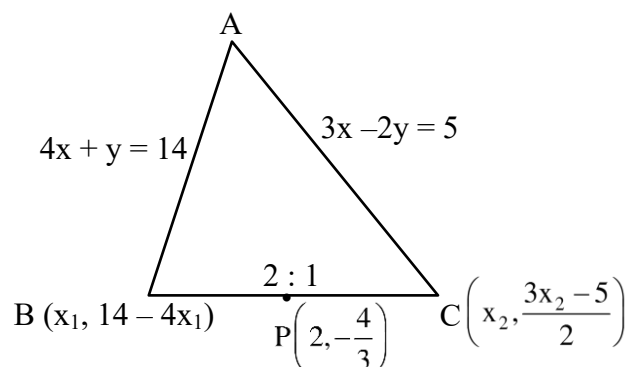
(2)  $x - 3y - 6 = 0$

(3)  $x + 3y + 2 = 0$

(4)  $x + 6y + 6 = 0$

Ans. (3)

Sol.



$$\frac{2x_2 + x_1}{3} = 2, \frac{2\left(\frac{3x_2 - 5}{2}\right) + (14 - 4x_1)}{3} = \frac{-4}{3}$$

$$2x_2 + x_1 = 6, 3x_2 - 4x_1 = -13$$

$$x_2 = 1, x_1 = 4$$

So,  $C(1, -1), B(4, -2)$

$$m = \frac{-1}{3}$$

Equation of BC :  $y + 1 = \frac{-1}{3}(x - 1)$

$$3y + 3 = -x + 1$$

$$x + 3y + 2 = 0$$

14. Let  $[t]$  be the greatest integer less than or equal to  $t$ . Let  $A$  be the set of all prime factors of 2310 and

$$f: A \rightarrow \mathbb{Z} \text{ be the function } f(x) = \left[ \log_2 \left( x^2 + \left[ \frac{x^3}{5} \right] \right) \right].$$

The number of one-to-one functions from  $A$  to the range of  $f$  is :

- (1) 20 (2) 120  
(3) 25 (4) 24

**Ans. (2)**

**Sol.**  $N = 2310 = 231 \times 10$

$$= 3 \times 11 \times 7 \times 2 \times 5$$

$$A = \{2, 3, 5, 7, 11\}$$

$$f(x) = \left[ \log_2 \left( x^2 + \left[ \frac{x^3}{5} \right] \right) \right]$$

$$f(2) = [\log_2(5)] = 2$$

$$f(3) = [\log_2(14)] = 3$$

$$f(5) = [\log_2(25 + 25)] = 5$$

$$f(7) = [\log_2(117)] = 6$$

$$f(11) = [\log_2 387] = 8$$

$$\text{Range of } f: B = \{2, 3, 5, 6, 8\}$$

$$\text{No. of one-one functions} = 5! = 120$$

15. Let  $z$  be a complex number such that  $|z + 2| = 1$

and  $\operatorname{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$ . Then the value of  $|\operatorname{Re}(\overline{z+2})|$

is :

- (1)  $\frac{\sqrt{6}}{5}$  (2)  $\frac{1+\sqrt{6}}{5}$   
(3)  $\frac{24}{5}$  (4)  $\frac{2\sqrt{6}}{5}$

**Ans. (4)**

**Sol.**  $|z + 2| = 1, \operatorname{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$

$$\text{Let } z + 2 = \cos\theta + i\sin\theta$$

$$\frac{1}{z+2} = \cos\theta - i\sin\theta$$

$$\Rightarrow \frac{z+1}{z+2} = 1 - \frac{1}{z+2} = 1 - (\cos\theta - i\sin\theta)$$

$$= (1 - \cos\theta) + i\sin\theta$$

$$\operatorname{Im}\left(\frac{z+1}{z+2}\right) = \sin\theta, \sin\theta = \frac{1}{5}$$

$$\cos\theta = \pm \sqrt{1 - \frac{1}{25}} = \pm \frac{2\sqrt{6}}{5}$$

$$|\operatorname{Re}(\overline{z+2})| = \frac{2\sqrt{6}}{5}$$

16. If the set  $R = \{(a, b) ; a + 5b = 42, a, b \in \mathbb{N}\}$

has  $m$  elements and  $\sum_{n=1}^m (1 + i^{n!}) = x + iy$ , where

$I = \sqrt{-1}$ , then the value of  $m + x + y$  is :

- (1) 8 (2) 12  
(3) 4 (4) 5

**Ans. (2)**

**Sol.**  $a + 5b = 42, a, b \in \mathbb{N}$

$$a = 42 - 5b, b = 1, a = 37$$

$$b = 2, a = 32$$

$$b = 3, a = 27$$

$\vdots$

$$b = 8, a = 2$$

$$R \text{ has "8" elements} \Rightarrow m = 8$$

$$\sum_{n=1}^8 (1 + i^{n!}) = x + iy$$

$$\text{for } n \geq 4, i^{n!} = 1$$

$$\Rightarrow (1 - i) + (1 - i^{2!}) + (1 - i^{3!})$$

$$= 1 - I + 2 + 1 + 1$$

$$= 5 - I = x + iy$$

$$m + x + y = 8 + 5 - 1 = 12$$

17. For the function  $f(x) = (\cos x) - x + 1$ ,  $x \in \mathbb{R}$ , between the following two statements

(S1)  $f(x) = 0$  for only one value of  $x$  is  $[0, \pi]$ .

(S2)  $f(x)$  is decreasing in  $\left[0, \frac{\pi}{2}\right]$  and increasing in

$$\left[\frac{\pi}{2}, \pi\right].$$

- (1) Both (S1) and (S2) are correct  
 (2) Only (S1) is correct  
 (3) Both (S1) and (S2) are incorrect  
 (4) Only (S2) is correct

**Ans. (2)**

**Sol.**  $f(x) = \cos x - x + 1$

$$f'(x) = -\sin x - 1$$

$f$  is decreasing  $\forall x \in \mathbb{R}$

$$f(x) = 0$$

$$f(0) = 2, f(\pi) = -\pi$$

$f$  is strictly decreasing in  $[0, \pi]$  and  $f(0) \cdot f(\pi) < 0$

$\Rightarrow$  only one solution of  $f(x) = 0$

S1 is correct and S2 is incorrect.

18. The set of all  $\alpha$ , for which the vector

$$\vec{a} = \alpha \hat{i} + 6\hat{j} - 3\hat{k} \quad \text{and} \quad \vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k} \quad \text{are}$$

inclined at an obtuse angle for all  $t \in \mathbb{R}$  is :

(1)  $[0, 1)$

(2)  $(-2, 0]$

(3)  $\left(-\frac{4}{3}, 0\right]$

(4)  $\left(-\frac{4}{3}, 1\right)$

**Ans. (3)**

**Sol.**  $\vec{a} = \alpha \hat{i} + 6\hat{j} - 3\hat{k}$

$$\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$$

$$\text{so } \vec{a} \cdot \vec{b} < 0, \forall t \in \mathbb{R}$$

$$\alpha t^2 - 12 + 6\alpha t < 0$$

$$\alpha t^2 + 6\alpha t - 12 < 0, \forall t \in \mathbb{R}$$

$$\alpha < 0, \text{ and } D < 0$$

$$36\alpha^2 + 48\alpha < 0$$

$$12\alpha(3\alpha + 4) < 0$$

$$\frac{-4}{3} < \alpha < 0$$

$$\text{also for } \alpha = 0, \vec{a} \cdot \vec{b} < 0$$

$$\text{hence } \alpha \in \left(-\frac{4}{3}, 0\right]$$

19. Let  $y = y(x)$  be the solution of the differential equation  $(1 + y^2)e^{\tan x} dx + \cos^2 x (1 + e^{2\tan x}) dy = 0$ ,

$y(0) = 1$ . Then  $y\left(\frac{\pi}{4}\right)$  is equal to :

(1)  $\frac{2}{e}$

(2)  $\frac{1}{e^2}$

(3)  $\frac{1}{e}$

(4)  $\frac{2}{e^2}$

**Ans. (3)**

**Sol.**  $(1 + y^2)e^{\tan x} dx + \cos^2 x (1 + e^{2\tan x}) dy = 0$

$$\int \frac{\sec^2 x e^{\tan x}}{1 + e^{2\tan x}} dx + \int \frac{dy}{1 + y^2} = C$$

$$\Rightarrow \tan^{-1}(e^{\tan x}) + \tan^{-1} y = C$$

$$\text{for } x = 0, y = 1, \tan^{-1}(1) + \tan^{-1} 1 = C$$

$$C = \frac{\pi}{2}$$

$$\tan^{-1}(e^{\tan x}) + \tan^{-1} y = \frac{\pi}{2}$$

$$\text{Put } x = \pi, \tan^{-1} e + \tan^{-1} y = \frac{\pi}{2}$$

$$\tan^{-1} y = \cot^{-1} e$$

$$y = \frac{1}{e}$$

20. Let  $H : \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$  be the hyperbola, whose

eccentricity is  $\sqrt{3}$  and the length of the latus rectum is  $4\sqrt{3}$ . Suppose the point  $(\alpha, 6)$ ,  $\alpha > 0$  lies on  $H$ . If  $\beta$  is the product of the focal distances of the point  $(\alpha, 6)$ , then  $\alpha^2 + \beta$  is equal to :

(1) 170

(2) 171

(3) 169

(4) 172

**Ans. (2)**

**Sol.** H :  $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ ,  $e = \sqrt{3}$

$$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{3} \Rightarrow \frac{a^2}{b^2} = 2$$

$$a^2 = 2b^2$$

$$\text{length of L.R.} = \frac{2a^2}{b} = 4\sqrt{3}$$

$$a = \sqrt{6}$$

$$P(\alpha, 6) \text{ lie on } \frac{y^2}{3} - \frac{x^2}{6} = 1$$

$$12 - \frac{\alpha^2}{6} = 1 \Rightarrow \alpha^2 = 66$$

$$\text{Foci} = (0, \pm be) = (0, 3) \text{ \& } (0, -3)$$

Let  $d_1$  &  $d_2$  be focal distances of  $P(\alpha, 6)$

$$d_1 = \sqrt{\alpha^2 + (6 + be)^2}, d_2 = \sqrt{\alpha^2 + (6 - be)^2}$$

$$d_1 = \sqrt{66 + 81}, d_2 = \sqrt{66 + 9}$$

$$\beta = d_1 d_2 = \sqrt{147 \times 75} = 105$$

$$\alpha^2 + \beta = 66 + 105 = 171$$

## SECTION-B

- 21.** Let  $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ . If the sum of the diagonal elements of  $A^{13}$  is  $3^n$ , then  $n$  is equal to \_\_\_\_\_.

**Ans. (7)**

**Sol.**  $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$$

$$A^7 = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix} \begin{bmatrix} -54 & 27 \\ -27 & -27 \end{bmatrix} = \begin{bmatrix} 3^6 \times 2 & -27^2 \\ 27^2 & 3^6 \end{bmatrix}$$

$$3^7 = 3^n \Rightarrow n = 7$$

- 22.** If the orthocentre of the triangle formed by the lines  $2x + 3y - 1 = 0$ ,  $x + 2y - 1 = 0$  and  $ax + by - 1 = 0$ , is the centroid of another triangle, whose circumcentre and orthocentre respectively are  $(3, 4)$  and  $(-6, -8)$ , then the value of  $|a - b|$  is \_\_\_\_\_.

**Ans. (16)**

**Sol.**  $2x + 3y - 1 = 0$

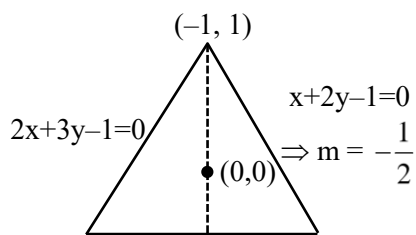
$$x + 2y - 1 = 0$$

$$ax + by - 1 = 0$$

$$\begin{array}{c} \text{2} \qquad \qquad \text{1} \\ \bullet \qquad \bullet \qquad \bullet \\ (-6, -8) \quad G \quad 0(3, 4) \\ \text{H} \qquad \qquad (6, 6) \end{array}$$

$$\left( \frac{6-6}{3}, \frac{8-8}{3} \right)$$

$$= (0, 0)$$



$$ax + by - 1 = 0$$

$$\left( \frac{1-0}{-1-0} \right) \left( \frac{-a}{b} \right) = -1$$

$$\Rightarrow -a = b$$

$$\Rightarrow ax - ay - 1 = 0$$

$$ax - a \left( 1 - \frac{2x}{3} \right) - 1 = 0$$

$$x \left( a + \frac{2a}{3} \right) = \frac{a}{3}$$

$$x = \frac{a+3}{5a}$$

$$2 \left( \frac{a+3}{5a} \right) + 3y - 1 = 0$$

$$y = \frac{1 - \frac{2a+6}{5a}}{3} = \frac{3a-6}{3 \times 5a}$$

$$y = \frac{a-2}{5a}$$

$$\frac{\left( \frac{a-2}{5a} \right)}{\left( \frac{a+3}{5a} \right)} = 2 \Rightarrow a-2 = 2a+6$$

$$a = -8$$

$$b = 8$$

$$-8x + 8y - 1 = 0$$

$$|a - b| = 16$$

23. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables  $X$  and  $Y$  respectively denote the number of blue and Yellow balls. If  $\bar{X}$  and  $\bar{Y}$  are the means of  $X$  and  $Y$  respectively, then  $7\bar{X} + 4\bar{Y}$  is equal to \_\_\_\_\_.

**Ans. (17)**

**Sol.**

| Blue balls | 0                                       | 1                                       | 2                                       | 3                                       | 4 | 5 |
|------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|---|---|
| Prob.      | $\frac{{}^5C_0 \cdot {}^4C_1}{{}^9C_3}$ | $\frac{{}^5C_1 \cdot {}^4C_2}{{}^9C_3}$ | $\frac{{}^5C_2 \cdot {}^4C_1}{{}^9C_3}$ | $\frac{{}^5C_3 \cdot {}^4C_0}{{}^9C_3}$ | 0 | 0 |

$$7\bar{X} = \frac{{}^5C_1 \cdot {}^4C_2 + {}^5C_2 \cdot {}^4C_1 \times 2 + {}^5C_3 \cdot {}^4C_0 \times 3}{{}^9C_3} \times 7$$

$$\frac{30 + 80 + 30}{84} \times 7$$

$$= \frac{140}{12} = \frac{70}{6} = \frac{35}{3}$$

| yellow | 0 | 1                       | 2                       | 3                       | 4 |
|--------|---|-------------------------|-------------------------|-------------------------|---|
|        |   | ${}^5C_2 \cdot {}^4C_1$ | ${}^5C_1 \cdot {}^4C_2$ | ${}^5C_0 \cdot {}^4C_3$ | 0 |

$$4\bar{Y} = \frac{40 + 60 + 12}{84} \times 4 = \frac{112}{21} = \frac{16}{3}$$

24. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to \_\_\_\_\_.

**Ans. (36)**

**Sol. 2, 3, 4, 5, 7**

total number of three digit numbers not divisible by 3 will be formed by using the digits

(4, 5, 7)

(3, 4, 7)

(2, 5, 7)

(2, 4, 7)

(2, 4, 5)

(2, 3, 5)

number of ways =  $6 \times 3! = 36$

25. Let the positive integers be written in the form :

$$\begin{array}{ccccccc}
 & & & & 1 & & \\
 & & & & 2 & & 3 \\
 & & 4 & & 5 & & 6 \\
 & \ddots & 7 & & 8 & & 9 & & 10 & \ddots \\
 & & & & & & & & & 
 \end{array}$$

If the  $k^{\text{th}}$  row contains exactly  $k$  numbers for every natural number  $k$ , then the row in which the number 5310 will be, is \_\_\_\_\_.

**Ans. (103)**

**Sol.**  $S = 1 + 2 + 4 + 7 + \dots + T_n$

$$S = 1 + 2 + 4 + \dots$$

$$T_n = 1 + 1 + 2 + 3 + \dots + (T_n - T_{n-1})$$

$$T_n = 1 + \left( \frac{n-1}{2} \right) [2 + (n-2) \times 1]$$

$$T_n = 1 + 1 + \frac{n(n-1)}{2}$$

$$n = 100 \quad T_n = 1 + \frac{100 \times 99}{2} = 4950 + 1$$

$$n = 101 \quad T_n = 1 + \frac{101 \times 100}{2} = 5050 + 1 = 5051$$

$$n = 102 \quad T_n = 1 + \frac{102 \times 101}{2} = 5151 + 1 = 5152$$

$$n = 103 \quad T_n = 1 + \frac{103 \times 102}{2} = 5254$$

$$n = 104 \quad T_n = 1 + \frac{104 \times 103}{2} = 5357$$

26. If the range of  $f(\theta) = \frac{\sin^4 \theta + 3 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$ ,  $\theta \in \mathbb{R}$  is

$[\alpha, \beta]$ , then the sum of the infinite G.P., whose first term is 64 and the common ratio is  $\frac{\alpha}{\beta}$ , is equal to

\_\_\_\_\_.

**Ans. (96)**

**Sol.**  $f(\theta) = \frac{\sin^4 \theta + 3 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$

$$f(\theta) = 1 + \frac{2 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$$

$$f(\theta) = \frac{2 \cos^2 \theta}{\cos^4 \theta - \cos^2 \theta + 1} + 1$$

$$f(\theta) = \frac{2}{\cos^2 \theta + \sec^2 \theta - 1} + 1$$

$$f(\theta)|_{\min.} = 1$$

$$f(\theta)|_{\max.} = 3$$

$$S = \frac{64}{1 - 1/3} = 96$$

27. Let  $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1) {}^n C_r$

$$\text{and } \beta = \left( \sum_{r=0}^n \frac{{}^n C_r}{r+1} \right) + \frac{1}{n+1}. \text{ If } 140 < \frac{2\alpha}{\beta} < 281,$$

then the value of  $n$  is \_\_\_\_\_.

**Ans. (5)**

**Sol.**  $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1) {}^n C_r$

$$\alpha = 4 \sum_{r=0}^n r^2 \cdot \frac{n}{r} \cdot {}^{n-1} C_{r-1} + 2 \sum_{r=0}^n r \cdot \frac{n}{r} \cdot {}^{n-1} C_{r-1} + \sum_{r=0}^n {}^n C_r$$

$$+ 4n \sum_{r=0}^n {}^{n-1} C_{r-1} + 2n \sum_{r=0}^n {}^{n-1} C_{r-1} + \sum_{r=0}^n {}^n C_r$$

$$\alpha = 4n(n-1) \cdot 2^{n-2} + 4n \cdot 2^{n-1} + 2n \cdot 2^{n-1} + 2^n$$

$$\alpha = 2^{n-2} [4n(n-1) + 8n + 4n + 4]$$

$$\alpha = 2^{n-2} [4n^2 + 8n + 4]$$

$$\alpha = 2n(n+1)^2$$

$$\beta = \sum_{r=0}^n \frac{{}^n C_r}{r+1} + \frac{1}{n+1}$$

$$= \sum_{r=0}^n \frac{{}^{n+1} C_{r+1}}{n+1} + \frac{1}{n+1}$$

$$= \frac{1}{n+1} (1 + {}^{n+1} C_1 + \dots + {}^{n+1} C_{n+1})$$

$$= \frac{2^{n+1}}{n+1}$$

$$\frac{2\alpha}{\beta} = \frac{2^{n+1}(n+1)^2}{2^{n+1}} \cdot (n+1) = (n+1)^3$$

$$140 < (n+1)^3 < 281$$

$$n = 4 \Rightarrow (n+1)^3 = 125$$

$$n = 5 \Rightarrow (n+1)^3 = 216$$

$$n = 6 \Rightarrow (n+1)^3 = 343$$

$$\therefore n = 5$$

28. Let  $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$ ,  $\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$  and  $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$  be three given vectors. If  $\vec{r}$  is a vector such that  $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$  and  $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$ , then  $\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$  is equal to \_\_\_\_\_.

**Ans. (569)**

**Sol.**  $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$

$\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$

$\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$

$\vec{b} + \vec{c} = 20\hat{i} + 5\hat{j} - 12\hat{k}$

$\vec{b} - \vec{c} = -14\hat{i} + 9\hat{j} - 14\hat{k}$

$(\vec{r} - (\vec{b} + \vec{c})) \times \vec{a} = 0$

$\vec{r} - (\vec{b} + \vec{c}) = \lambda \vec{a}$

$\vec{r} = \lambda \vec{a} + \vec{b} + \vec{c}$

But  $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow (\lambda \vec{a} + \vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow \lambda \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b} - \lambda \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} - \vec{c} \cdot \vec{c} = 0$

$\lambda = \frac{\vec{c} \cdot \vec{c} - \vec{b} \cdot \vec{b}}{\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}} = \frac{294 - 227}{-389 - 204} = \frac{-67}{593}$

$\therefore \vec{r} = \vec{b} + \vec{c} - \frac{67}{593} \vec{a}$

$\Rightarrow 593\vec{r} + 67\vec{a} = 593(\vec{b} + \vec{c})$

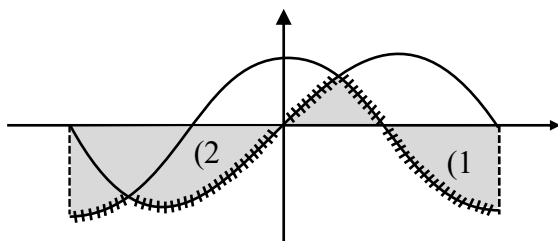
$\Rightarrow |\vec{b} + \vec{c}|^2 = 569$

29. Let the area of the region enclosed by the curve  $y = \min\{\sin x, \cos x\}$  and the x-axis between  $x = -\pi$  to  $x = \pi$  be A. Then  $A^2$  is equal to \_\_\_\_\_.

**Ans. (16)**

**Sol.**  $y = \min\{\sin x, \cos x\}$

x-axis      x = -\pi      x = \pi



$$\int_0^{\pi/4} \sin x = (\cos x)_{\pi/4}^0 = 1 - \frac{1}{\sqrt{2}}$$

$$\int_{-\pi}^{-3\pi/4} (\sin x - \cos x) = (-\cos x - \sin x)_{-\pi}^{-3\pi/4} = (\cos x + \sin x)_{-3\pi/4}^{-\pi}$$

$$= (-1 + 0) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= -1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$\int_{\pi/4}^{\pi/2} \cos x dx = (\sin x)_{\pi/4}^{\pi/2} = 1 - \frac{1}{\sqrt{2}}$$

$A = 4$

$A^2 = 16$

30. The value of

$$\lim_{x \rightarrow 0} 2 \left( \frac{1 - \cos x \sqrt{\cos 2x} \sqrt[3]{\cos 3x} \dots \sqrt[10]{\cos 10x}}{x^2} \right) \text{ is}$$

\_\_\_\_\_.

**Ans. (55)**

**Sol.**

$$\lim_{x \rightarrow 0} 2 \left( \frac{\left(1 - \left(1 - \frac{x^2}{2!}\right)\right) \left(1 - \frac{4x^2}{2!}\right) \left(1 - \frac{9x^2}{2!}\right) \dots \left(1 - \frac{100x^2}{2!}\right)}{x^2} \right)$$

By expansion

$$\lim_{x \rightarrow 0} \frac{2 \left( 1 - \left(1 - \frac{x^2}{2}\right) \left(1 - \frac{1}{2} \cdot \frac{4x^2}{2}\right) \left(1 - \frac{1}{3} \cdot \frac{9x^2}{2}\right) \dots \left(1 - \frac{1}{10} \cdot \frac{100x^2}{2}\right) \right)}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{2 \left( 1 - \left(1 - \frac{x^2}{2}\right) \left(1 - \frac{2x^2}{2}\right) \left(1 - \frac{3x^2}{2}\right) \dots \left(1 - \frac{10x^2}{2}\right) \right)}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{2 \left( 1 - 1 + x^2 \left( \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \dots + \frac{10}{2} \right) \right)}{x^2}$$

$$2 \left( \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \dots + \frac{10}{2} \right)$$

$$1 + 2 + \dots + 10 = \frac{10 \times 11}{2} = 55$$

## PHYSICS

## TEST PAPER WITH SOLUTION

### SECTION-A

- 31.** Three bodies A, B and C have equal kinetic energies and their masses are 400 g, 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is :

- (1)  $1:\sqrt{3}:2$                       (2)  $1:\sqrt{3}:\sqrt{2}$   
 (3)  $\sqrt{2}:\sqrt{3}:1$                       (4)  $\sqrt{3}:\sqrt{2}:1$

**Ans. (1)**

**Sol.**  $KE = \frac{P^2}{2m}$

$P \propto \sqrt{m}$

Hence,  $P_A : P_B : P_C$

$= \sqrt{400} : \sqrt{1200} : \sqrt{1600} = 1 : \sqrt{3} : 2$

- 32.** Average force exerted on a non-reflecting surface at normal incidence is  $2.4 \times 10^{-4} \text{ N}$ . If  $360 \text{ W/cm}^2$  is the light energy flux during span of 1 hour 30 minutes. Then the area of the surface is:

- (1)  $0.2 \text{ m}^2$                       (2)  $0.02 \text{ m}^2$   
 (3)  $20 \text{ m}^2$                       (4)  $0.1 \text{ m}^2$

**Ans. (2)**

**Sol.** Pressure =  $\frac{I}{C} = \frac{F}{A}$

$\Rightarrow \frac{360}{10^{-4} \times 3 \times 10^8} = \frac{2.4 \times 10^{-4}}{A}$

$\Rightarrow A = 2 \times 10^{-2} \text{ m}^2 = 0.02 \text{ m}^2$

- 33.** A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is:

(Assume  $h = 6.63 \times 10^{-34} \text{ J s}$ ,  $m_e = 9.0 \times 10^{-31} \text{ kg}$  and  $m_p = 1836 \text{ times } m_e$ )

- (1)  $1 : 1836$                       (2)  $1 : \frac{1}{1836}$   
 (3)  $1 : \frac{1}{\sqrt{1836}}$                       (4)  $1 : \sqrt{1836}$

**Ans. (1)**

**Sol.**  $\lambda$  is same for both

$P = \frac{h}{\lambda}$  same for both

$P = \sqrt{2mK}$

Hence,

$K \propto \frac{1}{m}$

$\Rightarrow \frac{KE_p}{KE_e} = \frac{m_e}{m_p} = \frac{1}{1836}$

- 34.** A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature ( $27^\circ\text{C}$ ). The ratio of specific heat of gases at constant volume respectively is:

- (1)  $\frac{7}{5}$                                       (2)  $\frac{3}{2}$   
 (3)  $\frac{3}{5}$                                       (4)  $\frac{5}{3}$

**Ans. (3)**

**Sol.**  $\frac{(C_v)_{\text{mono}}}{(C_v)_{\text{dia}}} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$

- 35.** In an expression  $a \times 10^b$  :

- (1) a is order of magnitude for  $b \leq 5$   
 (2) b is order of magnitude for  $a \leq 5$   
 (3) b is order of magnitude for  $5 < a \leq 10$   
 (4) b is order of magnitude for  $a \geq 5$

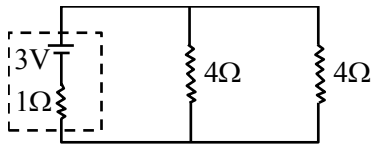
**Ans. (2)**

**Sol.**  $a \times 10^b$

if  $a \leq 5$  order is b

$a > 5$  order is  $b + 1$

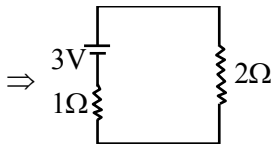
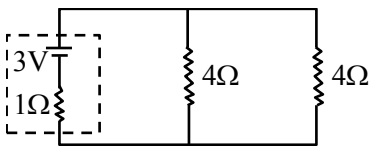
36. In the given circuit, the terminal potential difference of the cell is :



- (1) 2 V                      (2) 4 V  
(3) 1.5 V                    (4) 3 V

**Ans. (1)**

**Sol.**



$$i = \frac{3}{1+2} = 1\text{A}$$

$$v = E - ir$$

$$= 3 - 1 \times 1 = 2\text{V}$$

37. Binding energy of a certain nucleus is  $18 \times 10^8$  J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus:

- (1) 0.2  $\mu\text{g}$                       (2) 20  $\mu\text{g}$   
(3) 2  $\mu\text{g}$                       (4) 10  $\mu\text{g}$

**Ans. (2)**

**Sol.**  $\Delta mc^2 = 18 \times 10^8$

$$\Delta m \times 9 \times 10^{16} = 18 \times 10^8$$

$$\Delta m = 2 \times 10^{-8} \text{ kg} = 20 \mu\text{g}$$

38. Paramagnetic substances:

- A. align themselves along the directions of external magnetic field.  
B. attract strongly towards external magnetic field.  
C. has susceptibility little more than zero.  
D. move from a region of strong magnetic field to weak magnetic field.

Choose the **most appropriate** answer from the options given below:

- (1) A, B, C, D                      (2) B, D Only  
(3) A, B, C Only                    (4) A, C Only

**Ans. (4)**

**Sol.** A, C only

39. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take  $\pi = 3.14$ ) :

- (1) 139.4                      (2) 140.5  
(3) 220.0                      (4) 118.9

**Ans. (1)**

**Sol.**  $x_{\min} = \pi \times r_{\min}$

$$= \pi \times \frac{60}{100} \text{ m.}$$

$$x_{\text{second}} = 30 \times 2\pi \times r_{\text{second}}$$

$$= 30 \times 2\pi \times \frac{75}{100}$$

$$x = x_{\text{second}} - x_{\min}$$

$$= 139.4 \text{ m}$$

40. Young's modulus is determined by the equation given by  $Y = 49000 \frac{\text{m dyne}}{\ell \text{ cm}^2}$  where M is the mass

and  $\ell$  is the extension of wire used in the experiment. Now error in Young modules(Y) is estimated by taking data from M- $\ell$  plot in graph paper. The smallest scale divisions are 5 g and 0.02 cm along load axis and extension axis respectively. If the value of M and  $\ell$  are 500 g and 2 cm respectively then percentage error of Y is :

- (1) 0.2 %                      (2) 0.02 %  
(3) 2 %                      (4) 0.5 %

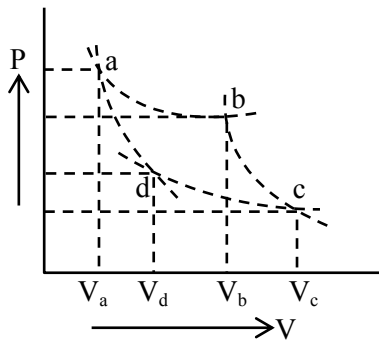
**Ans. (3)**

**Sol.**  $\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta \ell}{\ell}$   
 $= \frac{5}{500} + \frac{0.02}{2} = 0.01 + 0.01$

$\frac{\Delta Y}{Y} = 0.02 \Rightarrow \% \frac{\Delta Y}{Y} = 2\%$

- 41.** Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio  $\frac{V_a}{V_d}$  and the

ratio  $\frac{V_b}{V_c}$  is:



- (1)  $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$  (2)  $\frac{V_a}{V_d} \neq \frac{V_b}{V_c}$   
 (3)  $\frac{V_a}{V_d} = \frac{V_b}{V_c}$  (4)  $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^2$

**Ans. (3)**

**Sol.** For adiabatic process

$TV^{\gamma-1} = \text{constant}$

$T_a \cdot V_a^{\gamma-1} = T_d \cdot V_d^{\gamma-1}$

$\left(\frac{V_a}{V_d}\right)^{\gamma-1} = \frac{T_d}{T_a}$

$T_b \cdot V_b^{\gamma-1} = T_c \cdot V_c^{\gamma-1}$

$\left(\frac{V_b}{V_c}\right)^{\gamma-1} = \frac{T_c}{T_b}$

$\frac{V_a}{V_d} = \frac{V_b}{V_c} \quad \left( \because T_d = T_c \right)$   
 $\quad \quad \quad \left( T_a = T_b \right)$

- 42.** Two planets A and B having masses  $m_1$  and  $m_2$  move around the sun in circular orbits of  $r_1$  and  $r_2$  radii respectively. If angular momentum of A is  $L$  and that of B is  $3L$ , the ratio of time period  $\left(\frac{T_A}{T_B}\right)$  is:

- (1)  $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$  (2)  $\left(\frac{r_1}{r_2}\right)^3$   
 (3)  $\frac{1}{27} \left(\frac{m_2}{m_1}\right)^3$  (4)  $27 \left(\frac{m_1}{m_2}\right)^3$

**Ans. (3)**

**Sol.**  $\frac{\pi r_1^2}{T_A} = \frac{L}{2m_1} \dots\dots (1)$

$\frac{\pi r_2^2}{T_B} = \frac{3L}{2m_2} \dots\dots (2)$

$\Rightarrow \frac{T_A}{T_B} = 3 \cdot \frac{m_1}{m_2} \cdot \left(\frac{r_1}{r_2}\right)^2$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{T_A}{T_B}\right)^{\frac{4}{3}}$

$\Rightarrow \frac{1}{27} \cdot \left(\frac{m_2}{m_1}\right)^3 = \left(\frac{T_A}{T_B}\right)$

- 43.** A LCR circuit is at resonance for a capacitor  $C$ , inductance  $L$  and resistance  $R$ . Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now:

- (1) Zero (2) double  
 (3) same (4) halved

**Ans. (2)**

**Sol.** In resonance  $Z = R$

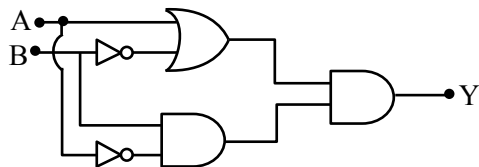
$I = \frac{V}{R}$

$R \rightarrow \text{halved}$

$\Rightarrow I \rightarrow 2I$

$I$  becomes doubled.

44. The output Y of following circuit for given inputs is :



- (1)  $A \cdot B(A + B)$  (2)  $A \cdot B$   
 (3) 0 (4)  $\bar{A} \cdot B$

**Ans. (3)**

**Sol.** By truth table

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

45. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is:

- (1)  $\sqrt{ab}$  (2) ab  
 (3)  $\frac{a}{b}$  (4)  $\frac{b}{a}$

**Ans. (3)**

**Sol.** Potential at surface will be same

$$\frac{Kq_1}{a} = \frac{Kq_2}{b}$$

$$\frac{q_1}{q_2} = \frac{a}{b}$$

46. Correct Bernoulli's equation is (symbols have their usual meaning) :

- (1)  $P + mgh + \frac{1}{2}mv^2 = \text{constant}$   
 (2)  $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$   
 (3)  $P + \rho gh + \rho v^2 = \text{constant}$   
 (4)  $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

**Ans. (2)**

**Sol.**  $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

47. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is:

- (1) 150 N (2) 3 N  
 (3) 30 N (4) 300 N

**Ans. (3)**

**Sol.**  $F = \frac{\Delta P}{\Delta t} = \frac{mv - 0}{0.1}$   
 $= \frac{150 \times 10^{-3} \times 20}{0.1} = 30 \text{ N}$

48. A stationary particle breaks into two parts of masses  $m_A$  and  $m_B$  which move with velocities  $v_A$  and  $v_B$  respectively. The ratio of their kinetic energies ( $K_B : K_A$ ) is :

- (1)  $v_B : v_A$  (2)  $m_B : m_A$   
 (3)  $m_B v_B : m_A v_A$  (4) 1 : 1

**Ans. (1)**

**Sol.** Initial momentum is zero.

Hence  $|P_A| = |P_B|$

$\Rightarrow m_A v_A = m_B v_B$

$$\frac{(KE)_A}{(KE)_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{v_A}{v_B}$$

$$\frac{(KE)_B}{(KE)_A} = \frac{v_B}{v_A}$$

49. Critical angle of incidence for a pair of optical media is  $45^\circ$ . The refractive indices of first and second media are in the ratio:

- (1)  $\sqrt{2} : 1$  (2) 1 : 2  
 (3)  $1 : \sqrt{2}$  (4) 2 : 1

**Ans. (1)**

**Sol.**  $\sin \theta_c = \frac{\mu_R}{\mu_d} = \frac{\mu_2}{\mu_1}$

$$\sin 45^\circ = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}}{1}$$

- 50.** The diameter of a sphere is measured using a vernier caliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is 2 cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is:

(1) 2.5 g/cm<sup>3</sup>                      (2) 1.7 g/cm<sup>3</sup>

(3) 2.2 g/cm<sup>3</sup>                      (4) 2.0 g/cm<sup>3</sup>

**Ans. (4)**

**Sol.** Given 9MSD = 10VSD

$$\text{mass} = 8.635 \text{ g}$$

$$\text{LC} = 1 \text{ MSD} - 1 \text{ VSD}$$

$$\text{LC} = 1 \text{ MSD} - \frac{9}{10} \text{ MSD}$$

$$\text{LC} = \frac{1}{10} \text{ MSD}$$

$$\text{LC} = 0.01 \text{ cm}$$

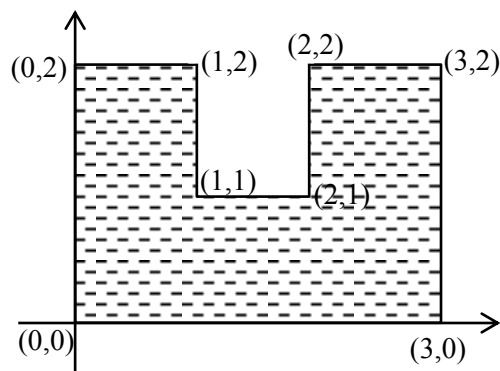
$$\begin{aligned} \text{Reading of diameter} &= \text{MSR} + \text{LC} \times \text{VSR} \\ &= 2 \text{ cm} + (0.01) \times (2) \\ &= 2.02 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Volume of sphere} &= \frac{4}{3} \pi \left( \frac{d}{2} \right)^3 = \frac{4}{3} \pi \left( \frac{2.02}{2} \right)^3 \\ &= 4.32 \text{ cm}^3 \end{aligned}$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{8.635}{4.32} = 1.998 \sim 2.00 \text{ g}$$

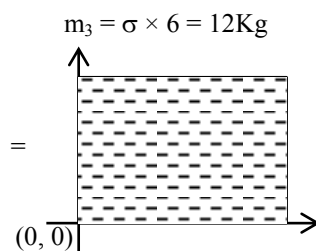
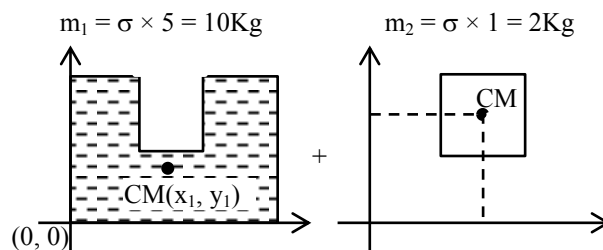
## SECTION-B

- 51.** A uniform thin metal plate of mass 10 kg with dimensions is shown. The ratio of x and y coordinates of center of mass of plate in  $\frac{n}{9}$ . The value of n is \_\_\_\_\_.



**Ans. (15)**

**Sol.**  $m_1 = \sigma \times 5 = 10 \text{ Kg}$



$$\Rightarrow m_1 x_1 + m_2 x_2 = m_3 x_3$$

$$10x_1 + 2(1.5) = 12(1.5) \Rightarrow x_1 = 1.5 \text{ cm}$$

$$\Rightarrow m_1 y_1 + m_2 y_2 = m_3 y_3$$

$$10y_1 + 2(1.5) = 12 \times 1 \Rightarrow y_1 = 0.9 \text{ cm}$$

$$\frac{x_1}{y_1} = \frac{1.5}{0.9} = \frac{15}{9}$$

$$n = 15$$

52. An electron with kinetic energy 5 eV enters a region of uniform magnetic field of 3  $\mu\text{T}$  perpendicular to its direction. An electric field  $E$  is applied perpendicular to the direction of velocity and magnetic field. The value of  $E$ , so that electron moves along the same path, is \_\_\_\_\_  $\text{NC}^{-1}$ .

(Given, mass of electron =  $9 \times 10^{-31}$  kg, electric charge =  $1.6 \times 10^{-19}$  C)

**Ans. (4)**

**Sol.** For the given condition of moving undeflected, net force should be zero.

$$qE = qvB$$

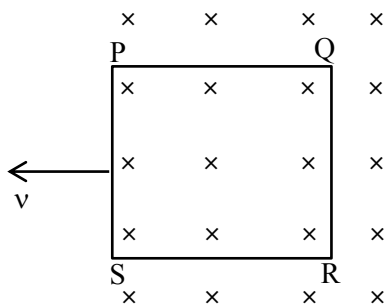
$$E = vB$$

$$= \sqrt{\frac{2 \times KE}{m}} \times B$$

$$= \sqrt{\frac{2 \times 5 \times 1.6 \times 10^{-19}}{9 \times 10^{-31}}} \times 3 \times 10^{-6}$$

$$= 4 \text{ N/C}$$

53. A square loop PQRS having 10 turns, area  $3.6 \times 10^{-3} \text{ m}^2$  and resistance 100  $\Omega$  is slowly and uniformly being pulled out of a uniform magnetic field of magnitude  $B = 0.5 \text{ T}$  as shown. Work done in pulling the loop out of the field in 1.0 s is \_\_\_\_\_  $\times 10^{-6} \text{ J}$ .



**Ans. (3)**

**Sol.**  $\epsilon = NB\ell v$

$$i = \frac{\epsilon}{R} = \frac{NB\ell v}{R}$$

$$F = N(i\ell B) = \frac{N^2 B^2 \ell^2 v}{R}$$

$$W = F \times \ell = \frac{N^2 B^2 \ell^3}{R} \left( \frac{\ell}{t} \right)$$

$$A = \ell^2$$

$$W = \frac{(10 \times 10)(0.5)^2 \times (3.6 \times 10^{-3})^2}{100 \times 1}$$

$$W = 3.24 \times 10^{-6} \text{ J}$$

54. Resistance of a wire at 0  $^{\circ}\text{C}$ , 100  $^{\circ}\text{C}$  and  $t$   $^{\circ}\text{C}$  is found to be 10  $\Omega$ , 10.2  $\Omega$  and 10.95  $\Omega$  respectively. The temperature  $t$  in Kelvin scale is \_\_\_\_\_.

**Ans. (748)**

**Sol.**  $R = R_0(1 + \alpha\Delta T)$

$$\frac{\Delta R}{R_0} = \alpha\Delta T$$

**Case-I**

$$0^{\circ}\text{C} \rightarrow 100^{\circ}\text{C}$$

$$\frac{10.2 - 10}{10} = \alpha(100 - 0) \quad \dots (1)$$

**Case-II**

$$0^{\circ}\text{C} \rightarrow t^{\circ}\text{C}$$

$$\frac{10.95 - 10}{10} = \alpha(t - 0) \quad \dots (2)$$

$$\Rightarrow \frac{t}{100} = \frac{0.95}{0.2} = 475^{\circ}\text{C}$$

$$t = 475 + 273 = 748 \text{ K}$$

55. An electric field,  $\vec{E} = \frac{2\hat{i} + 6\hat{j} + 8\hat{k}}{\sqrt{6}}$  passes through the surface of 4  $\text{m}^2$  area having unit vector  $\hat{n} = \left( \frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}} \right)$ . The electric flux for that surface is \_\_\_\_\_ V m.

**Ans. (12)**

**Sol.**  $\phi = \vec{E} \cdot \vec{A}$

$$= \left( \frac{2\hat{i} + 6\hat{j} + 8\hat{k}}{\sqrt{6}} \right) \cdot 4 \left( \frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}} \right)$$

$$= \frac{4}{6} \times (4 + 6 + 8) = 12 \text{ Vm}$$

56. A liquid column of height 0.04 cm balances excess pressure of soap bubble of certain radius. If density of liquid is  $8 \times 10^3 \text{ kg m}^{-3}$  and surface tension of soap solution is  $0.28 \text{ Nm}^{-1}$ , then diameter of the soap bubble is \_\_\_\_\_ cm.

(if  $g = 10 \text{ ms}^{-2}$ )

Ans. (7)

Sol.  $\rho gh = \frac{4S}{R}$

$$\Rightarrow R = \frac{4 \times 0.28}{8 \times 10^3 \times 10 \times 4 \times 10^{-4}}$$

$$\Rightarrow \frac{0.28}{8} \text{ m} = \frac{28}{8} \text{ cm}$$

$$\Rightarrow R = 3.5 \text{ cm}$$

Diameter = 7 cm

57. A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is  $\left(\frac{a-1}{a}\right)$  then the value of a is \_\_\_\_\_.

Ans. (16)

Sol. For closed organ pipe

$$f_c = (2n+1) \frac{v}{4\ell} = \frac{15v}{4\ell}$$

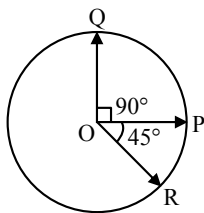
For open organ pipe

$$f_o = (n+1) \frac{v}{2\ell} = \frac{8v}{2\ell}$$

$$\frac{f_c}{f_o} = \frac{15}{16} = \frac{a-1}{a}$$

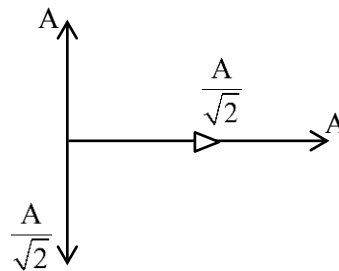
$$\Rightarrow a = 16$$

58. Three vectors  $\vec{OP}$ ,  $\vec{OQ}$  and  $\vec{OR}$  each of magnitude A are acting as shown in figure. The resultant of the three vectors is  $A\sqrt{x}$ . The value of x is \_\_\_\_\_.



Ans. (3)

Sol.



$$\vec{R} = \left(A + \frac{A}{\sqrt{2}}\right)\hat{i} + \left(A - \frac{A}{\sqrt{2}}\right)\hat{j}$$

$$|\vec{R}| = \sqrt{\left(A + \frac{A}{\sqrt{2}}\right)^2 + \left(A - \frac{A}{\sqrt{2}}\right)^2} = \sqrt{3}A$$

59. A parallel beam of monochromatic light of wavelength 600 nm passes through single slit of 0.4 mm width. Angular divergence corresponding to second order minima would be \_\_\_\_\_  $\times 10^{-3}$  rad.

Ans. (6)

Sol.  $\sin \theta \approx \theta = \frac{2\lambda}{b}$

$$= \frac{2 \times 600 \times 10^{-9}}{4 \times 10^{-4}} = 3 \times 10^{-3} \text{ rad}$$

$$\text{Total divergence} = (3 + 3) \times 10^{-3} = 6 \times 10^{-3} \text{ rad}$$

60. In an alpha particle scattering experiment distance of closest approach for the  $\alpha$  particle is  $4.5 \times 10^{-14} \text{ m}$ . If target nucleus has atomic number 80, then maximum velocity of  $\alpha$ -particle is \_\_\_\_\_  $\times 10^5 \text{ m/s}$  approximately.

$$\left(\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ SI unit, mass of } \alpha \text{ particle} = 6.72 \times 10^{-27} \text{ kg}\right)$$

Ans. (156)

Sol.  $v = \sqrt{\frac{4KZe^2}{mr_{\min}}}$

$$= \sqrt{\frac{4 \times 9 \times 10^9 \times 80}{6.72 \times 10^{-27} \times 4.5 \times 10^{-14}}} \times 1.6 \times 10^{-19}$$

$$= 9.759 \times 10^{25} \times 1.6 \times 10^{-19}$$

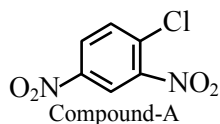
$$= 156 \times 10^5 \text{ m/s}$$

## CHEMISTRY

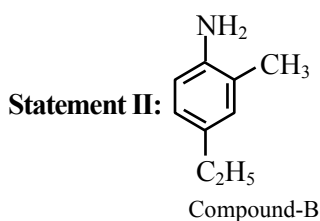
### SECTION-A

61. Given below are two statements:

**Statement I :**



IUPAC name of Compound A is 4-chloro-1,3-dinitrobenzene:



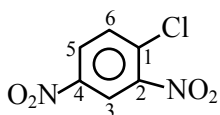
IUPAC name of Compound B is 4-ethyl-2-methylaniline.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) Both Statement I and Statement II are correct
- (2) Statement I is incorrect but Statement II is correct
- (3) Statement I is correct but Statement II is incorrect
- (4) Both Statement I and Statement II are incorrect

**Ans. (2)**

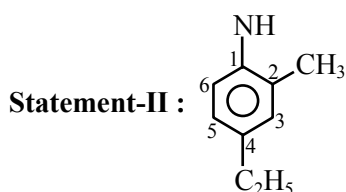
**Sol. Statement I :**



IUPAC name

⇒ 1-chloro-2,4-dinitrobenzene

⇒ statement-I is incorrect



⇒ 4-ethyl-2-methylaniline

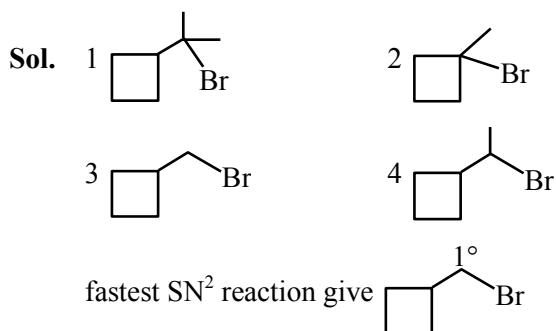
⇒ statement-II is correct

## TEST PAPER WITH SOLUTION

62. Which among the following compounds will undergo fastest  $S_N2$  reaction.

- (1)
- (2)
- (3)
- (4)

**Ans. (3)**



Rate of  $S_N2$  is  $Me - x > 1^\circ - x > 2^\circ - x > 3^\circ - x$

63. Combustion of glucose ( $C_6H_{12}O_6$ ) produces  $CO_2$  and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is: [Molar mass of glucose in  $g\ mol^{-1} = 180$ ]

- (1) 480
- (2) 960
- (3) 800
- (4) 32

**Ans. (2)**

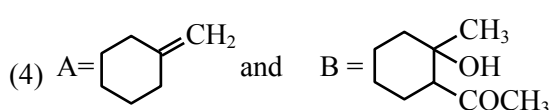
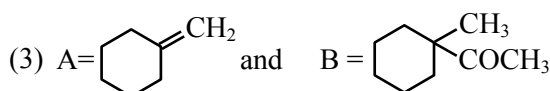
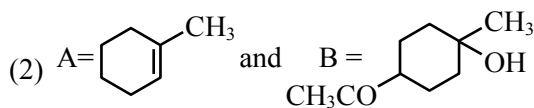
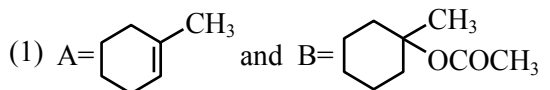
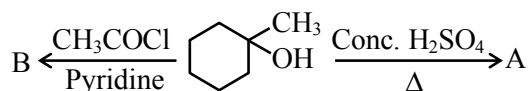
**Sol.**  $C_6H_{12}O_{6(s)} + 6O_{2(g)} \longrightarrow 6CO_{2(g)} + 6H_2O_{(l)}$

$$\frac{900}{180}$$

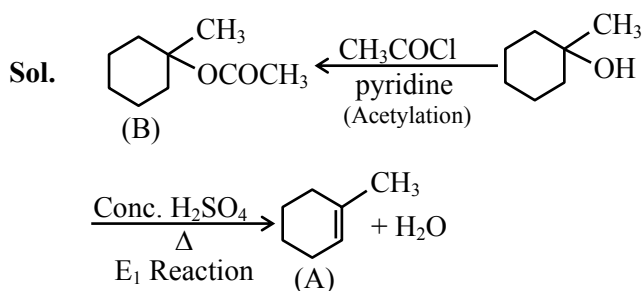
$$= 5\ mol \quad 30\ mol$$

$$\text{Mass of } O_2 \text{ required} = 30 \times 32 = 960\ gm$$

64. Identify the major products A and B respectively in the following set of reactions.



Ans. (1)



65. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**:

**Assertion A** : The stability order of +1 oxidation state of Ga, In and Tl is  $\text{Ga} < \text{In} < \text{Tl}$ .

**Reason R** : The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the *correct* answer from the options given below :

- (1) Both **A** and **R** are true and **R** is the correct explanation of **A**.
- (2) **A** is true but **R** is false.
- (3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.
- (4) **A** is false but **R** is true.

Ans. (1)

Sol. The relative stability of +1 oxidation state progressively increases for heavier elements due to inert pair effect.

$$\therefore \text{Stability of } A\ell^{+1} < \text{Ga}^{+1} < \text{In}^{+1} < \text{Tl}^{+1}$$

66. Match List I with List-II

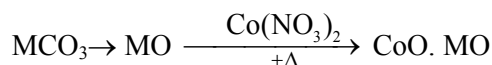
| List-I<br>(Name of the test) |                      | List-II<br>(Reaction sequence involved)<br>[M is metal] |                                                                                                                                          |
|------------------------------|----------------------|---------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------|
| A                            | Borax bead test      | I.                                                      | $\text{MCO}_3 \rightarrow \text{MO}$<br>$\xrightarrow{+\Delta} \text{Co(NO}_3)_2 \rightarrow \text{CoO} \cdot \text{MO}$                 |
| B.                           | Charcoal cavity test | II.                                                     | $\text{MCO}_3 \rightarrow \text{MCl}_2 \rightarrow \text{M}^{2+}$                                                                        |
| C.                           | Cobalt nitrate test  | III                                                     | $\text{MSO}_4 \xrightarrow[\Delta]{\text{Na}_2\text{B}_4\text{O}_7}$<br>$\text{M(BO}_2)_2 \rightarrow \text{MBO}_2 \rightarrow \text{M}$ |
| D.                           | Flame test           | IV                                                      | $\text{MSO}_4 \xrightarrow[\Delta]{\text{Na}_2\text{CO}_3} \text{MCO}_3 \rightarrow$<br>$\text{MO} \rightarrow \text{M}$                 |

Choose the **correct** answer from the option below :

- (1) A-III, B-I, C-IV, D-II
- (2) A-III, B-II, C-IV, D-I
- (3) A-III, B-I, C-II, D-IV
- (4) A-III, B-IV, C-I, D-II

Ans. (4)

Sol. **Cobalt nitrate test**



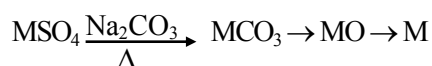
**Flame test**



**Borax Bead test**



**Charcoal cavity test**



67. Match **List I** and with **List II**

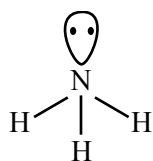
| List-I (Molecule) |                  | List-II(Shape) |                      |
|-------------------|------------------|----------------|----------------------|
| A                 | NH <sub>3</sub>  | I.             | Square pyramid       |
| B.                | BrF <sub>5</sub> | II.            | Tetrahedral          |
| C.                | PCl <sub>5</sub> | III            | Trigonal pyramidal   |
| D.                | CH <sub>4</sub>  | IV             | Trigonal bipyramidal |

Choose the **correct** answer from the option below :

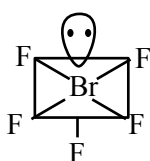
- (1) A-IV, B-III, C-I, D-II  
 (2) A-II, B-IV, C-I, D-III  
 (3) A-III, B-I, C-IV, D-II  
 (4) A-III, B-IV, C-I, D-II

**Ans. (3)**

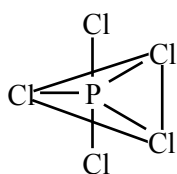
**Sol.**



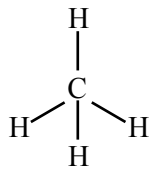
Trigonal pyramidal



Square pyramidal

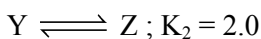


Trigonal bipyramidal

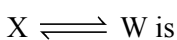


Tetrahedral

68. For the given hypothetical reactions, the equilibrium constants are as follows:

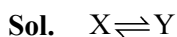


The equilibrium constant for the reaction



- (1) 6.0                                      (2) 12.0  
 (3) 8.0                                      (4) 7.0

**Ans. (3)**



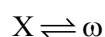
$$K_1 = 1$$



$$K_2 = 2$$



$$K_3 = 4$$

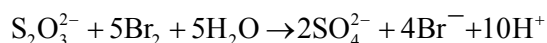
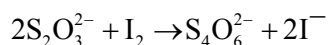


$$K_1 \cdot K_2 \cdot K_3$$

$$K = 1 \times 2 \times 4$$

$$K = 8$$

69. Thiosulphate reacts differently with iodine and bromine in the reaction given below :



Which of the following statement justifies the above dual behaviour of thiosulphate?

- (1) Bromine undergoes oxidation and iodine undergoes reduction by iodine in these reactions  
 (2) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reaction  
 (3) Bromine is a stronger oxidant than iodine  
 (4) Bromine is a weaker oxidant than iodine

**Ans. (3)**

**Sol.** In the reaction of  $S_2O_3^{2-}$  with  $I_2$ , oxidation state of sulphur changes to +2 to +2.5

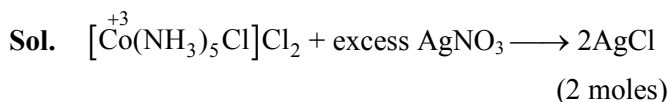
In the reaction of  $S_2O_3^{2-}$  with  $Br_2$ , oxidation state of sulphur changes from +2 to +6.

$\therefore$  Both  $I_2$  and  $Br_2$  are oxidant (oxidising agent) and  $Br_2$  is stronger oxidant than  $I_2$ .

70. An octahedral complex with the formula  $CoCl_3nNH_3$  upon reaction with excess of  $AgNO_3$  solution given 2 moles of  $AgCl$ . Consider the oxidation state of Co in the complex is 'x'. The value of "x + n" is \_\_\_\_\_.

- (1) 3                                      (2) 6  
 (3) 8                                      (4) 5

**Ans. (3)**

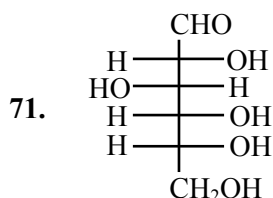


$$x + 0 - 1 - 2 = 0$$

$$x = +3$$

$$n = 5$$

$$\therefore x + n = 8$$

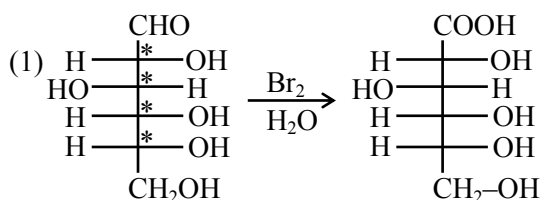


The **incorrect** statement regarding the given structure is

- (1) Can be oxidized to a dicarboxylic acid with  $\text{Br}_2$  water
- (2) despite the presence of  $-\text{CHO}$  does not give Schiff's test
- (3) has 4-asymmetric carbon atom
- (4) will coexist in equilibrium with 2 other cyclic structure

**Ans. (1)**

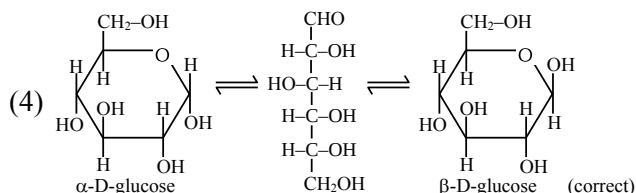
**Sol.**



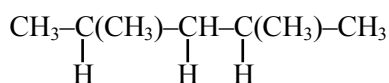
statement 1 is incorrect (monocarboxylic acid)

(2) correct

(3) c.c. is 4 (correct)

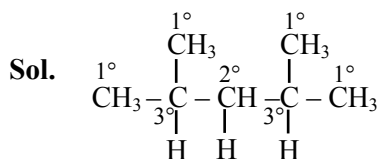


72. In the given compound, the number of  $2^\circ$  carbon atom/s is \_\_\_\_.



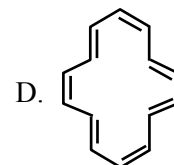
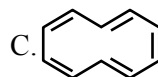
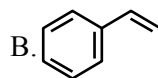
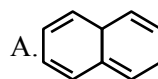
- (1) Three
- (2) One
- (3) Two
- (4) Four

**Ans. (2)**



only one  $2^\circ$  carbon is present in this compound.

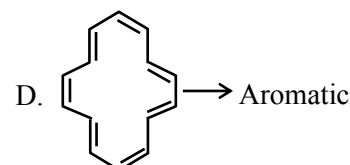
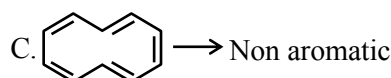
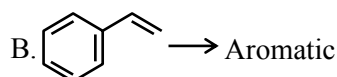
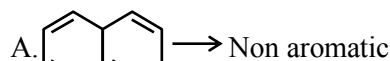
73. Which of the following are aromatic?



- (1) B and D only
- (2) A and C only
- (3) A and B only
- (4) C and D only

**Ans. (1)**

**Sol.**



74. Among the following halogens

$\text{F}_2$ ,  $\text{Cl}_2$ ,  $\text{Br}_2$  and  $\text{I}_2$

Which can undergo disproportionation reaction?

- (1) Only  $\text{I}_2$
- (2)  $\text{Cl}_2$ ,  $\text{Br}_2$  and  $\text{I}_2$
- (3)  $\text{F}_2$ ,  $\text{Cl}_2$  and  $\text{Br}_2$
- (4)  $\text{F}_2$  and  $\text{Cl}_2$

**Ans. (2)**

**Sol.**  $\text{F}_2$  do not disproportionate because fluorine do not exist in positive oxidation state however  $\text{Cl}_2$ ,  $\text{Br}_2$  &  $\text{I}_2$  undergoes disproportionation.

75. Given below are two statements:

**Statement I :**  $\text{N}(\text{CH}_3)_3$  and  $\text{P}(\text{CH}_3)_3$  can act as ligands to form transition metal complexes.

**Statement II:** As N and P are from same group, the nature of bonding of  $\text{N}(\text{CH}_3)_3$  and  $\text{P}(\text{CH}_3)_3$  is always same with transition metals.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) Statement I is incorrect but Statement II is correct
- (2) Both Statement I and Statement II are correct
- (3) Statement I is correct but Statement II is incorrect
- (4) Both Statement I and Statement II are incorrect

**Ans. (3)**

**Sol.**  $\text{N}(\text{CH}_3)_3$  and  $\text{P}(\text{CH}_3)_3$  both are Lewis base and acts as ligand, However,  $\text{P}(\text{CH}_3)_3$  has a  $\pi$ -acceptor character.

76. Match **List I** with **List II**

| List-I (Elements) |        | List-II (Properties in their respective groups) |                                                             |
|-------------------|--------|-------------------------------------------------|-------------------------------------------------------------|
| A                 | Cl, S  | I.                                              | Elements with highest electronegativity                     |
| B.                | Ge, As | II.                                             | Elements with largest atomic size                           |
| C.                | Fr, Ra | III                                             | Elements which show properties of both metals and non metal |
| D.                | F, O   | IV                                              | Elements with highest negative electron gain enthalpy       |

Choose the **correct** answer from the options given below :

- (1) A-II, B-III, C-IV, D-I
- (2) A-III, B-II, C-I, D-IV
- (3) A-IV, B-III, C-II, D-I
- (4) A-II, B-I, C-IV, D-III

**Ans. (3)**

**Sol.** Elements with highest electronegativity  $\rightarrow$  F, O

Elements with largest atomic size  $\rightarrow$  Fr, Ra

Elements which shows properties of both metal and non-metals i.e. metalloids  $\rightarrow$  Ge, As

Elements with highest negative electron gain enthalpy  $\rightarrow$  Cl, S

77. Iron (III) catalyses the reaction between iodide and persulphate ions, in which

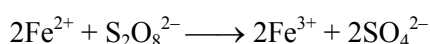
- A.  $\text{Fe}^{3+}$  oxidises the iodide ion
- B.  $\text{Fe}^{3+}$  oxidises the persulphate ion
- C.  $\text{Fe}^{2+}$  reduces the iodide ion
- D.  $\text{Fe}^{2+}$  reduces the persulphate ion

Choose the **most appropriate** answer from the options given below:

- (1) B and C only
- (2) B only
- (3) A only
- (4) A and D only

**Ans. (4)**

**Sol.**  $2\text{Fe}^{3+} + 2\text{I}^- \longrightarrow 2\text{Fe}^{2+} + \text{I}_2$



$\text{Fe}^{+3}$  oxidises  $\text{I}^-$  to  $\text{I}_2$  and convert itself into  $\text{Fe}^{+2}$ . This  $\text{Fe}^{+2}$  reduces  $\text{S}_2\text{O}_8^{2-}$  to  $\text{SO}_4^{2-}$  and converts itself into  $\text{Fe}^{+3}$ .

78. Match **List I** with **List II**

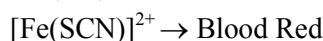
| List-I (Compound) |                                                                   | List-II (Colour) |               |
|-------------------|-------------------------------------------------------------------|------------------|---------------|
| A                 | $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3 \cdot x\text{H}_2\text{O}$ | I.               | Violet        |
| B.                | $[\text{Fe}(\text{CN})_5\text{NOS}]^{4-}$                         | II.              | Blood Red     |
| C.                | $[\text{Fe}(\text{SCN})]^{2+}$                                    | III.             | Prussian Blue |
| D.                | $(\text{NH}_4)_3\text{PO}_4 \cdot 12\text{MoO}_3$                 | IV.              | Yellow        |

Choose the **correct** answer from the options given below :

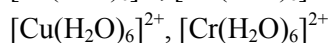
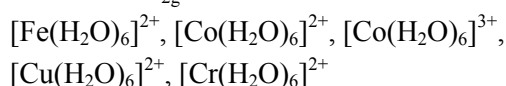
- (1) A-III, B-I, C-II, D-IV
- (2) A-IV, B-I, C-II, D-III
- (3) A-II, B-III, C-IV, D-I
- (4) A-I, B-II, C-III, D-IV

**Ans. (1)**

**Sol.**  $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3 \cdot x\text{H}_2\text{O} \rightarrow$  Prussian Blue



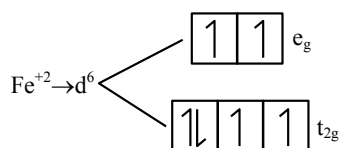
79. Number of complexes with even number of electrons in  $t_{2g}$  orbitals is -



- (1) 1
- (2) 3
- (3) 2
- (4) 5

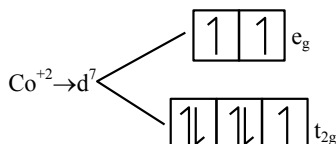
**Ans. (2)**

**Sol.**  $[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$



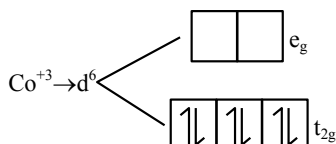
Electron in  $t_{2g} = 4(\text{even})$

$[\text{Co}(\text{H}_2\text{O})_6]^{2+}$



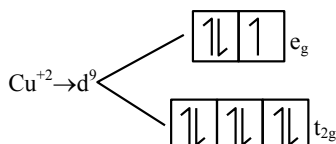
Electron in  $t_{2g} = 5(\text{odd})$

$[\text{Co}(\text{H}_2\text{O})_6]^{3+}$



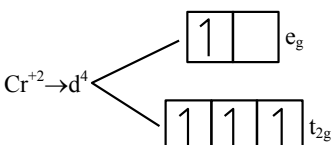
Electron in  $t_{2g} = 6(\text{even})$

$[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$



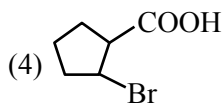
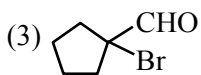
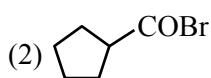
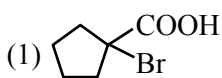
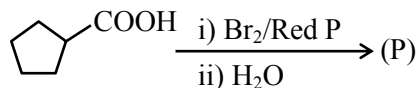
Electron in  $t_{2g} = 6(\text{even})$

$[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$



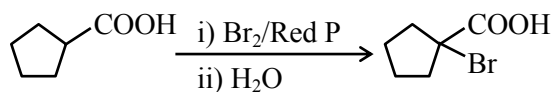
Electron in  $t_{2g} = 3(\text{odd})$

**80.** Identify the product (P) in the following reaction:



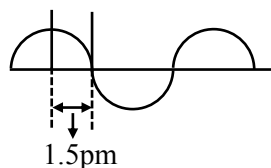
**Ans. (1)**

**Sol.** HVZ Reaction



## SECTION-B

**81.** A hypothetical electromagnetic wave is shown below.



The frequency of the wave is  $x \times 10^{19}$  Hz.

$x = \underline{\hspace{2cm}}$  (nearest integer)

**Ans. (5)**

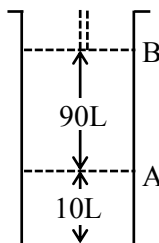
**Sol.**  $\lambda = 1.5 \times 4 \text{ pm}$   
 $= 6 \times 10^{-12} \text{ meter}$

$$\lambda \nu = c$$

$$6 \times 10^{-12} \times \nu = 3 \times 10^8$$

$$\nu = 5 \times 10^{19} \text{ Hz}$$

**82.**



Consider the figure provided.

1 mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at  $18^\circ\text{C}$ . If the piston is moved to position B, keeping the temperature unchanged, then 'x' L atm work is done in this reversible process.

$x = \underline{\hspace{2cm}}$  L atm. (nearest integer)

[Given : Absolute temperature =  $^\circ\text{C} + 273.15$ ,  
 $R = 0.08206 \text{ L atm mol}^{-1} \text{ K}^{-1}$ ]

**Ans. (55)**

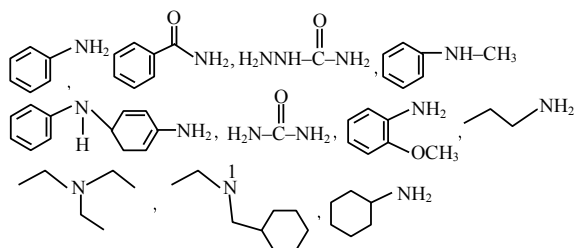
**Sol.**  $\omega = -nRT \ln \left( \frac{V_2}{V_1} \right)$

$$= -1 \times 0.08206 \times 291.15 \ln \left( \frac{100}{10} \right)$$

$$= -55.0128$$

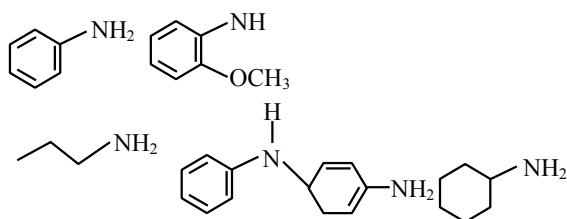
Work done by system  $\approx 55 \text{ atm lit.}$

83. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is \_\_\_\_.

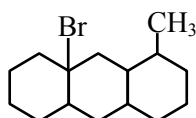


Ans. (5)

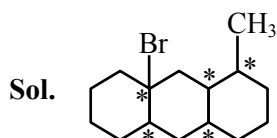
Sol. Primary amine give an ionic solid upon reaction with Hinsberg reagent which is soluble in NaOH.



84. The number of optical isomers in following compound is : \_\_\_\_.



Ans. (32)



Sol.

Total chiral centre = 5

No. of optical isomers =  $2^5 = 32$ .

85. The 'spin only' magnetic moment value of  $\text{MO}_4^{2-}$  is \_\_\_\_ BM. (Where M is a metal having least metallic radii. among Sc, Ti, V, Cr, Mn and Zn).  
(Given atomic number : Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25 and Zn = 30)

Ans. (0)

Sol. Metal having least metallic radii among Sc, Ti, V, Cr, Mn & Zn is Cr.

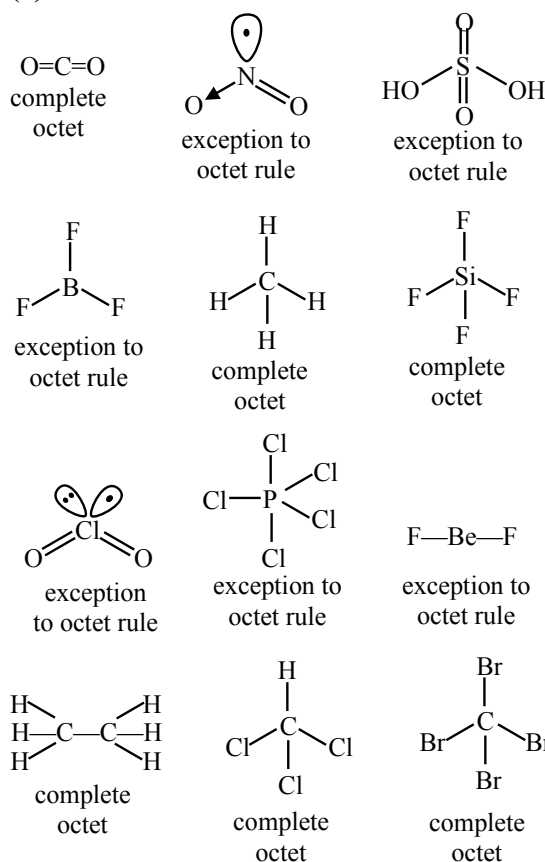
Spin only magnetic moment of  $\text{CrO}_4^{2-}$ .

Here  $\text{Cr}^{+6}$  is in  $d^0$  configuration (diamagnetic).

86. Number of molecules from the following which are exceptions to octet rule is \_\_\_\_.  
 $\text{CO}_2$ ,  $\text{NO}_2$ ,  $\text{H}_2\text{SO}_4$ ,  $\text{BF}_3$ ,  $\text{CH}_4$ ,  $\text{SiF}_4$ ,  $\text{ClO}_2$ ,  $\text{PCl}_5$ ,  $\text{BeF}_2$ ,  $\text{C}_2\text{H}_6$ ,  $\text{CHCl}_3$ ,  $\text{CBr}_4$

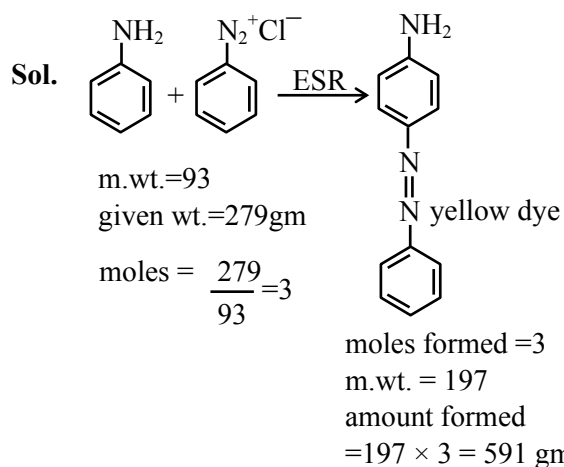
Ans. (6)

Sol.

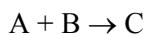


87. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the maximum amount of aniline yellow formed will be \_\_\_\_ g. (nearest integer)  
(consider complete conversion)

Ans. (591)



88. Consider the following reaction



The time taken for A to become  $1/4^{\text{th}}$  of its initial concentration is twice the time taken to become  $1/2$  of the same. Also, when the change of concentration of B is plotted against time, the resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis.

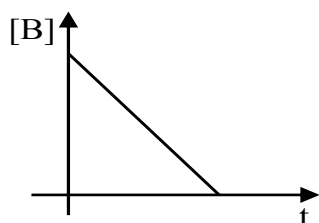
The overall order of the reaction is \_\_\_\_.

Ans. (1)

Sol. For  $1^{\text{st}}$  order reaction

$$75\% \text{ life} = 2 \times 50\% \text{ life}$$

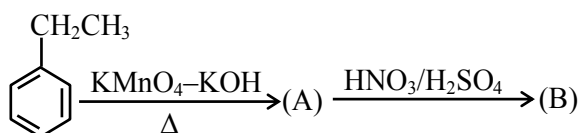
So order with respect to A will be first order.



So order with respect to B will be zero.

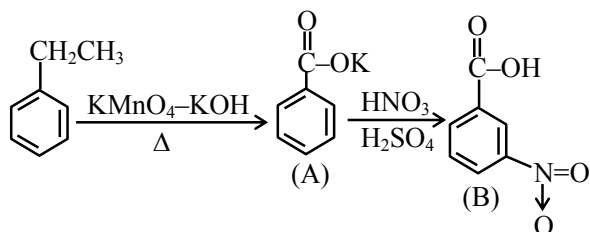
$$\text{Overall order of reaction} = 1 + 0 = 1$$

89. Major product B of the following reaction has \_\_\_\_  $\pi$ -bond.



Ans. (5)

Sol. Major product B is  $\rightarrow$

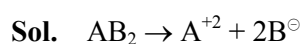


Total number of  $\pi$  bonds in B are 5

90. A solution containing 10g of an electrolyte  $\text{AB}_2$  in 100g of water boils at  $100.52^\circ\text{C}$ . The degree of ionization of the electrolyte ( $\alpha$ ) is \_\_\_\_  $\times 10^{-1}$ . (nearest integer)

[Given : Molar mass of  $\text{AB}_2 = 200\text{g mol}^{-1}$ .  $K_b$  (molal boiling point elevation const. of water)  $= 0.52\text{ K kg mol}^{-1}$ , boiling point of water  $= 100^\circ\text{C}$  ;  $\text{AB}_2$  ionises as  $\text{AB}_2 \rightarrow \text{A}^{2+} + 2\text{B}^-$ ]

Ans. (5)



$$i = 1 + (3 - 1) \alpha$$

$$i = 1 + 2\alpha$$

$$\Delta T_b = k_b \cdot m$$

$$0.52 = 0.52 (1 + 2\alpha) \frac{10}{\frac{200}{100} \cdot \frac{1000}{1000}}$$

$$1 = (1 + 2\alpha) \frac{10}{20}$$

$$2 = 1 + 2\alpha$$

$$\alpha = 0.5$$

$$\text{Ans. } \alpha = 5 \times 10^{-1}$$