

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Tuesday 30th January, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

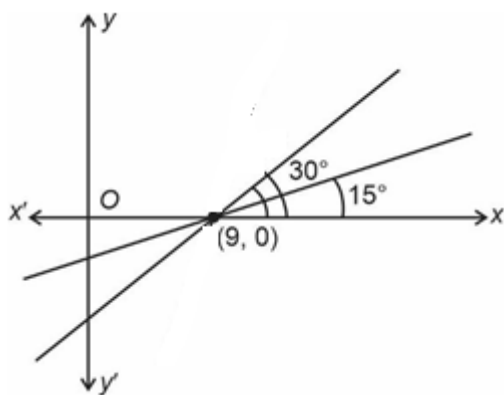
SECTION-A

1. A line passing through the point A(9, 0) makes an angle of 30° with the positive direction of x-axis. If this line is rotated about A through an angle of 15° in the clockwise direction, then its equation in the new position is

- (1) $\frac{y}{\sqrt{3}-2} + x = 9$ (2) $\frac{x}{\sqrt{3}-2} + y = 9$
 (3) $\frac{x}{\sqrt{3}+2} + y = 9$ (4) $\frac{y}{\sqrt{3}+2} + x = 9$

Ans. (1)

Sol.



$$\text{Eq}^n : y - 0 = \tan 15^\circ (x - 9) \Rightarrow y = (2 - \sqrt{3})(x - 9)$$

2. Let S_n denote the sum of first n terms an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is :

- (1) 395
 (2) 390
 (3) 405
 (4) 410

Ans. (1)

Sol. $S_{20} = \frac{20}{2}[2a + 19d] = 790$

$$2a + 19d = 79 \quad \dots\dots(1)$$

$$S_{10} = \frac{10}{2}[2a + 9d] = 145$$

$$2a + 9d = 29 \quad \dots\dots(2)$$

From (1) and (2) $a = -8, d = 5$

$$S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$$

$$= \frac{15}{2}[-16 + 70] - \frac{5}{2}[-16 + 20]$$

$$= 405 - 10$$

$$= 395$$

3. If $z = x + iy$, $xy \neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then $|z^2|$ is equal to :

- (1) 9
 (2) 1
 (3) 4
 (4) $\frac{1}{4}$

Ans. (2)

Sol. $z^2 = -i\bar{z}$

$$|z^2| = |i\bar{z}|$$

$$|z^2| = |z|$$

$$|z|^2 - |z| = 0$$

$$|z|(|z| - 1) = 0$$

$$|z| = 0 \text{ (not acceptable)}$$

$$\therefore |z| = 1$$

$$\therefore |z|^2 = 1$$

4. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$; $\vec{a} \cdot \vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to :

(1) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(2) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(4) $\cos^{-1}\left(\frac{2}{3}\right)$

Ans. (3)

Sol. Given $|\vec{a}|=1, |\vec{b}|=4, \vec{a} \cdot \vec{b}=2$

$$\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$$

Dot product with $\vec{a}$ on both sides

$$\vec{c} \cdot \vec{a} = -6 \quad \dots (1)$$

Dot product with $\vec{b}$ on both sides

$$\vec{b} \cdot \vec{c} = -48 \quad \dots (2)$$

$$\vec{c} \cdot \vec{c} = 4|\vec{a} \times \vec{b}|^2 + 9|\vec{b}|^2$$

$$|\vec{c}|^2 = 4[|\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2] + 9|\vec{b}|^2$$

$$|\vec{c}|^2 = 4[(1)(4)^2 - (2)^2] + 9(16)$$

$$|\vec{c}|^2 = 4[12] + 144$$

$$|\vec{c}|^2 = 48 + 144$$

$$|\vec{c}|^2 = 192$$

$$\therefore \cos \theta = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}| |\vec{c}|}$$

$$\therefore \cos \theta = \frac{-48}{\sqrt{192} \cdot 4}$$

$$\therefore \cos \theta = \frac{-48}{8\sqrt{3} \cdot 4}$$

$$\therefore \cos \theta = \frac{-3}{2\sqrt{3}}$$

$$\therefore \cos \theta = \frac{-\sqrt{3}}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

5. The maximum area of a triangle whose one vertex is at $(0, 0)$ and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and $(-x, y)$ where $y > 0$ is :

(1) 88

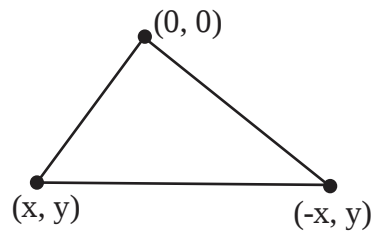
(2) 122

(3) 92

(4) 108

Ans. (4)

Sol.



Area of Δ

$$= \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2} (xy + xy) = |xy|$$

$$\text{Area}(\Delta) = |xy| = \left| x(-2x^2 + 54) \right|$$

$$\frac{d(\Delta)}{dx} = \left| (-6x^2 + 54) \right| \Rightarrow \frac{d\Delta}{dx} = 0 \text{ at } x = 3$$

$$\text{Area} = 3(-2 \times 9 + 54) = 108$$

6. The value of $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^3}{(n^2 + k^2)(n^2 + 3k^2)}$ is :

$$(1) \frac{(2\sqrt{3} + 3)\pi}{24}$$

$$(2) \frac{13\pi}{8(4\sqrt{3} + 3)}$$

$$(3) \frac{13(2\sqrt{3} - 3)\pi}{8}$$

$$(4) \frac{\pi}{8(2\sqrt{3} + 3)}$$

Ans. (2)

$$\text{Sol.} \quad \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^3}{n^4 \left(1 + \frac{k^2}{n^2}\right) \left(1 + \frac{3k^2}{n^2}\right)}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{n^3}{\left(1 + \frac{k^2}{n^2}\right) \left(1 + \frac{3k^2}{n^2}\right)}$$

$$= \int_0^1 \frac{dx}{3(1+x^2) \left(\frac{1}{3} + x^2\right)}$$

$$\begin{aligned}
&= \int_0^1 \frac{1}{3} \times \frac{3}{2} \frac{\left(x^2 + 1\right) - \left(x^2 + \frac{1}{3}\right)}{\left(1 + x^2\right)\left(x^2 + \frac{1}{3}\right)} dx \\
&= \frac{1}{2} \int_0^1 \left[\frac{1}{x^2 + \left(\frac{1}{\sqrt{3}}\right)^2} - \frac{1}{1 + x^2} \right] dx \\
&= \frac{1}{2} \left[\sqrt{3} \tan^{-1}(\sqrt{3}x) \right]_0^1 - \frac{1}{2} \left[\tan^{-1}x \right]_0^1 \\
&= \frac{\sqrt{3}}{2} \left(\frac{\pi}{3} \right) - \frac{1}{2} \left(\frac{\pi}{4} \right) = \frac{\pi}{2\sqrt{3}} - \frac{\pi}{8} \\
&= \frac{13\pi}{8(4\sqrt{3} + 3)}
\end{aligned}$$

7. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be a non constant twice differentiable such that $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as

$$f(x) = \frac{1}{2} [g(x) + g(2-x)], \text{ then}$$

- (1) $f''(x) = 0$ for atleast two x in $(0, 2)$
- (2) $f''(x) = 0$ for exactly one x in $(0, 1)$
- (3) $f''(x) = 0$ for no x in $(0, 1)$
- (4) $f'\left(\frac{3}{2}\right) + f'\left(\frac{1}{2}\right) = 1$

Ans. (1)

$$\text{Sol. } f'(x) = \frac{g'(x) - g'(2-x)}{2}, f'\left(\frac{3}{2}\right) = \frac{g'\left(\frac{3}{2}\right) - g'\left(\frac{1}{2}\right)}{2} = 0$$

$$\text{Also } f'\left(\frac{1}{2}\right) = \frac{g'\left(\frac{1}{2}\right) - g'\left(\frac{3}{2}\right)}{2} = 0, f'\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow f'\left(\frac{3}{2}\right) = f'\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow \text{roots in } \left(\frac{1}{2}, 1\right) \text{ and } \left(1, \frac{3}{2}\right)$$

$$\Rightarrow f''(x) \text{ is zero at least twice in } \left(\frac{1}{2}, \frac{3}{2}\right)$$

8. The area (in square units) of the region bounded by the parabola $y^2 = 4(x-2)$ and the line $y = 2x - 8$

- (1) 8
- (2) 9
- (3) 6
- (4) 7

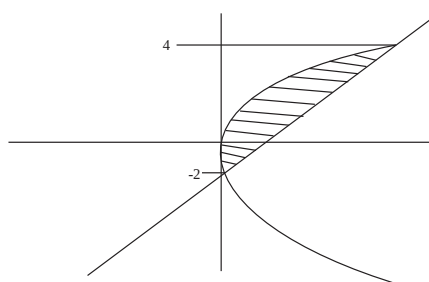
Ans. (2)

Sol. Let $X = x - 2$

$$y^2 = 4x, \quad y = 2(x+2) - 8$$

$$y^2 = 4x, \quad y = 2x - 4$$

$$A = \int_{-2}^4 \frac{y^2}{4} - \frac{y+4}{2}$$



$$= 9$$

9. Let $y = y(x)$ be the solution of the differential equation $\sec x \, dy + \{2(1-x) \tan x + x(2-x)\} \, dx = 0$ such that $y(0) = 2$. Then $y(2)$ is equal to :

- (1) 2
- (2) $2\{1 - \sin(2)\}$
- (3) $2\{\sin(2) + 1\}$
- (4) 1

Ans. (1)

$$\text{Sol. } \frac{dy}{dx} = 2(x-1)\sin x + (x^2 - 2x)\cos x$$

Now both side integrate

$$y(x) = \int 2(x-1)\sin x \, dx + \left[(x^2 - 2x)(\sin x) - \int (2x-2)\sin x \, dx \right]$$

$$y(x) = (x^2 - 2x)\sin x + \lambda$$

$$y(0) = 0 + \lambda \Rightarrow 2 = \lambda$$

$$y(x) = (x^2 - 2x)\sin x + 2$$

$$y(2) = 2$$

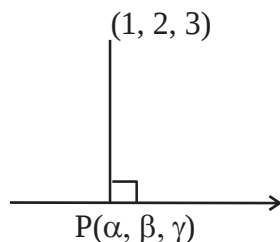
β, γ be the foot of perpendicular from the point $(1, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

then $19(\alpha + \beta + \gamma)$ is equal to :

- (1) 102
- (2) 101
- (3) 99
- (4) 100

Ans. (2)

Sol.



Let foot $P(5k-3, 2k+1, 3k-4)$

DR's $\rightarrow AP: 5k-4, 2k-1, 3k-7$

DR's $\rightarrow$ Line: 5, 2, 3

Condition of perpendicular lines $(25k-20) + (4k-2) + (9k-21)=0$

Then $k = \frac{43}{38}$

Then $19(\alpha + \beta + \gamma) = 101$

11. Two integers x and y are chosen with replacement from the set $\{0, 1, 2, 3, \dots, 10\}$. Then the probability that $|x - y| > 5$ is :

- (1) $\frac{30}{121}$
- (2) $\frac{62}{121}$
- (3) $\frac{60}{121}$
- (4) $\frac{31}{121}$

Ans. (1)

Sol. If $x = 0, y = 6, 7, 8, 9, 10$

If $x = 1, y = 7, 8, 9, 10$

If $x = 2, y = 8, 9, 10$

If $x = 3, y = 9, 10$

If $x = 4, y = 10$

If $x = 5, y = \text{no possible value}$

Total possible ways $= (5 + 4 + 3 + 2 + 1) \times 2$

$= 30$

Required probability $= \frac{30}{11 \times 11} = \frac{30}{121}$

12. If the domain of the function

$f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + (\log_e(3-x))^{-1}$ is

$[-\alpha, \beta) - \{y\}$, then $\alpha + \beta + \gamma$ is equal to :

- (1) 12
- (2) 9
- (3) 11
- (4) 8

Ans. (3)

Sol. $-1 \leq \left| \frac{2-|x|}{4} \right| \leq 1$

$$\Rightarrow \left| \frac{2-|x|}{4} \right| \leq 1$$

$$-4 \leq 2 - |x| \leq 4$$

$$-6 \leq -|x| \leq 2$$

$$-2 \leq |x| \leq 6$$

$$|x| \leq 6$$

$$\Rightarrow x \in [-6, 6] \quad \dots(1)$$

Now, $3 - x \neq 1$

$$\text{And } x \neq 2 \quad \dots(2)$$

$$\text{and } 3 - x > 0$$

$$x < 3 \quad \dots(3)$$

From (1), (2) and (3)

$$\Rightarrow x \in [-6, 3) - \{2\}$$

$$\alpha = 6$$

$$\beta = 3$$

$$\gamma = 2$$

$$\alpha + \beta + \gamma = 11$$

13. Consider the system of linear equation $x + y + z = 4\mu, x + 2y + 2\lambda z = 10\mu, x + 3y + 4\lambda^2 z = \mu^2 + 15$, where $\lambda, \mu \in \mathbb{R}$. Which one of the following statements is NOT correct ?

- (1) The system has unique solution if $\lambda \neq \frac{1}{2}$ and $\mu \neq 1, 15$

(2) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

(3) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(4) The system is consistent if $\lambda \neq \frac{1}{2}$

Ans. (2)

Sol. $x + y + z = 4\mu$, $x + 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$,

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda - 1)^2$$

For unique solution $\Delta \neq 0$, $2\lambda - 1 \neq 0$, $\left(\lambda \neq \frac{1}{2}\right)$

Let $\Delta = 0$, $\lambda = \frac{1}{2}$

$$\Delta_y = 0, \Delta_x = \Delta_z = \begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2 + 15 & 3 & 1 \end{vmatrix}$$

$$= (\mu - 15)(\mu - 1)$$

For infinite solution $\lambda = \frac{1}{2}$, $\mu = 1$ or 15

14. If the circles $(x+1)^2 + (y+2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then

(1) $5 < r < 9$

(2) $0 < r < 7$

(3) $3 < r < 7$

(4) $\frac{1}{2} < r < 7$

Ans. (3)

Sol. If two circles intersect at two distinct points

$$\Rightarrow |r_1 - r_2| < C_1 C_2 < r_1 + r_2$$

$$|r - 2| < \sqrt{9 + 16} < r + 2$$

$$|r - 2| < 5 \text{ and } r + 2 > 5$$

$$-5 < r - 2 < 5 \quad r > 3 \dots\dots\dots(2)$$

$$-3 < r < 7 \quad \dots\dots\dots(1)$$

From (1) and (2)

$$3 < r < 7$$

15. If the length of the minor axis of ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{\sqrt{5}}{3}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{\sqrt{3}}$

(4) $\frac{2}{\sqrt{5}}$

Ans. (4)

Sol. $2b = ae$

$$\frac{b}{a} = \frac{e}{2}$$

$$e = \sqrt{1 - \frac{e^2}{4}}$$

$$e = \frac{2}{\sqrt{5}}$$

16. Let M denote the median of the following frequency distribution.

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then 20 M is equal to :

(1) 416

(2) 104

(3) 52

(4) 208

Ans. (4)

Sol.

Class	Frequency	Cumulative frequency
0-4	3	3
4-8	9	12
8-12	10	22
12-16	8	30
16-20	6	36

$$M = 1 + \left(\frac{\frac{N}{2} - C}{f} \right) h$$

$$M = 8 + \frac{18-12}{10} \times 4$$

$$M = 10.4$$

$$20M = 208$$

17. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then

$$\frac{1}{5} f'(0) \text{ is equal to } \underline{\hspace{2cm}}$$

(1) 0

(2) 1

(3) 2

(4) 6

Ans. (1)

Sol. $\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^2 4x & \sin^2 2x \end{vmatrix}$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$$

$$f(x) = 45$$

$$f'(x) = 0$$

18. Let A (2, 3, 5) and C(-3, 4, -2) be opposite vertices of a parallelogram ABCD if the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$ then the area of the parallelogram is equal to

(1) $\frac{1}{2}\sqrt{410}$

(2) $\frac{1}{2}\sqrt{474}$

(3) $\frac{1}{2}\sqrt{586}$

(4) $\frac{1}{2}\sqrt{306}$

Ans. (2)

Sol. Area = $|\overrightarrow{AC} \times \overrightarrow{BD}|$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \frac{1}{2} |-17\hat{i} - 8\hat{j} + 11\hat{k}| = \frac{1}{2} \sqrt{474}$$

19. If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, $n \in \mathbb{N}$, then the roots of the equation $x^2 + nx + (n-3) = 0$ belong to :

(1) $(0, \infty)$

(2) $(-\infty, 0)$

(3) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

(4) $\mathbb{Z}$

Ans. (2)

Sol. $2\sin^3 x + 2\sin x \cdot \cos^2 x + 4\sin x - 4 = 0$

$$2\sin^3 x + 2\sin x \cdot (1 - \sin^2 x) + 4\sin x - 4 = 0$$

$$6\sin x - 4 = 0$$

$$\sin x = \frac{2}{3}$$

$$n = 5 \text{ (in the given interval)}$$

$$x^2 + 5x + 2 = 0$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

$$\text{Required interval } (-\infty, 0)$$

20. Let $f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ be a differentiable function

$$\text{such that } f(0) = \frac{1}{2}, \text{ If the } \lim_{x \rightarrow 0} \frac{x \int_0^x f(t) dt}{e^{x^2} - 1} = \alpha,$$

then $8\alpha^2$ is equal to :

(1) 16

(2) 2

(3) 1

(4) 4

Ans. (2)

Sol. $\lim_{x \rightarrow 0} \frac{x \int_0^x f(t) dt}{\left(\frac{e^{x^2} - 1}{x^2} \right) \times x^2}$

$$\lim_{x \rightarrow 0} \frac{\int_0^x f(t) dt}{x} \quad \left(\lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{x^2} = 1 \right)$$

$$= \lim_{x \rightarrow 0} \frac{f(x)}{1} \quad (\text{using L Hospital})$$

$$f(0) = \frac{1}{2}$$

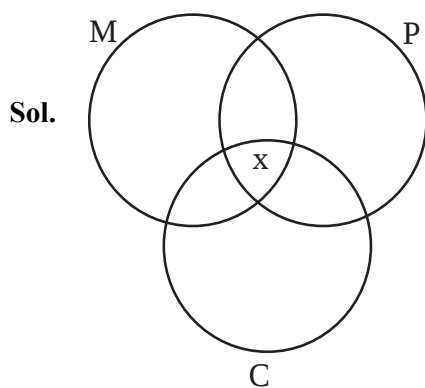
$$\alpha = \frac{1}{2}$$

$$8\alpha^2 = 2$$

SECTION-B

- 21.** A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is _____.

Ans. (10)

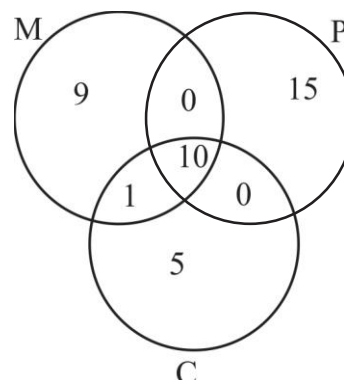


$$11 - x \geq 0 \quad (\text{Maths and Physics})$$

$$x \leq 11$$

$x = 11$ does not satisfy the data.

For $x = 10$



Hence maximum number of students passed in all the three subjects is 10.

- 22.** If d_1 is the shortest distance between the lines $x + 1 = 2y = -12z$, $x = y + 2 = 6z - 6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$, $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is :

Ans. (16)

Sol. $L_1: \frac{x+1}{1} = \frac{y}{1/2} = \frac{z}{-1/12}$, $L_2: \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{6}$

d_1 = shortest distance between L_1 & L_2

$$= \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|(\vec{b}_1 \times \vec{b}_2)|}$$

$$d_1 = 2$$

$$L_3: \frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}, \quad L_4: \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

d_2 = shortest distance between L_3 & L_4

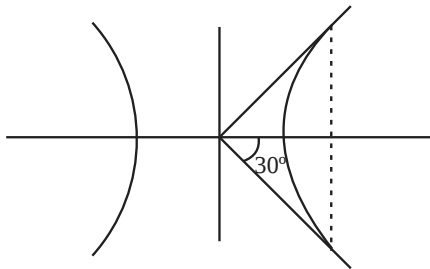
$$d_2 = \frac{12}{\sqrt{3}} \quad \text{Hence}$$

$$= \frac{32\sqrt{3}d_1}{d_2} = \frac{32\sqrt{3} \times 2}{\frac{12}{\sqrt{3}}} = 16$$

23. Let the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{l}{m}(1 + \sqrt{n})$, where l and m are co-prime numbers, then $l^2 + m^2 + n^2$ is equal to _____

Ans. (182)

Sol. LR subtends 60° at centre



$$\Rightarrow \tan 30^\circ = \frac{b^2/a}{ae} = \frac{b^2}{a^2e} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow e = \frac{\sqrt{3}b^2}{9}$$

$$\text{Also, } e^2 = 1 + \frac{b^2}{9} \Rightarrow 1 + \frac{b^2}{9} = \frac{3b^4}{81}$$

$$\Rightarrow b^4 = 3b^2 + 27$$

$$\Rightarrow b^4 - 3b^2 - 27 = 0$$

$$\Rightarrow b^2 = \frac{3}{2}(1 + \sqrt{13})$$

$$\Rightarrow \ell = 3, m = 2, n = 13$$

$$\Rightarrow \ell^2 + m^2 + n^2 = 182$$

24. Let $A = \{1, 2, 3, \dots, 7\}$ and let $P(1)$ denote the power set of A . If the number of functions $f: A \rightarrow P(A)$ such that $a \in f(a), \forall a \in A$ is m^n , m and $n \in \mathbb{N}$ and m is least, then $m + n$ is equal to _____.

Ans. (44)

Sol. $f: A \rightarrow P(A)$

$$a \in f(a)$$

That means 'a' will connect with subset which contain element 'a'.

Total options for 1 will be 2^6 . (Because 2^6 subsets contains 1)

Similarly, for every other element

$$\text{Hence, total is } 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 = 2^{42}$$

$$\text{Ans. } 2+42 = 44$$

25. The value $9 \int_0^9 \left[\sqrt{\frac{10x}{x+1}} \right] dx$, where $[t]$ denotes the greatest integer less than or equal to t , is _____.

Ans. (155)

$$\text{Sol. } \frac{10x}{x+1} = 1 \Rightarrow x = \frac{1}{9}$$

$$\frac{10x}{x+1} = 4 \Rightarrow x = \frac{2}{3}$$

$$\frac{10x}{x+1} = 9 \Rightarrow x = 9$$

$$I = 9 \left(\int_0^{1/9} 0 dx + \int_{1/9}^{2/3} 1 dx + \int_{2/3}^9 2 dx \right)$$

$$= 155$$

26. Number of integral terms in the expansion of

$$\left\{ 7 \left(\frac{1}{2} \right) + 11 \left(\frac{1}{6} \right) \right\}^{824} \text{ is equal to } \underline{\hspace{2cm}}.$$

Ans. (138)

Sol. General term in expansion of $((7)^{1/2} + (11)^{1/6})^{824}$ is

$$t_{r+1} = {}^{824}C_r (7)^{\frac{824-r}{2}} (11)^{r/6}$$

For integral term, r must be multiple of 6.

$$\text{Hence } r = 0, 6, 12, \dots, 822$$

27. Let $y = y(x)$ be the solution of the differential equation $(1 - x^2) dy = [xy + (x^3 + 2)\sqrt{3(1 - x^2)}] dx$, $-1 < x < 1, y(0) = 0$. If $y\left(\frac{1}{2}\right) = \frac{m}{n}$, m and n are co-prime numbers, then $m + n$ is equal to _____.

Ans. (97)

Sol.
$$\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{(x^3+2)\sqrt{3(1-x^2)}}{1-x^2}$$

IF = $e^{-\int \frac{x}{1-x^2} dx} = e^{\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$

$y\sqrt{1-x^2} = \sqrt{3} \int (x^3+2) dx$

$y\sqrt{1-x^2} = \sqrt{3} \left(\frac{x^4}{4} + 2x \right) + c$

$\Rightarrow y(0) = 0 \quad \therefore c = 0$

$y\left(\frac{1}{2}\right) = \frac{65}{32} = \frac{m}{n}$

$m + n = 97$

28. Let $\alpha, \beta \in \mathbb{N}$ be roots of equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin \mathbb{N}$. If λ assumes the minimum possible value, then $\frac{(\sqrt{\alpha-1} + \sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$ is equal to :

Ans. (60)

Sol. $x^2 - 70x + \lambda = 0$

$\alpha + \beta = 70$

$\alpha\beta = \lambda$

$\therefore \alpha(70 - \alpha) = \lambda$

Since, 2 and 3 does not divide λ

$\therefore \alpha = 5, \beta = 65, \lambda = 325$

By putting value of α, β, λ we get the required value 60.

29. If the function $f(x) = \begin{cases} \frac{1}{|x|} & , |x| \geq 2 \\ ax^2 + 2b & , |x| < 2 \end{cases}$ is differentiable on $\mathbb{R}$, then $48(a + b)$ is equal to _____.

Ans. (15)

Sol. $f(x) = \begin{cases} \frac{1}{x} & ; x \geq 2 \\ ax^2 + 2b & ; -2 < x < 2 \\ -\frac{1}{x} & ; x \leq -2 \end{cases}$

Continuous at $x = 2 \quad \Rightarrow \frac{1}{2} = \frac{a}{4} + 2b$

Continuous at $x = -2 \quad \Rightarrow \frac{1}{2} = \frac{a}{4} + 2b$

Since, it is differentiable at $x = 2$

$-\frac{1}{x^2} = 2ax$

Differentiable at $x = 2 \quad \Rightarrow \frac{-1}{4} = 4a \Rightarrow a = \frac{-1}{16}, b = \frac{3}{8}$

30. Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \dots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to _____.

Ans. (353)

Sol. $\alpha = 1^2 + 4^2 + 8^2 + \dots$

$t_n = an^2 + bn + c$

$$1 = a + b + c$$

$$4 = 4a + 2b + c$$

$$8 = 9a + 3b + c$$

On solving we get, $a = \frac{1}{2}$, $b = \frac{3}{2}$, $c = -1$

$$\alpha = \sum_{n=1}^{10} \left(\frac{n^2}{2} + \frac{3n}{2} - 1 \right)^2$$

$$4\alpha = \sum_{n=1}^{10} (n^2 + 3n - 2)^2, \quad \beta = \sum_{n=1}^{10} n^4$$

$$4\alpha - \beta = \sum_{n=1}^{10} (6n^3 + 5n^2 - 12n + 4) = 55(353) + 40$$

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

31. Match List-I with List-II.

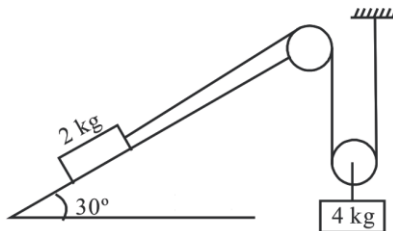
	List-I		List-II
A.	Coefficient of viscosity	I.	$[M L^2 T^{-2}]$
B.	Surface Tension	II.	$[M L^2 T^{-1}]$
C.	Angular momentum	III.	$[M L^{-1} T^{-1}]$
D.	Rotational kinetic energy	IV.	$[M L^0 T^{-2}]$

- (1) A-II, B-I, C-IV, D-III
 (2) A-I, B-II, C-III, D-IV
 (3) A-III, B-IV, C-II, D-I
 (4) A-IV, B-III, C-II, D-I

Ans. (3)

Sol. $F = \eta A \frac{dv}{dy}$
 $[MLT^{-2}] = \eta [L^2][T^{-1}]$
 $\eta = [ML^{-1}T^{-1}]$
 $S.T = \frac{F}{\ell} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$
 $L = mvr = [ML^2T^{-1}]$
 $K.E = \frac{1}{2} I \omega^2 = [ML^2T^{-2}]$

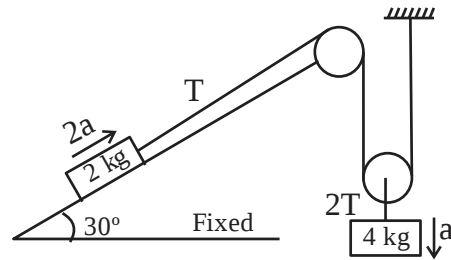
32. All surfaces shown in figure are assumed to be frictionless and the pulleys and the string are light. The acceleration of the block of mass 2 kg is :



- (1) g (2) $\frac{g}{3}$
 (3) $\frac{g}{2}$ (4) $\frac{g}{4}$

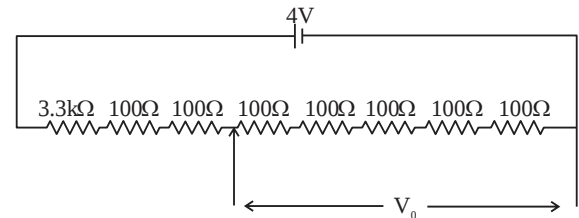
Ans. (2)

Sol.



$40 - 2T = 4a$
 $T - 10 = 4a \Rightarrow 20 = 12a$
 $\Rightarrow a = 5/3 \Rightarrow 2a = \frac{g}{3}$

33. A potential divider circuit is shown in figure. The output voltage V_0 is



- (1) 4V (2) 2 mV
 (3) 0.5 V (4) 12 mV

Ans. (3)

Sol. $R_{eq} = 4000 \Omega$

$i = \frac{4}{4000} = \frac{1}{1000} A$

$V_0 = i.R = \frac{1}{1000} \times 500 = 0.5V$

34. Young's modulus of material of a wire of length 'L' and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved then Young's modulus will be :

- (1) $\frac{Y}{4}$ (2) 4Y
 (3) Y (4) 2Y

Ans. (3)

Sol. Young's modulus depends on the material not length and cross sectional area. So young's modulus remains same.

35. The work function of a substance is 3.0 eV. The longest wavelength of light that can cause the emission of photoelectrons from this substance is approximately:

- (1) 215 nm (2) 414 nm
(3) 400 nm (4) 200 nm

Ans. (2)

Sol. For P.E.E. : $\lambda \leq \frac{hc}{W_e}$

$$\lambda \leq \frac{1240 \text{ nm-eV}}{3 \text{ eV}}$$

$$\lambda \leq 413.33 \text{ nm}$$

$$\lambda_{\text{max}} \approx 414 \text{ nm for P.E.E.}$$

36. The ratio of the magnitude of the kinetic energy to the potential energy of an electron in the 5th excited state of a hydrogen atom is :

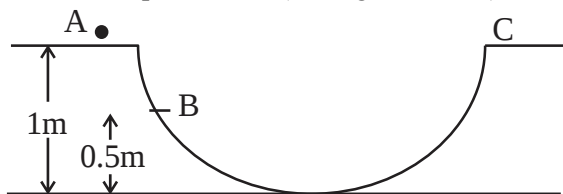
- (1) 4 (2) $\frac{1}{4}$
(3) $\frac{1}{2}$ (4) 1

Ans. (3)

Sol. $\frac{1}{2}|PE| = KE$ for each value of n (orbit)

$$\therefore \frac{KE}{|PE|} = \frac{1}{2}$$

37. A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed toward right. The speed of the particle when it reaches the point B is : (Take $g = 10 \text{ m/s}^2$).



- (1) 20 m/s (2) $\sqrt{10} \text{ m/s}$
(3) $2\sqrt{10} \text{ m/s}$ (4) 10 m/s

Ans. (2)

Sol. By COME

$$KE_A + U_A = KE_B + U_B$$

$$0 + mg(1) = \frac{1}{2}mv^2 + mg \times 0.5$$

$$v = \sqrt{g} = \sqrt{10} \text{ m/s}$$

38. The electric field of an electromagnetic wave in free space is represented as $\vec{E} = E_0 \cos(\omega t - kz) \hat{i}$. The corresponding magnetic induction vector will be :

$$(1) \vec{B} = E_0 C \cos(\omega t - kz) \hat{j}$$

$$(2) \vec{B} = \frac{E_0}{C} \cos(\omega t - kz) \hat{j}$$

$$(3) \vec{B} = E_0 C \cos(\omega t + kz) \hat{j}$$

$$(4) \vec{B} = \frac{E_0}{C} \cos(\omega t + kz) \hat{j}$$

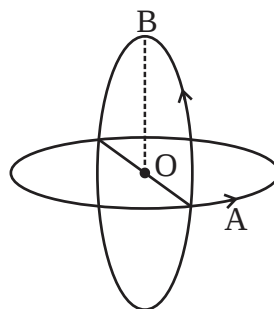
Ans. (2)

Sol. Given $\vec{E} = E_0 \cos(\omega t - kz) \hat{i}$

$$\vec{B} = \frac{E_0}{C} \cos(\omega t - kz) \hat{j}$$

$$\hat{C} = \hat{E} \times \hat{B}$$

39. Two insulated circular loop A and B radius 'a' carrying a current of 'I' in the anti clockwise direction as shown in figure. The magnitude of the magnetic induction at the centre will be :



$$(1) \frac{\sqrt{2}\mu_0 I}{a}$$

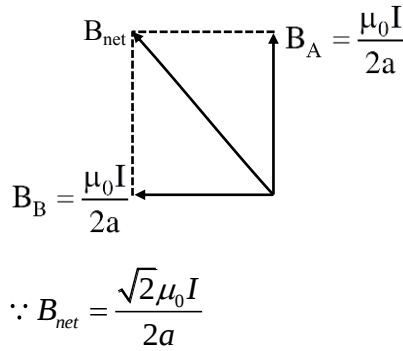
$$(2) \frac{\mu_0 I}{2a}$$

$$(3) \frac{\mu_0 I}{\sqrt{2}a}$$

$$(4) \frac{2\mu_0 I}{a}$$

Ans. (3)

Sol.



- 40.** The diffraction pattern of a light of wavelength 400 nm diffracting from a slit of width 0.2 mm is focused on the focal plane of a convex lens of focal length 100 cm. The width of the 1st secondary maxima will be :

- (1) 2 mm (2) 2 cm
(3) 0.02 mm (4) 0.2 mm

Ans. (1)

Sol. Width of 1st secondary maxima = $\frac{\lambda}{a} \cdot D$

Here

$$a = 0.2 \times 10^{-3} \text{ m}$$

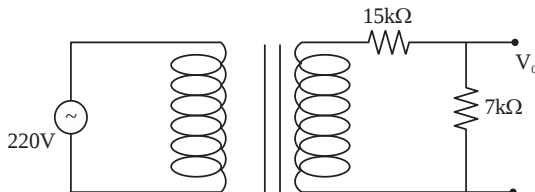
$$\lambda = 400 \times 10^{-9} \text{ m}$$

$$D = 100 \times 10^{-2}$$

Width of 1st secondary maxima

$$= \frac{400 \times 10^{-9}}{0.2 \times 10^{-3}} \times 100 \times 10^{-2} \\ = 2 \text{ mm}$$

- 41.** Primary coil of a transformer is connected to 220 V ac. Primary and secondary turns of the transforms are 100 and 10 respectively. Secondary coil of transformer is connected to two series resistance shown in shown in figure. The output voltage (V_0) is :



- (1) 7 V (2) 15 V
(3) 44 V (4) 22 V

Ans. (1)

Sol. $\frac{\epsilon_1}{\epsilon_2} = \frac{N_1}{N_2} = \frac{100}{10} \Rightarrow \epsilon_2 = 22 \text{ V}$

$$I = \frac{22}{22 \times 10^3} = 1 \text{ mA}, V_0 = 7 \text{ V}$$

- 42.** The gravitational potential at a point above the surface of earth is $-5.12 \times 10^7 \text{ J/kg}$ and the acceleration due to gravity at that point is 6.4 m/s^2 . Assume that the mean radius of earth to be 6400 km. The height of this point above the earth's surface is :

- (1) 1600 km
(2) 540 km
(3) 1200 km
(4) 1000 km

Ans. (1)

Sol. $-\frac{GM_E}{R_E + h} = -5.12 \times 10^7 \dots (i)$

$$\frac{GM_E}{(R_E + h)^2} = 6.4 \dots (ii)$$

By (i) and (ii)

$$\Rightarrow h = 16 \times 10^5 \text{ m} = 1600 \text{ km}$$

- 43.** An electric toaster has resistance of 60Ω at room temperature (27°C). The toaster is connected to a 220 V supply. If the current flowing through it reaches 2.75 A, the temperature attained by toaster is around : (if $\alpha = 2 \times 10^{-4} / ^\circ\text{C}$)

- (1) 694°C
(2) 1235°C
(3) 1694°C
(4) 1667°C

Ans. (3)

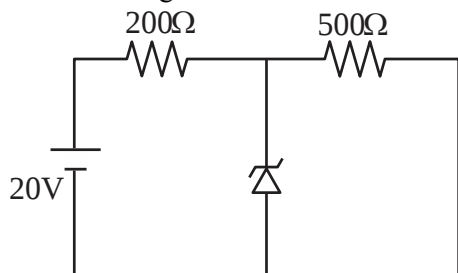
Sol. $R_{T=27} = 60 \Omega, R_T = \frac{220}{2.75} = 80 \Omega$

$$R = R_0 (1 + \alpha \Delta T)$$

$$80 = 60 [1 + 2 \times 10^{-4} (T - 27)]$$

$$T \approx 1694^\circ\text{C}$$

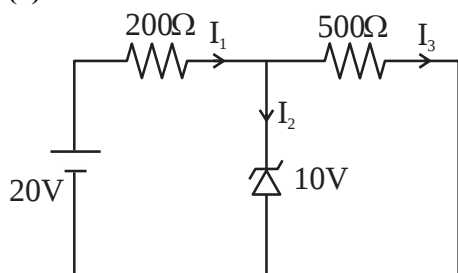
44. A Zener diode of breakdown voltage 10V is used as a voltage regulator as shown in the figure. The current through the Zener diode is



- (1) 50 mA (2) 0
(3) 30 mA (4) 20 mA

Ans. (3)

Sol.



Zener is in breakdown region.

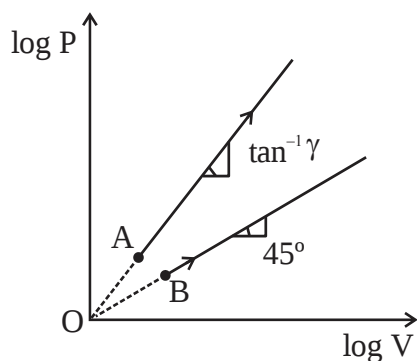
$$I_3 = \frac{10}{500} = \frac{1}{50}$$

$$I_1 = \frac{10}{200} = \frac{1}{20}$$

$$I_2 = I_1 - I_3$$

$$I_2 = \left(\frac{1}{20} - \frac{1}{50} \right) = \left(\frac{3}{100} \right) = 30 \text{ mA}$$

45. Two thermodynamical process are shown in the figure. The molar heat capacity for process A and B are C_A and C_B . The molar heat capacity at constant pressure and constant volume are represented by C_P and C_V , respectively. Choose the correct statement.



- (1) $C_B = \infty, C_A = 0$
(2) $C_A = 0$ and $C_B = \infty$
(3) $C_P > C_V > C_A = C_B$
(4) $C_A > C_P > C_V$

Ans. (Bonus)

Sol. For process A

$$\log P = \gamma \log V \Rightarrow P = V^\gamma, (\gamma > 1)$$

$$PV^{-\gamma} = \text{Constant}$$

$$C_A = C_V + \frac{R}{1+\gamma} \dots (i)$$

Likewise for process B $\rightarrow PV^{-1} = \text{Constant}$

$$C_B = C_V + \frac{R}{1+1}$$

$$C_B = C_V + \frac{R}{2} \dots (ii)$$

$$C_P = C_V + R \dots (iii)$$

By (i), (ii) & (iii)

$$C_P > C_B > C_A > C_V \text{ [No answer matching]}$$

46. The electrostatic potential due to an electric dipole at a distance 'r' varies as :

- (1) r (2) $\frac{1}{r^2}$
(3) $\frac{1}{r^3}$ (4) $\frac{1}{r}$

Ans. (2)

Sol. $V = \frac{kP \cos \theta}{r^2}$

& can also checked dimensionally

47. A spherical body of mass 100 g is dropped from a height of 10 m from the ground. After hitting the ground, the body rebounds to a height of 5m. The impulse of force imparted by the ground to the body is given by : (given $g = 9.8 \text{ m/s}^2$)

- (1) 4.32 kg ms^{-1} (2) 43.2 kg ms^{-1}
(3) 23.9 kg ms^{-1} (4) 2.39 kg ms^{-1}

Ans. (4)

Sol. $\vec{I} = \Delta \vec{P} = \vec{P}_f - \vec{P}_i$

$$M = 0.1 \text{ kg}$$

$$I = \Delta P = 0.1 \left(\sqrt{2 \times 9.8 \times 5} - \left(-\sqrt{2 \times 9.8 \times 10} \right) \right)$$

$$= 0.1 (14 + 7\sqrt{2}) \approx 2.39 \text{ kg ms}^{-1}$$

48. A particle of mass m projected with a velocity ' u ' making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is :

(1) $\frac{\sqrt{3}}{16} \frac{mu^3}{g}$ (2) $\frac{\sqrt{3}}{2} \frac{mu^2}{g}$
 (3) $\frac{mu^3}{\sqrt{2}g}$ (4) zero

Ans. (1)

Sol. $L = mu \cos \theta H$

$$= mu \cos \theta \times \frac{u^2 \sin^2 \theta}{2g}$$

$$= \frac{mu^3}{2g} \times \frac{\sqrt{3}}{2} \times \left(\frac{1}{2}\right)^2 = \frac{\sqrt{3}mu^3}{16g}$$

49. At which temperature the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C ?

(1) 80 K (2) -73 K
 (3) 4 K (4) 20 K

Ans. (4)

Sol. $\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R(320)}{32}}$

$$T = \frac{320}{16} = 20 \text{ K}$$

50. A series L,R circuit connected with an ac source $E = (25 \sin 1000 t) \text{ V}$ has a power factor of $\frac{1}{\sqrt{2}}$. If

the source of emf is changed to $E = (20 \sin 2000 t) \text{ V}$, the new power factor of the circuit will be :

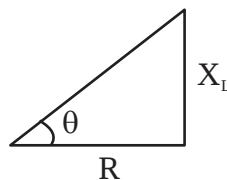
(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{3}}$
 (3) $\frac{1}{\sqrt{5}}$ (4) $\frac{1}{\sqrt{7}}$

Ans. (3)

Sol. $E = 25 \sin (1000 t)$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

LR circuit



$$\text{Initially } \frac{R}{\omega_1 L} = \frac{1}{\tan \theta} = \frac{1}{\tan 45^\circ} = 1$$

$$X_L = \omega_1 L$$

$$\omega_2 = 2\omega_1, \text{ given}$$

$$\tan \theta' = \frac{\omega_2 L}{R} = \frac{2\omega_1 L}{R}$$

$$\tan \theta' = 2$$

$$\cos \theta' = \frac{1}{\sqrt{5}}$$

SECTION-B

51. The horizontal component of earth's magnetic field at a place is $3.5 \times 10^{-5} \text{ T}$. A very long straight conductor carrying current of $\sqrt{2} \text{ A}$ in the direction from South east to North West is placed. The force per unit length experienced by the conductor is $\times 10^{-6} \text{ N/m}$.

Ans. (35)

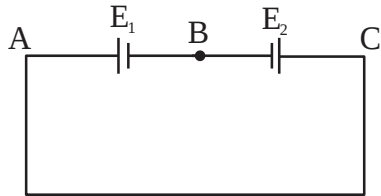
Sol. $B_H = 3.5 \times 10^{-5} \text{ T}$

$$F = i \ell B \sin \theta, \quad i = \sqrt{2} \text{ A}$$

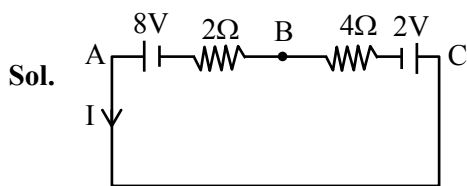
$$\frac{F}{\ell} = i B \sin \theta = \sqrt{2} \times 3.5 \times 10^{-5} \times \frac{1}{\sqrt{2}}$$

$$= 35 \times 10^{-6} \text{ N/m}$$

52. Two cells are connected in opposition as shown. Cell E_1 is of 8 V emf and 2Ω internal resistance; the cell E_2 is of 2 V emf and 4Ω internal resistance. The terminal potential difference of cell E_2 is:



Ans. (6)



$$I = \frac{8-2}{2+4} = \frac{6}{6} = 1A$$

Applying Kirchhoff from C to B

$$V_C - 2 - 4 \times 1 = V_B$$

$$V_C - V_B = 6V$$

$$= 6V$$

53. A electron of hydrogen atom on an excited state is having energy $E_n = -0.85$ eV. The maximum number of allowed transitions to lower energy level is

Ans. (6)

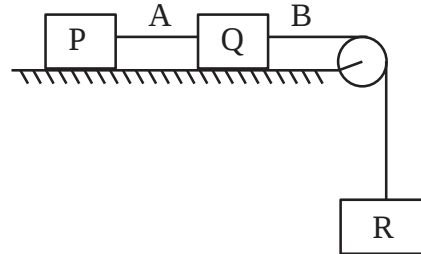
Sol. $E_n = -\frac{13.6}{n^2} = -0.85$

$$\Rightarrow n = 4$$

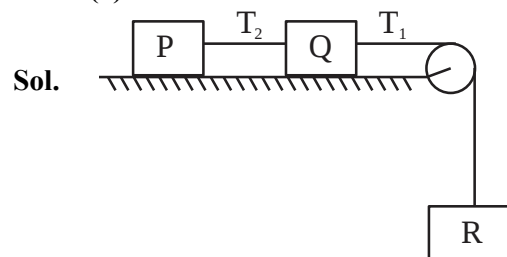
No of transition

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

54. Each of three blocks P, Q and R shown in figure has a mass of 3 kg. Each of the wire A and B has cross-sectional area 0.005 cm^2 and Young's modulus $2 \times 10^{11} \text{ N m}^{-2}$. Neglecting friction, the longitudinal strain on wire B is $\text{_____} \times 10^{-4}$. (Take $g = 10 \text{ m/s}^2$)



Ans. (2)



Sol.

$$a = \frac{10}{3} \text{ m/s}^2$$

$$30 - T_1 = 3 \times a$$

$$T_1 = 20 \text{ N}$$

$$\text{strain} = \frac{\text{stress}}{Y}$$

$$= 2 \times 10^{-4}$$

55. The distance between object and its two times magnified real image as produced by a convex lens is 45 cm. The focal length of the lens used is _____ cm.

Ans. (10)

Sol. $\frac{v}{u} = -2$

$$v = -2u \text{ ... (i)}$$

$$v - u = 45 \text{ ... (ii)}$$

$$\Rightarrow u = -15 \text{ cm}$$

$$v = 30 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$f = +10 \text{ cm}$$

56. The displacement and the increase in the velocity of a moving particle in the time interval of t to $(t + 1)$ s are 125 m and 50 m/s, respectively. The distance travelled by the particle in $(t + 2)^{\text{th}}$ s is ____ m.

Ans. (175)

Sol. Considering acceleration is constant

$$v = u + at$$

$$u + 50 = u + a \Rightarrow a = 50 \text{ m/s}^2$$

$$125 = ut + \frac{1}{2}at^2$$

$$125 = u + \frac{a}{2}$$

$$\Rightarrow u = 100 \text{ m/s}$$

$$\therefore S_{n^{\text{th}}} = u + \frac{a}{2}[2n - 1]$$

$$= 175 \text{ m}$$

57. A capacitor of capacitance C and potential V has energy E . It is connected to another capacitor of capacitance $2C$ and potential $2V$. Then the loss of energy is $\frac{x}{3}E$, where x is ____.

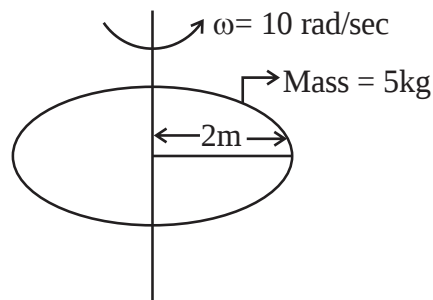
Ans. (2)

$$\text{Sol. Energy loss} = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$

$$= \frac{2}{3}E$$

$$\therefore x = 2$$

58. Consider a Disc of mass 5 kg, radius 2m, rotating with angular velocity of 10 rad/s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is ____ J.



Ans. (250)

$$\text{Sol. } \vec{L}_i = I\omega_i = \frac{MR^2}{2} \cdot \omega = 100 \text{ kgm}^2/\text{s}$$

$$E_i = \frac{1}{2} \cdot \frac{MR^2}{2} \cdot \omega^2 = 500 \text{ J}$$

$$\vec{L}_i = \vec{L}_f \Rightarrow 100 = 2I\omega_f$$

$$\omega_f = 5 \text{ rad/sec}$$

$$E_f = 2 \times \frac{1}{2} \cdot \frac{5(2)^2}{2} \cdot (5)^2 = 250 \text{ J}$$

$$\Delta E = 250 \text{ J}$$

59. In a closed organ pipe, the frequency of fundamental note is 30 Hz. A certain amount of water is now poured in the organ pipe so that the fundamental frequency is increased to 110 Hz. If the organ pipe has a cross-sectional area of 2 cm^2 , the amount of water poured in the organ tube is ____ g. (Take speed of sound in air is 330 m/s)

Ans. (400)

Sol. $\frac{V}{4\ell_1} = 30 \Rightarrow \ell_1 = \frac{11}{4}m$

$$\frac{V}{4\ell_2} = 110 \Rightarrow \ell_2 = \frac{3}{4}m$$

$$\Delta\ell = 2m,$$

Change in volume = $A\Delta\ell = 400\text{ cm}^3$

M = 400 g ; $(\because \rho = 1\text{ g / cm}^3)$

60. A ceiling fan having 3 blades of length 80 cm each is rotating with an angular velocity of 1200 rpm. The magnetic field of earth in that region is 0.5 G and angle of dip is 30° . The emf induced across the blades is $N\pi \times 10^{-5}\text{ V}$. The value of N is _____ .

Ans. (32)

Sol. $B_v = B \sin 30 = \frac{1}{4} \times 10^{-4}$

$$\omega = 2\pi \times f = \frac{2\pi}{60} \times 1200 \text{ rad / s}$$

$$\varepsilon = \frac{1}{2} B_v \omega \ell^2$$

$$= 32\pi \times 10^{-5} \text{ V}$$

CHEMISTRY

SECTION-A

61. Given below are two statements:

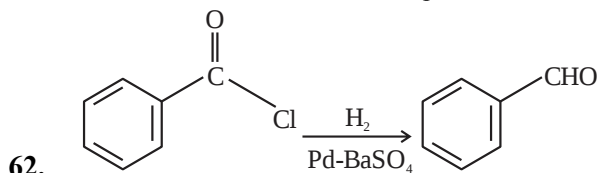
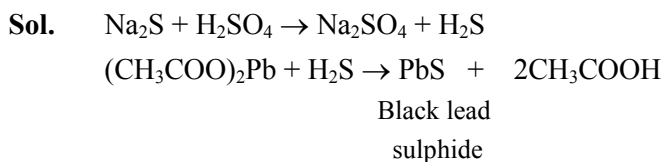
Statement-I: The gas liberated on warming a salt with dil H_2SO_4 , turns a piece of paper dipped in lead acetate into black, it is a confirmatory test for sulphide ion.

Statement-II: In statement-I the colour of paper turns black because of formation of lead sulphite.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Both Statement-I and Statement-II are false
- (2) Statement-I is false but Statement-II is true
- (3) Statement-I is true but Statement-II is false
- (4) Both Statement-I and Statement-II are true.

Ans. (3)

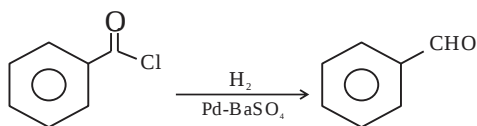


This reduction reaction is known as:

- (1) Rosenmund reduction
- (2) Wolff-Kishner reduction
- (3) Stephen reduction
- (4) Etard reduction

Ans. (1)

Sol.



It is known as rosenmund reduction that is the partial reduction of acid chloride to aldehyde

TEST PAPER WITH SOLUTION

63. Sugar which does not give reddish brown precipitate with Fehling's reagent is:

- | | |
|-------------|-------------|
| (1) Sucrose | (2) Lactose |
| (3) Glucose | (4) Maltose |

Ans. (1)

Sol. Sucrose do not contain hemiacetal group.

Hence it does not give test with Fehling solution.

While all other give positive test with Fehling solution

64. Given below are the two statements: one is labeled as Assertion (A) and the other is labeled as Reason (R).

Assertion (A): There is a considerable increase in covalent radius from N to P. However from As to Bi only a small increase in covalent radius is observed.

Reason (R): covalent and ionic radii in a particular oxidation state increases down the group.

In the light of the above statement, choose the most appropriate answer from the options given below:

- (1) (A) is false but (R) is true
- (2) Both (A) and (R) are true but (R) is not the correct explanation of (A)
- (3) (A) is true but (R) is false
- (4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Ans. (2)

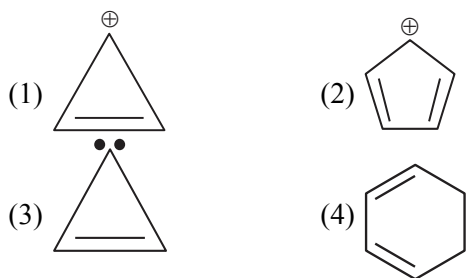
Sol. According to NCERT,

Statement-I : Factual data,

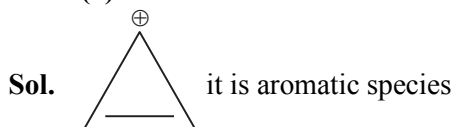
Statement-II is true.

But correct explanation is presence of completely filled d and f-orbitals of heavier members

65. Which of the following molecule/species is most stable?



Ans. (1)



66. Diamagnetic Lanthanoid ions are:

- (1) Nd^{3+} and Eu^{3+} (2) La^{3+} and Ce^{4+}
 (3) Nd^{3+} and Ce^{4+} (4) Lu^{3+} and Eu^{3+}

Ans. (2)

Sol. Ce : $[\text{Xe}] 4f^1 5d^1 6s^2$; Ce^{4+} diamagnetic
 La : $[\text{Xe}] 4f^0 5d^1 6s^2$; La^{3+} diamagnetic

67. Aluminium chloride in acidified aqueous solution forms an ion having geometry

- (1) Octahedral
 (2) Square Planar
 (3) Tetrahedral
 (4) Trigonal bipyramidal

Ans. (1)

Sol. AlCl_3 in acidified aqueous solution forms octahedral geometry $[\text{Al}(\text{H}_2\text{O})_6]^{3+}$

68. Given below are two statements:

Statement-I: The orbitals having same energy are called as degenerate orbitals.

Statement-II: In hydrogen atom, 3p and 3d orbitals are not degenerate orbitals.

In the light of the above statements, choose the **most appropriate** answer from the options given

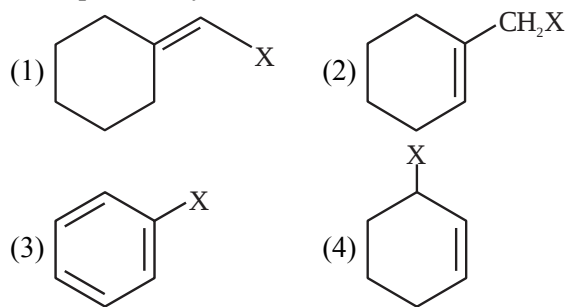
- (1) Statement-I is true but Statement-II is false
 (2) Both Statement-I and Statement-II are true.
 (3) Both Statement-I and Statement-II are false
 (4) Statement-I is false but Statement-II is true

Ans. (1)

Sol. For single electron species the energy depends upon principal quantum number 'n' only. So, statement II is false.

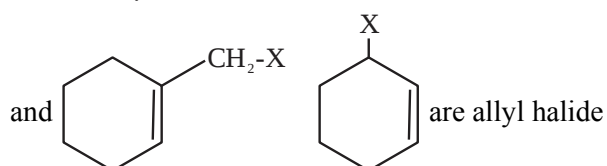
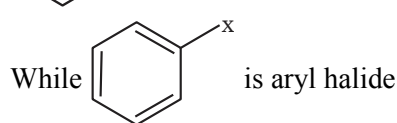
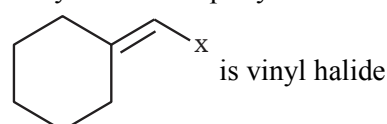
Statement I is correct definition of degenerate orbitals.

69. Example of vinylic halide is

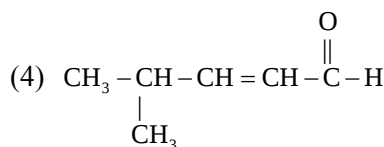
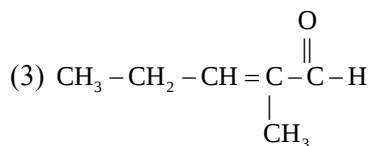
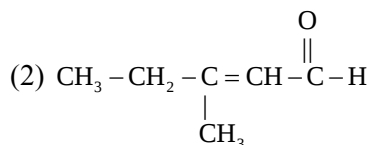
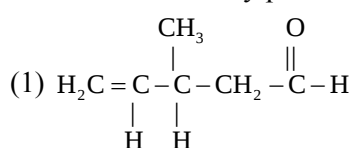


Ans. (1)

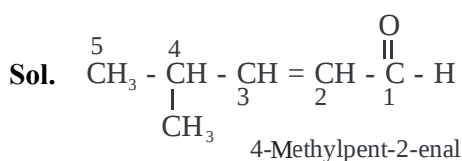
Sol. Vinyl carbon is sp^2 hybridized aliphatic carbon



70. Structure of 4-Methylpent-2-enal is



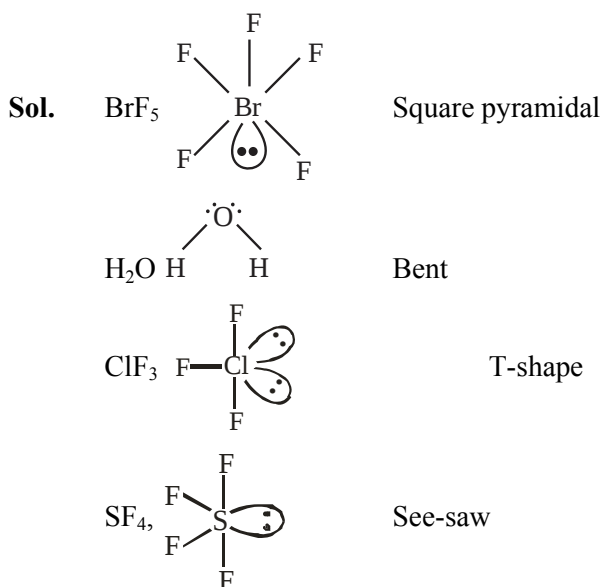
Ans. (4)



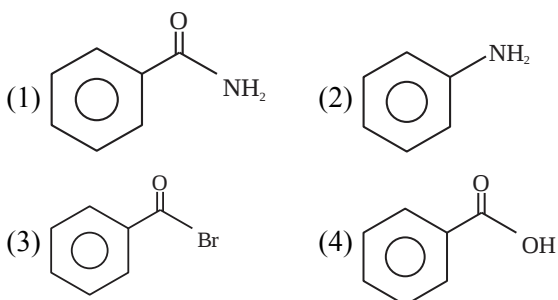
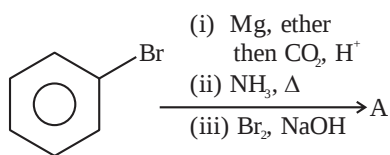
71. Match List-I with List-II

List-I	List-II
Molecule	Shape
(A) BrF_5	(I) T-shape
(B) H_2O	(II) See saw
(C) ClF_3	(III) Bent
(D) SF_4	(IV) Square pyramidal
(1) (A)-I, (B)-II, (C)-IV, (D)-III	
(2) (A)-II, (B)-I, (C)-III, (D)-IV	
(3) (A)-III, (B)-IV, (C)-I, (D)-II	
(4) (A)-IV, (B)-III, (C)-I, (D)-II	

Ans. (4)

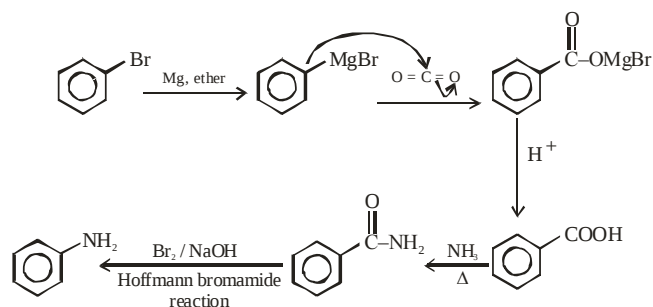


72. The final product A, formed in the following multistep reaction sequence is:

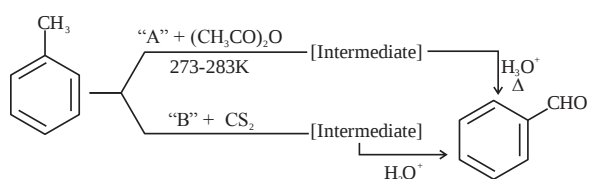


Ans. (2)

Sol.



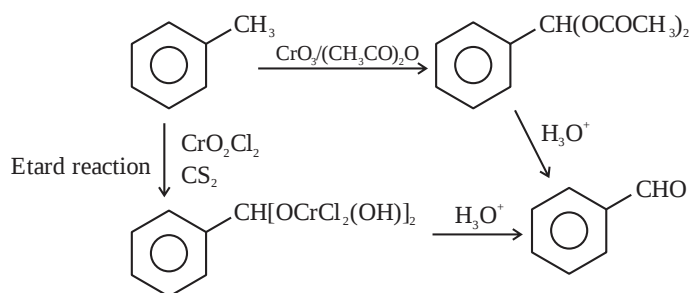
73. In the given reactions identify the reagent A and reagent B



- | | |
|----------------------------------|------------------------------|
| (1) A- CrO_3 | B- CrO_3 |
| (2) A- CrO_3 | B- CrO_2Cl_2 |
| (3) A- CrO_2Cl_2 | B- CrO_2Cl_2 |
| (4) A- CrO_2Cl_2 | B- CrO_3 |

Ans. (2)

Sol.



74. Given below are two statements one is labeled as **Assertion (A)** and the other is labeled as **Reason (R)**.

Assertion (A): $\text{CH}_2 = \text{CH} - \text{CH}_2 - \text{Cl}$ is an example of allyl halide

Reason (R): Allyl halides are the compounds in which the halogen atom is attached to sp^2 hybridised carbon atom.

In the light of the two above statements, choose the **most appropriate** answer from the options given below:

- (1) (A) is true but (R) is false
- (2) Both (A) and (R) are true but (R) is **not** the correct explanation of (A)
- (3) (A) is false but (R) is true
- (4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Ans. (1)

Sol. $\text{CH}_2 = \text{CH} - \text{CH}_2 - \text{Cl}$



It is allyl carbon and sp^3 hybridized

75. What happens to freezing point of benzene when small quantity of naphthalene is added to benzene?

- (1) Increases
- (2) Remains unchanged
- (3) First decreases and then increases
- (4) Decreases

Ans. (4)

Sol. On addition of naphthalene to benzene there is depression in freezing point of benzene.

76. Match List-I with List-II

List-I	List-II
Species	Electronic distribution
(A) Cr^{+2}	(I) 3d^8
(B) Mn^+	(II) $3\text{d}^3 4\text{s}^1$
(C) Ni^{+2}	(III) 3d^4
(D) V^+	(IV) $3\text{d}^5 4\text{s}^1$

Choose the correct answer from the options given below:

- (1) (A)-I, (B)-II, (C)-III, (D)-IV
- (2) (A)-III, (B) - IV, (C) - I, (D)-II
- (3) (A)-IV, (B)-III, (C)-I, (D)-II
- (4) (A)-II, (B)-I, (C)-IV, (D)-III

Ans. (2)

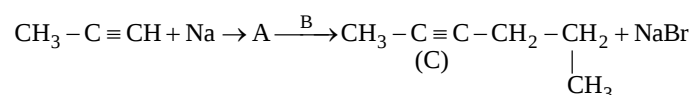
Sol. $_{24}\text{Cr} \rightarrow [\text{Ar}] 3\text{d}^5 4\text{s}^1$; $\text{Cr}^{2+} \rightarrow [\text{Ar}] 3\text{d}^4$

$_{25}\text{Mn} \rightarrow [\text{Ar}] 3\text{d}^5 4\text{s}^2$; $\text{Mn}^+ \rightarrow [\text{Ar}] 3\text{d}^5 4\text{s}^1$

$_{28}\text{Ni} \rightarrow [\text{Ar}] 3\text{d}^8 4\text{s}^2$; $\text{Ni}^{2+} \rightarrow [\text{Ar}] 3\text{d}^8$

$_{23}\text{V} \rightarrow [\text{Ar}] 3\text{d}^3 4\text{s}^2$; $\text{V}^+ \rightarrow [\text{Ar}] 3\text{d}^3 4\text{s}^1$

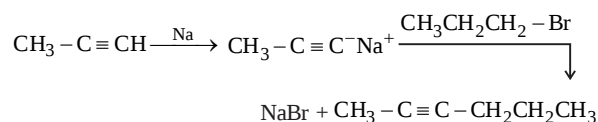
77. Compound A formed in the following reaction reacts with B gives the product C. Find out A and B.



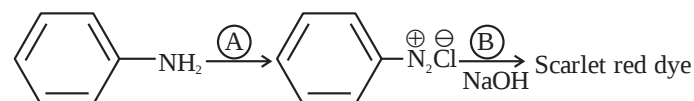
- (1) $\text{A} = \text{CH}_3 - \text{C} \equiv \text{CNa}^+$, $\text{B} = \text{CH}_3 - \text{CH}_2 - \text{CH}_2 - \text{Br}$
- (2) $\text{A} = \text{CH}_3 - \text{CH} = \text{CH}_2$, $\text{B} = \text{CH}_3 - \text{CH}_2 - \text{CH}_2 - \text{Br}$
- (3) $\text{A} = \text{CH}_3 - \text{CH}_2 - \text{CH}_3$, $\text{B} = \text{CH}_3 - \text{C} \equiv \text{CH}$
- (4) $\text{A} = \text{CH}_3 - \text{C} \equiv \text{CNa}^+$, $\text{B} = \text{CH}_3 - \text{CH}_2 - \text{CH}_3$

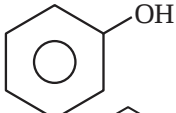
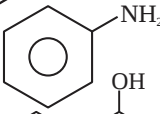
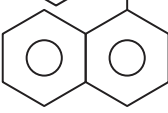
Ans. (1)

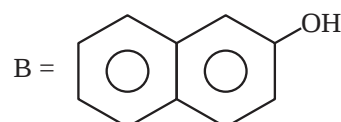
Sol.



78. Following is a confirmatory test for aromatic primary amines. Identify reagent (A) and (B)

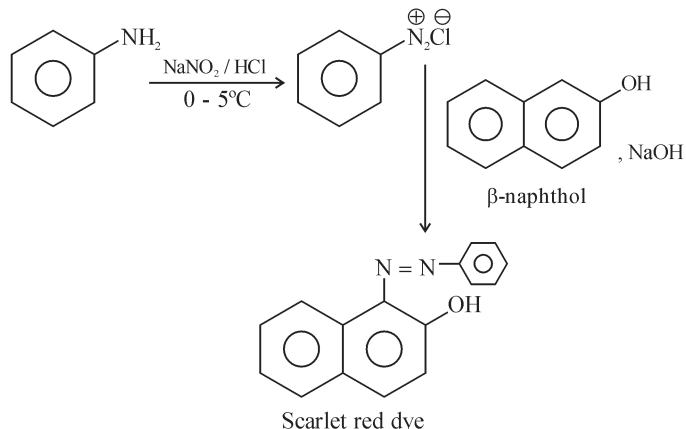


- (1) $\text{A} = \text{HNO}_3/\text{H}_2\text{SO}_4$; $\text{B} =$ 
- (2) $\text{A} = \text{NaNO}_2 + \text{HCl}$, $0 - 5^\circ\text{C}$; $\text{B} =$ 
- (3) $\text{A} = \text{NaNO}_2 + \text{HCl}$, $0 - 5^\circ\text{C}$; $\text{B} =$ 
- (4) $\text{A} = \text{NaNO}_2 + \text{HCl}$, $0 - 5^\circ\text{C}$;



Ans. (4)

Sol.



79. The Lassaigne's extract is boiled with dil HNO_3 before testing for halogens because,

- (1) AgCN is soluble in HNO_3
- (2) Silver halides are soluble in HNO_3
- (3) Ag_2S is soluble in HNO_3
- (4) Na_2S and NaCN are decomposed by HNO_3

Ans. (4)

Sol. If nitrogen or sulphur is also present in the compound, the sodium fusion extract is first boiled with concentrated nitric acid to decompose cyanide or sulphide of sodium during Lassaigne's test

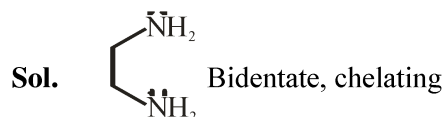
80. Choose the correct Statements from the following:

- (A) Ethane-1,2-diamine is a chelating ligand.
- (B) Metallic aluminium is produced by electrolysis of aluminium oxide in presence of cryolite.
- (C) Cyanide ion is used as ligand for leaching of silver.
- (D) Phosphine act as a ligand in Wilkinson catalyst.
- (E) The stability constants of Ca^{2+} and Mg^{2+} are similar with EDTA complexes.

Choose the correct answer from the options given below:

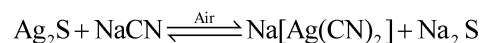
- (1) (B), (C), (E) only
- (2) (C), (D), (E) only
- (3) (A), (B), (C) only
- (4) (A), (D), (E) only

Ans. (3)



Based on Hall-Heroult's process

$[\text{Rh}(\text{PPh}_3)_3\text{Cl}]$ Wilkinson's catalyst



Ca^{++} ion forms more stable complex with EDTA

SECTION-B

81. The rate of first order reaction is $0.04 \text{ mol L}^{-1} \text{ s}^{-1}$ at 10 minutes and $0.03 \text{ mol L}^{-1} \text{ s}^{-1}$ at 20 minutes after initiation. Half life of the reaction is _____ minutes. (Given $\log 2 = 0.3010$, $\log 3 = 0.4771$)

Ans. (24)

Sol. $0.04 = k[A]_0 e^{-k \times 10 \times 60} \quad \dots(1)$

$0.03 = k[A]_0 e^{-k \times 20 \times 60} \quad \dots(2)$

$(1)/(2)$

$$\frac{4}{3} = e^{600k(2-1)}$$

$$\frac{4}{3} = e^{600k}$$

$$\ln \frac{4}{3} = 600k$$

$$\ln \frac{4}{3} = 600 \times \frac{\ln 2}{t_{1/2}}$$

$$t_{1/2} = 600 \times \frac{\ln 2}{\ln \frac{4}{3}} \text{ sec}$$

$$t_{1/2} = 600 \times \frac{\log 2}{\log 4 - \log 3} \text{ sec.} = 10 \times \frac{0.3010}{0.6020 - 0.477} \text{ min}$$

$$t_{1/2} = 24.08 \text{ min}$$

Ans. 24

82. The pH at which $\text{Mg}(\text{OH})_2$ [$K_{\text{sp}} = 1 \times 10^{-11}$] begins to precipitate from a solution containing 0.10 M Mg^{2+} ions is _____

Ans. (09)

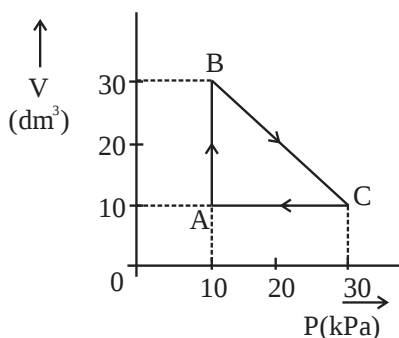
Sol. Precipitation when $Q_{\text{sp}} = K_{\text{sp}}$

$$[\text{Mg}^{2+}][\text{OH}^-]^2 = 10^{-11}$$

$$0.1 \times [\text{OH}^-]^2 = 10^{-11} \Rightarrow [\text{OH}^-] = 10^{-5}$$

$$\Rightarrow \text{pOH} = 5 \quad \Rightarrow \text{pH} = 9$$

83.



An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path $A \rightarrow B \rightarrow C \rightarrow A$ as shown in the diagram. The total work done in the process is _____ J.

Ans. (200)

Sol. Work done is given by area enclosed in the P vs V cyclic graph or V vs P cyclic graph.

Sign of work is positive for clockwise cyclic process for V vs P graph.

$$W = \frac{1}{2} \times (30 - 10) \times (30 - 10) = 200 \text{ kPa} - \text{dm}^3$$

$$= 200 \times 1000 \text{ Pa} - \text{L} = 2 \text{ L-bar} = 200 \text{ J}$$

84. if IUPAC name of an element is “Unununnium” then the element belongs to nth group of periodic table. The value of n is _____

Ans. (11)

Sol. 111 belongs to 11th group

85. The total number of molecular orbitals formed from 2s and 2p atomic orbitals of a diatomic molecule

Ans. (08)

Sol. Two molecular orbitals $\sigma 2s$ and $\sigma^* 2s$.

Six molecular orbitals $\sigma 2p_z$ and $\sigma^* 2p_z$.

$\pi 2p_x$, $\pi 2p_y$ and $\pi^* 2p_x$, $\pi^* 2p_y$

86. On a thin layer chromatographic plate, an organic compound moved by 3.5 cm, while the solvent moved by 5 cm. The retardation factor of the organic compound is _____ $\times 10^{-1}$

Ans. (07)

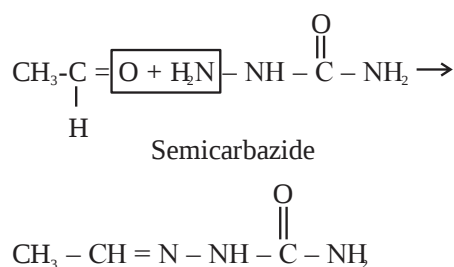
Sol. Retardation factor = $\frac{\text{Distance travelled by sample/organic compound}}{\text{Distance travelled by solvent}}$

$$= \frac{3.5}{5} = 7 \times 10^{-1}$$

87. The compound formed by the reaction of ethanal with semicarbazide contains _____ number of nitrogen atoms.

Ans. (03)

Sol.



88. 0.05 cm thick coating of silver is deposited on a plate of 0.05 m² area. The number of silver atoms deposited on plate are _____ $\times 10^{23}$. (At mass Ag = 108, d = 7.9 g cm⁻³)

Ans. (11)

Sol. Volume of silver coating = $0.05 \times 0.05 \times 10000$
= 25 cm³

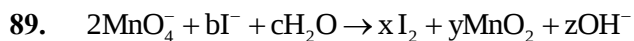
Mass of silver deposited = $25 \times 7.9 \text{ g}$

$$\text{Moles of silver atoms} = \frac{25 \times 7.9}{108}$$

$$\text{Number of silver atoms} = \frac{25 \times 7.9}{108} \times 6.023 \times 10^{23}$$

$$= 11.01 \times 10^{23}$$

Ans. 11



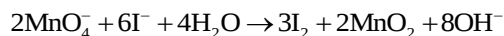
If the above equation is balanced with integer coefficients, the value of z is _____

Ans. (08)

Sol. **Reduction Half** **Oxidation Half**



Adding oxidation half and reduction half, net reaction is



$$\Rightarrow \text{z} = 8$$

$$\Rightarrow \text{Ans } 8$$

90. The mass of sodium acetate (CH_3COONa) required to prepare 250 mL of 0.35 M aqueous solution is _____ g. (Molar mass of CH_3COONa is 82.02 g mol^{-1})

Ans. (7)

Sol. Moles = Molarity $\times$ Volume in litres

$$= 0.35 \times 0.25$$

Mass = moles $\times$ molar mass

$$= 0.35 \times 0.25 \times 82.02 = 7.18 \text{ g}$$

Ans. 7