

Physics

SECTION - A

1. Match List I with List II

List I	List II
A. Surface tension	I. $\text{kgm}^{-1} \text{s}^{-1}$
B. Pressure	II. kgms^{-1}
C. Viscosity	III. $\text{kgm}^{-1} \text{s}^{-2}$
D. Impulse	IV. kg s^{-2}

Choose the correct answer from the options given below:

(1) A-II , B-I , C-III , D-IV

(2) A-IV, B-III , C-I , D-II

(3) A-III , B-IV, C-I , D-II

(4) A-IV , B-III , C-II , D-I

Sol. 2

$$\text{Surface tension (S)} = \frac{F}{l} \rightarrow \text{kg} \frac{\text{M}}{\text{S}^2} \cdot \frac{1}{\text{M}} \rightarrow \text{Kg s}^{-2}$$

$$\text{Impulse (J)} = \int F dt \rightarrow \text{N} \cdot \text{s}$$

$$\rightarrow \text{Kg ms}^{-2} \cdot \text{s}$$

$$\rightarrow \text{Kg ms}^{-1}$$

$$\text{Pressure (P)} = \frac{F}{A} \rightarrow \text{Kgms}^{-2}, \text{m}^{-2}$$

$$\rightarrow \text{Kg ms}^{-1} \text{s}^{-2}$$

$$\text{Viscosity (}\eta\text{)} = \frac{F}{6\pi r v}$$

$$\rightarrow \frac{\text{kg ms}^{-2}}{\text{m} \cdot \text{ms}^{-1}}$$

$$\rightarrow \text{kg m}^{-1} \text{s}^{-1}$$

2. The ratio of the density of oxygen nucleus ($^{16}_8\text{O}$) and helium nucleus (^4_2He) is

(1) 4:1

(2) 2:1 (3) 1:1 (4) 8:1

Sol. 3

$$\rho = \frac{M}{V} \text{ and } V = \frac{4}{3}\pi r^3 \text{ when } r = R_0 A^{1/3}$$

$$\therefore \rho = \frac{M}{\frac{4}{3}\pi R_0^3 A}$$

$$\therefore \rho \propto \frac{M}{A}$$

$$\frac{\rho_{\text{O}}}{\rho_{\text{He}}} = \frac{M_{\text{O}}}{A_{\text{O}}} \times \frac{A_{\text{He}}}{M_{\text{He}}} = \frac{16}{8} \times \frac{2}{4} = 1$$

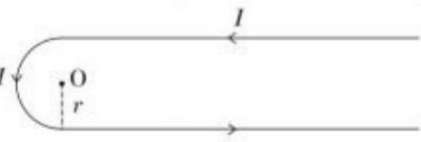
3. The root mean square velocity of molecules of gas is
- (1) Inversely proportional to square root of temperature $\left(\sqrt{\frac{1}{T}}\right)$
 - (2) Proportional to square of temperature (T^2)
 - (3) Proportional to temperature (T)
 - (4) Proportional to square root of temperature $(\sqrt{T})$

Sol. 4

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M_0}}$$

$$\therefore V_{\text{rms}} \propto \sqrt{T}$$

4. Match List I with List II

List I (Current configuration)	List II (Magnitude of Magnetic Field at point O)
A. 	I. $B_0 = \frac{\mu_0 I}{4\pi r} [\pi + 2]$
B. 	II. $B_0 = \frac{\mu_0}{4} \frac{I}{r}$
C. 	III. $B_0 = \frac{\mu_0 I}{2\pi r} [\pi - 1]$
D. 	IV. $B_0 = \frac{\mu_0 I}{4\pi r} [\pi + 1]$

Choose the correct answer from the options given below :

- (1) A-III, B-I, C-IV, D-II
- (2) A-I, B-III, C-IV, D-II
- (3) A-III, B-IV, C-I, D-II
- (4) A-II, B-I, C-IV, D-III

Sol. 1

$$\begin{aligned}
 \text{(A)} \quad B &= \frac{\mu_0 I}{4\pi r} \times 2 - \frac{\mu_0 I}{2r} \\
 &= \frac{\mu I}{2r} \left(\frac{1}{\pi} - 1 \right)
 \end{aligned}$$

$$= \frac{\mu I}{2\pi r} (1 - \pi) \odot$$

$$= \frac{\mu I}{2\pi r} (\pi - 1) \otimes$$

$$(B) \quad B = \frac{\mu_0 I}{4\pi r} \times \pi + \frac{\mu_0 I}{4\pi r} \times 2$$

$$= \frac{\mu_0 I}{4\pi r} (\pi + 2) \odot$$

$$(C) \quad B = \frac{\mu_0 I}{4\pi r} \cdot \pi + 0 + \frac{\mu_0 I}{4\pi r}$$

$$= \frac{\mu_0 I}{4\pi r} (\pi + 1) \otimes$$

$$(D) \quad B = \frac{\mu_0 I}{4r} \odot$$

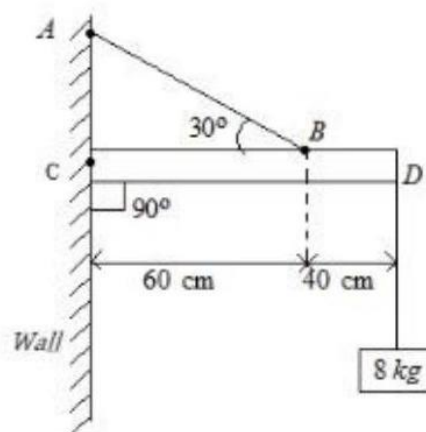
5. A message signal of frequency 5kHz is used to modulate a carrier signal of frequency 2MHz. The bandwidth for amplitude modulation is:

(1) 20 kHz (2) 5kHz (3) 10kHz (4) 2.5kHz

Sol. 3

Bandwidth = $2 \times$ highest of base band frequency
 $= 2 \times 5 = 10 \text{ kHz}$

6. An object of mass 8 kg hanging from one end of a uniform rod CD of mass 2 kg and length 1m pivoted at its end C on a vertical walls as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is:
 (Take $g = 10 \text{ m/s}^2$)



(1) 90 N (2) 30 N (3) 300 N (4) 240 N

Sol. 3

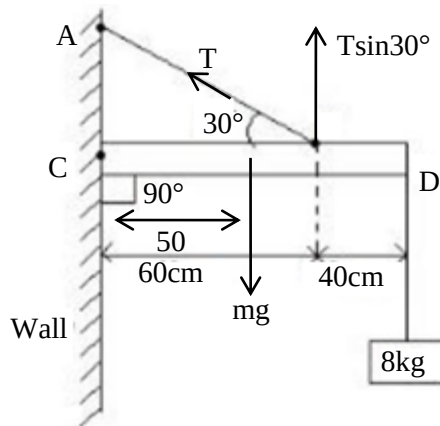
The rod is in equilibrium. So, net torque about any point will be zero.

$$\tau_c = 0$$

$$Mg \times 50 + 80 \times 100 = T \sin 30^\circ \times 30$$

$$20 \times 50 + 80 \times 100 = \frac{T}{2} \times 60$$

$$T = \frac{900}{3} = 300 \text{ N}$$



7. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Photodiodes are used in forward bias usually for measuring the light intensity.

Reason R: For a p-n junction diode, at applied voltage

V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm V \geq |V_0|$ where V_0 is the threshold voltage and V_z is the breakdown voltage.

In the light of the above statements, choose the correct answer from the options given below

- (1) Both A and R are true and R is correct explanation A
- (2) A is false but R is true
- (3) Both A and R are true but R is NOT the correct explanation A
- (4) A is true but R is false

Sol. 2

Photo diodes are not used in forward bias.

8. In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency ω_0 . The value of x is:

- (1) 4
- (2) 1/16
- (3) 16
- (4) $\frac{1}{4}$

Sol. 4

$$\omega = \frac{1}{\sqrt{LC}}$$

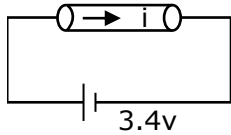
$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{L_2 C_2}{L_1 C_1}} = \sqrt{\frac{2L \cdot 8C}{L \cdot C}} = 4$$

$$\omega_2 = \frac{\omega_1}{4} = \frac{\omega_0}{4}$$

9. A uniform metallic wire carries a current 2A, when 3.4 V battery is connected across it. The mass of uniform metallic wires is 8.92×10^{-3} kg density is 8.92×10^3 kg/m³ and resistivity is $1.7 \times 10^{-8} \Omega - \text{m}$. The length of wire is:

- (1) $l = 10$ m (2) $l = 100$ m (3) $l = 5$ m (4) $l = 6.8$ m

Sol. 1



Given, $i = 2\text{A}$

$$v = 3.4 \text{ v}$$

$$v = iR$$

$$R = \frac{v}{i} = \frac{3.4}{2} = 1.7\Omega$$

$$\text{volume} = \frac{\text{mass}}{\text{Density}} = \frac{8.92 \times 10^{-3}}{8.92 \times 10^3} \text{m}^3 = 10^{-6} \text{m}^3$$

$$\Rightarrow A\ell = 10^{-6} \text{m}^3 \quad \text{--- (i)}$$

$$R = \frac{\rho\ell}{A}$$

$$\Rightarrow \frac{\rho}{R} = \frac{A}{\ell}$$

$$\frac{1.7 \times 10^{-8}}{1.7} = \frac{A}{\ell}$$

$$\frac{A}{\ell} = 10^{-8} \quad \text{--- (ii)}$$

$$\frac{\text{eq(i)}}{\text{eq(ii)}}$$

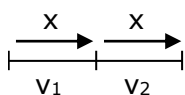
$$\ell^2 = 10^2$$

$$\ell = 10 \text{ m}$$

10. A car travels a distance of 'x' with speed v_1 and then same distance 'x' with speed v_2 in the same direction. The average speed of the car is:

- (1) $\frac{2v_1v_2}{v_1+v_2}$ (2) $\frac{2x}{v_1+v_2}$ (3) $\frac{v_1v_2}{2(v_1+v_2)}$ (4) $\frac{v_1+v_2}{2}$

Sol. 1



$$v_{\text{avg}} = \frac{\text{total Distance}}{\text{Total Time}}$$

$$= \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$$

$$= \frac{2v_1 v_2}{v_1 + v_2}$$

11. A car is moving with a constant speed of 20 m/s in a circular horizontal track of radius 40m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be: (Take $g = 10 \text{ m/s}^2$)

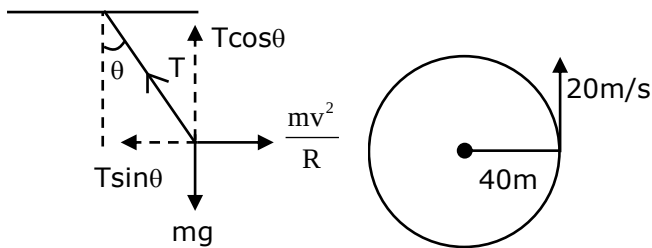
(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{6}$

Sol. 3



$$T \cos \theta = mg \quad \text{--- (i)}$$

$$T \sin \theta = \frac{mv^2}{R} \quad \text{--- (ii)}$$

$$\frac{\text{eq(i)}}{\text{eq(ii)}}; \frac{\cos \theta}{\sin \theta} = \frac{gR}{v^2}$$

$$\tan \theta = \frac{v^2}{Rg} = \frac{400}{40 \times 10} = 1$$

$$\theta = \frac{\pi}{4}$$

12. A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C . How long it will take to cool from 75°C to 69°C ?

(1) 1 minute

(2) 1.4 minutes

(3) 0.5 minute

(4) 2 minutes

Sol. 2

According to NLC,

$$-\frac{d\theta}{dt} = k\theta$$

$$\frac{12}{2} = K \left(\frac{98 + 86}{2} - 22 \right)$$

$$\Rightarrow 6 = K (92 - 22) = K \times 70$$

$$\Rightarrow K = \frac{6}{70} \quad \text{.... (i)}$$

$$\begin{aligned}\text{Now, } \frac{6}{t_2} &= \frac{6}{70} \left(\frac{75+69}{2} - 22 \right) \\ &= \frac{6}{70} \times (72 - 22) \\ t_2 &= \frac{6 \times 70}{6 \times 50} \\ \frac{7}{5} &= 1.4 \text{ min}\end{aligned}$$

13. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2m long and 0.2m in diameter. The magnetic intensity at the center of the solenoid when a current of 2A flows through it is?

- (1) $2.4 \times 10^3 \text{ A m}^{-1}$ (2) $1.2 \times 10^3 \text{ A m}^{-1}$
(3) $2.4 \times 10^{-3} \text{ A m}^{-1}$ (4) 1 A m^{-1}

Sol. 2

$$B = \mu_0 n I \text{ and } n = \frac{1200}{2} = 600$$

$$\text{Magnetic field intensity } H = \frac{B}{\mu_0} = nI = 600 \times 2 = 1200 = 1.2 \times 10^3 \text{ A m}^{-1}$$

14. In Young's double slits experiment, the position of 5th bright fringe from the central maximum is 5cm. The distance between slits and screen is 1m and wavelength of used monochromatic light is 600 nm. The separation between the slits is:

- (1) $48\mu\text{m}$ (2) $36\mu\text{m}$ (3) $12\mu\text{m}$ (4) $60\mu\text{m}$

Sol. 4

$$5\beta = 5 \text{ cm} \\ \Rightarrow \beta = 1 \text{ cm}$$

$$\frac{\lambda D}{d} = 1 \text{ cm} = \frac{1}{100} \text{ m}$$

$$\begin{aligned}\Rightarrow d &= 600 \times 10^{-9} \times 100 \times 1 \\ &= 60 \times 10^{-6} \text{ m} \\ &= 60 \mu\text{m}\end{aligned}$$

15. An electromagnetic wave is transporting energy in the negative z direction. At a certain point and certain time the direction of electric field of the wave is along positive y direction. What will be the direction of the magnetic field of the wave at the point and instant?

- (1) Negative direction of y (2) Positive direction of z
(3) Positive direction of x (4) Negative direction of x

Sol. 3

$$\vec{B} \perp \vec{r} \quad \vec{E} \quad \text{and} \quad \text{Direction of propagation is given by} \\ \vec{E} \times \vec{B}.$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

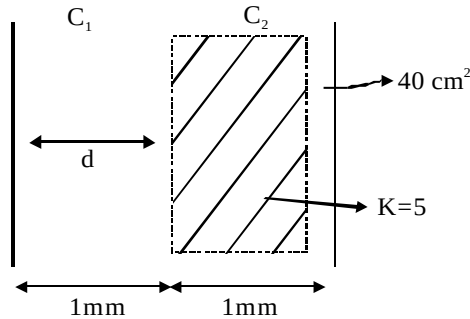
16. A parallel plate capacitor has plate area 40 cm^2 and plates separation 2 mm . The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5 . The capacitance of the system is:
- (1) $24\epsilon_0 \text{ F}$ (2) $\frac{10}{3}\epsilon_0 \text{ F}$ (3) $\frac{3}{10}\epsilon_0 \text{ F}$ (4) $10\epsilon_0 \text{ F}$

Sol. 2

$$C_1 = \frac{\epsilon_0 A}{d} = C_0$$

$$C_2 = K \frac{\epsilon_0 A}{d} = K\epsilon_0$$

$$\begin{aligned} C_{eq} &= \frac{C_1 C_2}{C_1 + C_2} = \frac{C_0 \times KC_0}{(K+1)\epsilon_0} = \frac{KC_0}{K+1} \\ &= \frac{5 \times \epsilon_0 \times 40 \times 10^{-4}}{1 \times 10^{-3} \times 6} \\ &= \frac{10}{3} \epsilon_0 \text{ F} \end{aligned}$$



17. Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100 g . The time period of the motion of the particle will be (approximately) (Take $g = 10 \text{ m s}^{-2}$, radius of earth $= 6400 \text{ km}$)
- (1) 12 hours (2) 1 hour 40 minutes
(3) 24 hours (4) 1 hour 24 minutes

Sol. 4

Inside earth, force is given by $F = -\frac{GM_e m x}{R_e^3}$

And $g_0(\text{on surface of earth}) = \frac{GM_e}{R_e^2}$

$$\therefore F = -\frac{g_0 m}{R_e} x$$

$$\Rightarrow a = -\frac{g_0}{R_e} x$$

$$\omega = \sqrt{\frac{g_0}{R_e}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{R_e}{g_0}} = 2\pi \sqrt{\frac{6400 \times 10^3}{10}} = 2 \times 3.13 \times 8 \times 10^2 \text{ sec} = 5024 \text{ sec} = 1.4 \text{ hr}$$

$$T = 1.4 \text{ hr} = 1 \text{ hr } 24 \text{ minutes}$$

18. Electron beam used in an electron microscope, when accelerated by a voltage of 20 kV , has a de-Broglie wavelength of λ_0 . If the voltage is increased to 40 kV , then the de-Broglie wavelength associated with the electron beam would be:
- (1) $3\lambda_0$ (2) $\frac{\lambda_0}{2}$ (3) $\frac{\lambda_0}{\sqrt{2}}$ (4) $9\lambda_0$

Sol. 3

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{V}}$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}}$$

$$\Rightarrow \frac{\lambda_0}{\lambda_2} = \sqrt{\frac{40}{20}} = \sqrt{2}$$

$$\Rightarrow \lambda_2 = \frac{\lambda_0}{\sqrt{2}}$$

19. A Carnot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be :

(1) 300 K (2) 900 K (3) 1000 K (4) 360 K

Sol. 3

$$\eta = 1 - \frac{T_L}{T_H}$$

$$0.5 = 1 - \frac{T_L}{600} \Rightarrow T_L = (1 - 0.5) \times 600 \text{ K} = 300 \text{ K}$$

$$\text{Now } 0.7 = 1 - \frac{300}{T_2}$$

$$\frac{300}{T_2} = 0.3 \Rightarrow T_2 = \frac{300}{0.3} = 1000 \text{ K}$$

20. T is the time period of simple pendulum on the earth's surface. Its time Period becomes x T when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be:

(1) 4 (2) 2 (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Sol. 2

$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$

$$\text{And } g_{eff \text{ above earth's surface}} = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = \frac{g_0}{4}$$

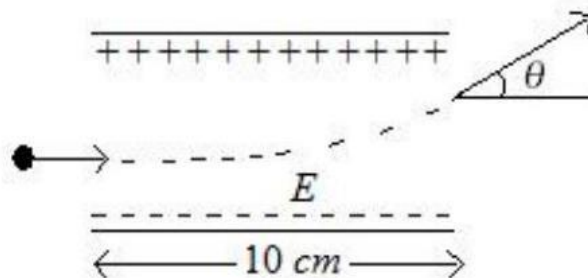
$$\text{Now } \frac{T_1}{T_2} = \sqrt{\frac{\frac{g_0}{4}}{g_0}} = \frac{1}{2}$$

$$T_2 = 2T_1$$

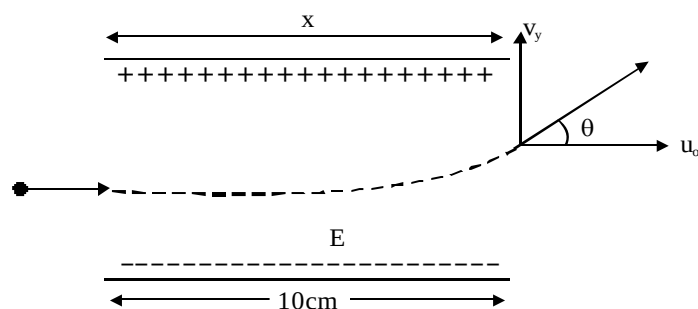
$$\therefore x = 2$$

SECTION – B

21. A uniform electric field of 10 N/C is created between two parallel charged plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5 eV. The length of each plate is 10 cm. The angle (θ) of deviation of the path of electron as it comes out of the field is _____ (in degree).



Sol. 45°



Force due to electric field is given by $\vec{F} = q\vec{E}$

$$\therefore F = eE$$

$$\Rightarrow a = \frac{eE}{m}$$

The electron will take a parabolic path i.e., projectile motion.

$$\text{Here, } s_x = 10\text{cm} = 0.1\text{m}$$

$$\therefore t = \frac{0.1}{u_x} \text{ --- (i)}$$

$$\text{Now } v_y = u_y + a_y t$$

$$\Rightarrow v_y = 0 + \frac{eE}{m} \times \frac{0.1}{u_x} \text{ --- (ii)}$$

$$\text{Also } KE = \frac{1}{2} mv^2 = \frac{1}{2} mu_x^2$$

$$mu_x^2 = 2 \times KE = 2 \times 0.5e = e \text{ --- (iii)}$$

$$\text{From eq (i), (ii) and (iii), } \tan\theta = \frac{v_y}{u_x} = \frac{eE}{m} \times \frac{0.1}{u_x} \times \frac{1}{u_x} = \frac{0.1eE}{mu_x^2} = \frac{0.1eE}{e} = 0.1 \times 10 = 1$$

$$\Rightarrow \tan\theta = 1$$

$$\therefore \theta = 45^\circ$$

22. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is _____.

Sol. 27

Bohr's energy is given by $E = -13.6 \times \frac{1}{n^2}$ for hydrogen atom.

$$\text{And } E = \frac{hc}{\lambda}$$

$$\text{For 1st condition, } \frac{hc}{\lambda_0} = 13.6 \left(\frac{1}{4} - \frac{1}{9} \right) = 13.6 \times \frac{5}{36} \text{ --- (i)}$$

$$\text{For 2nd condition, } \frac{hc}{\lambda} = 13.6 \left(\frac{1}{4} - \frac{1}{16} \right) = 13.6 \times \frac{3}{16} \text{ --- (ii)}$$

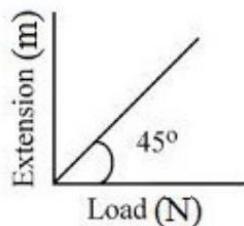
Dividing equation (i) by (ii),

$$\frac{\lambda}{\lambda_0} = \frac{5}{36} \times \frac{16}{3} = \frac{20}{27}$$

$$\Rightarrow \lambda = \frac{20}{27}\lambda_0$$

$$\Rightarrow n = 27$$

23. As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8cm and its diameter is 4 mm. The Young's modulus is found to be $x \times 10^4 \text{ Nm}^{-2}$. The value of x is _____



Sol. 5

$$\text{From graph, } \tan 45^\circ = \frac{\Delta l}{F}$$

$$\Rightarrow \frac{\Delta l}{F} = 1 \text{ --- (i)}$$

$$\text{Also, Young's modulus is given by } Y = \frac{FL}{A\Delta l} = \frac{l}{A} \times \frac{F}{\Delta l} = \frac{l}{A} \times 1$$

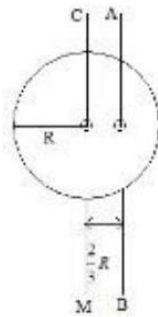
$$\therefore Y = \frac{l}{A} = \frac{62.8 \times 10^{-2}}{\pi \times 4 \times 10^{-6}} = 5 \times 10^4 \text{ Nm}^{-2}$$

$$\therefore x = 5$$

- 24 I_{CM} is the moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of disc. I_{AB} is its moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance $\frac{2}{3}R$ from center.

Where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is $x:9$.

The value of x is _____



Sol. 17

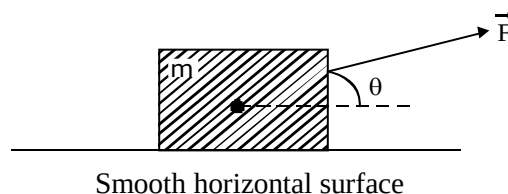
$$I_{CM} = \frac{MR^2}{2}$$

$$I_{AB} = \frac{MR^2}{2} + M\left(\frac{2}{3}R\right)^2 = \frac{MR^2}{2} + \frac{4MR^2}{9} = \frac{17MR^2}{18}$$

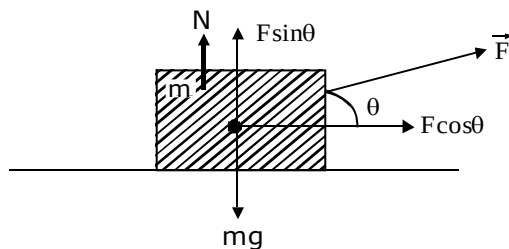
$$\text{As per question, } \frac{I_{AB}}{I_{CM}} = \frac{17}{9}$$

$$\therefore x = 17$$

25. An object of mass 'm' initially at rest on a smooth horizontal plane starts moving under the action of force $F = 2N$. In the process of its linear motion, the angle θ (as shown in figure) between the direction of force and horizontal varies as $\theta = kx$, where k is constant and x is the distance covered by the object from the initial position. The expression of kinetic energy of the object will be $E = \frac{n}{k} \sin \theta$, The value of n is _____.



Sol. 2



$$F_x = 2 \cos kx$$

$$F_y = 2 \sin kx - mg$$

According to Work Energy Theorem, $\Delta K = \Delta W$

Taking motion only along horizontal direction (X) i.e., linear motion as mentioned in question, $\Delta K = \int_0^x F_x dx$

$$K_f - K_i = \int_0^x 2 \cos kx dx = \frac{2 \sin kx}{k}$$

$$\text{Hence } K_i = 0, \therefore K_f = \frac{2 \sin kx}{k}$$

$$\therefore n = 2$$

26. An LCR series circuit of capacitance 62.5nF and resistance of 50Ω , is connected to an A.C. source of frequency 2.0kHz . For maximum value of amplitude of current in circuit, the value of inductance is _____ mH.

Take $\pi^2 = 10$)

Sol. 100

At maximum current, there will be condition of resonance.

$$\text{So, } \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{4 \times \pi^2 \times 4 \times 10^6 \times 62.5 \times 10^{-9}} H = 0.1 H = 100 \text{ mH}$$

27. The distance between two consecutive points with phase difference of 60° in a wave of frequency 500 Hz is 6.0 m . The velocity with which wave is traveling is _____ km/s

Sol. 18

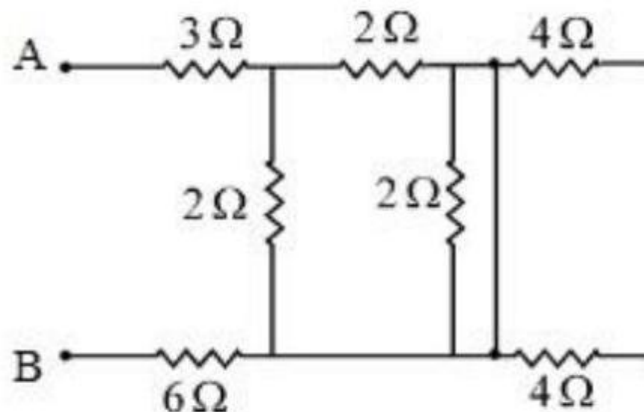
$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x$$

$$\Rightarrow \frac{\pi}{3} = \frac{2\pi}{\lambda} \times 6$$

$$\Rightarrow \lambda = 36 \text{ m}$$

$$\text{Now } v = f\lambda = 500 \times 36 \text{ m/s} = 18000 \text{ m/s} = 18 \text{ km/s}$$

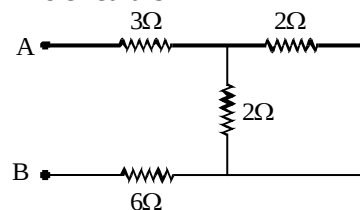
28. In the given circuit, the equivalent resistance between the terminal A and B is Ω .



Sol. 10

Due to short circuit, 3 resistances get vanished from the circuit.

The circuit is



$$R_{eq} = 3 + 3 + 1 = 10\Omega$$

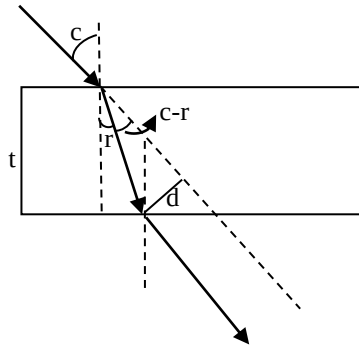
29. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, The unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$. The value of x is

Sol. 4

$$\begin{aligned}\text{Let } \vec{C} &= \vec{P} \times \vec{Q} = 3\sqrt{3}\hat{k} - 7.5\hat{j} - 4\sqrt{3}\hat{k} + 2.5\sqrt{3}\hat{i} + 8\hat{j} - 2\sqrt{3}\hat{i} \\ &= \frac{1}{2}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}) \\ |\vec{C}| &= \frac{1}{2}\sqrt{3 + 1 + 12} = \frac{1}{2} \times 4 = 2 \\ \therefore \hat{C} &= \frac{\vec{C}}{|\vec{C}|} = \frac{1}{4}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})\end{aligned}$$

30. A ray of light is incident from air on a glass plate having thickness $\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is _____ $\times 10^{-2}$ cm. (given $\sin 15^\circ = 0.26$)

Sol. 52



$$\sin c = \frac{1}{\sqrt{2}} \Rightarrow c = 45^\circ$$

Using Snell's law on 1st surface, $\sin c = \sqrt{2} \sin r$

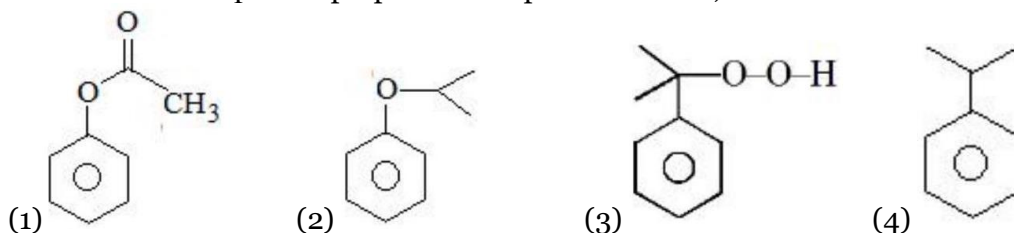
$$\Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30^\circ$$

$$d = t \sec r \times \sin(c - r) = \sqrt{3} \times \frac{2}{\sqrt{3}} \times 0.26 = 0.52 \text{ cm} = 52 \times 10^{-2} \text{ cm}$$

Chemistry

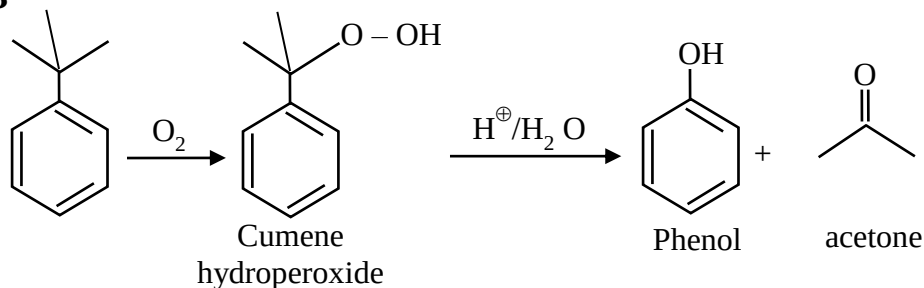
SECTION - A

31. In the cumene to phenol preparation in presence of air, the intermediate is

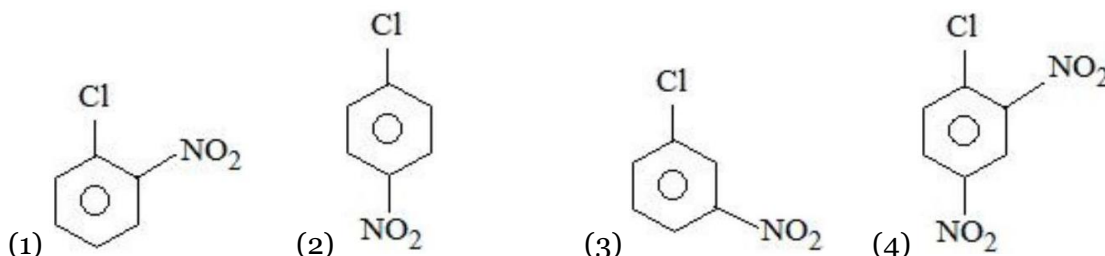


Sol.

3



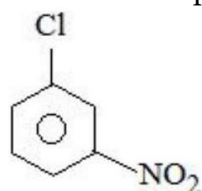
32. The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with OH^- is



Sol.

3

Rate of nucleophilic aromatic substitution decrease by e^- withdrawing group



$-\text{NO}_2$ of meta shows $-I$ effect which is less dominating than $-M$

33. Match List I with List II

LIST I Elements		LIST II Colour imparted to the flame	
A.	K	I.	Brick Red
B.	Ca	II.	Violet
C.	Sr	III.	Apple Green
D.	Ba	IV.	Crimson Red

Choose the correct answer from the options given below:

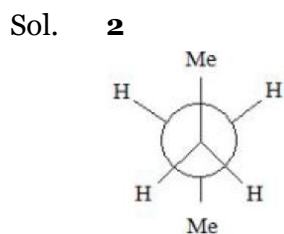
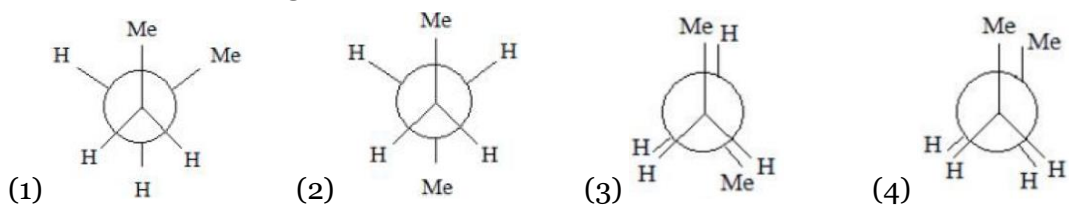
- (1) A-II, B-I, C-III, D-IV
(3) A-IV, B-III, C-II, D-I

- (2) A-II, B-I, C-IV, D-III
(4) A-II, B-IV, C-I, D-III

Sol. 2
Flame Test.

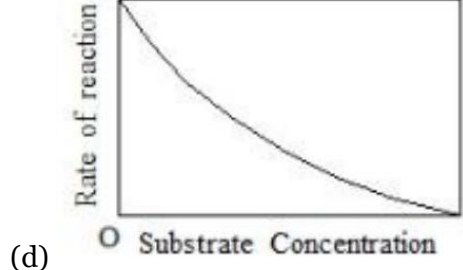
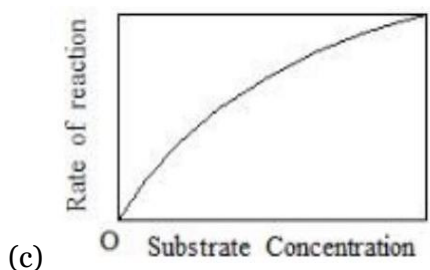
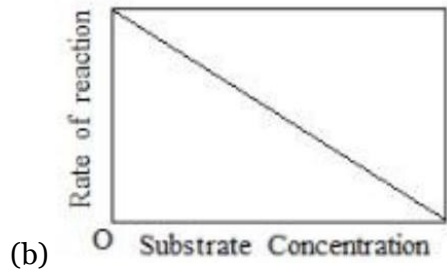
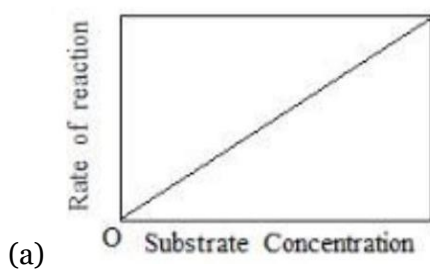
Metals	Colour of flame test
K	Violet
Ca	Brick Red
Sr	Crimson Red
Ba	Apple Green

34. Which of the following conformations will be the most stable ?



Anti position highly stable (bulky group maximum distance)

35. The variation of the rate of an enzyme catalyzed reaction with substrate concentration is correctly represented by graph



(1) (b)

(2) (a)

(3) (d)

(4) (c)

Sol. 4
Fact base.

36. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Acetal / Ketal is stable in basic medium.

Reason R : The high leaving tendency of alkoxide ion gives the stability to acetal/ ketal in basic medium.

In the light of the above statements, choose the correct answer from the options given below :

- (1) A is true but R is false
 (2) A is false but R is true
 (3) Both A and R are true but R is NOT the correct explanation of A
 (4) Both A and R are true and R is the correct explanation of A

Sol. **1**

Acetal and ketals are basically ether hence they must be stable in basic medium but should break down in acidic medium.

Hence assertion is correct.

Alkoxide ion (RO^-) is not considered a good leaving group hence reason must be false.

37. A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. Y is at $\frac{1}{3}$ rd of the total faces. The empirical formula of the compound is

- (1) $\text{XY}_{2.5}$ (2) $\text{X}_2\text{Y}_{1.5}$ (3) $\text{X}_{2.5}\text{Y}$ (4) $\text{X}_{1.5}\text{Y}_2$

Sol. **4**

$$\text{Number of X-atom per unit cell} = 1 + 4 \times \frac{1}{8} = \frac{3}{2}$$

$$\text{Number of Y-atoms per unit cell} = 2 \times \frac{1}{2} = 1$$

$\therefore$ Empirical formula of the solid is X_3Y_2 .

38. Match the List-I with List-II

List-I	List-II
Cations	Group reagents
A $\rightarrow \text{Pb}^{2+}, \text{Cu}^{2+}$	i) H_2S gas in presence of dilute HCl
B $\rightarrow \text{Al}^{3+}, \text{Fe}^{3+}$	ii) $(\text{NH}_4)_2\text{CO}_3$ in presence of NH_4OH
C $\rightarrow \text{Co}^{2+}, \text{Ni}^{2+}$	iii) NH_4OH in presence of NH_4Cl
D $\rightarrow \text{Ba}^{2+}, \text{Ca}^{2+}$	iv) H_2S in presence of NH_4OH

Correct match is -

- (1) A $\rightarrow$ iii, B $\rightarrow$ i, C $\rightarrow$ iv, D $\rightarrow$ ii
 (2) A $\rightarrow$ i, B $\rightarrow$ iii, C $\rightarrow$ ii, D $\rightarrow$ iv
 (3) A $\rightarrow$ iv, B $\rightarrow$ ii, C $\rightarrow$ iii, D $\rightarrow$ i
 (4) A $\rightarrow$ i, B $\rightarrow$ iii, C $\rightarrow$ iv, D $\rightarrow$ ii

Sol. **4**

Cations	Group No.	Group reagents
$\text{Pb}^{2+}, \text{Cu}^{2+}$	II	$\text{H}_2\text{S} + \text{HCl}$
$\text{Al}^{3+}, \text{Fe}^{3+}$	III	$\text{NH}_4\text{Cl} + \text{NH}_4\text{OH}$
$\text{Co}^{2+}, \text{Ni}^{2+}$	IV	$\text{NH}_4\text{OH} + \text{H}_2\text{S}$
$\text{Ba}^{2+}, \text{Ca}^{2+}$	V	$\text{NH}_4\text{OH}, \text{Na}_2\text{CO}_3$

39. Which of the following statements is incorrect for antibiotics?
 (1) An antibiotic must be a product of metabolism.
 (2) An antibiotic should promote the growth or survival of microorganisms.
 (3) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring antibiotic.
 (4) An antibiotic should be effective in low concentrations.

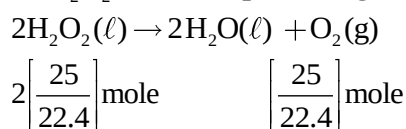
Sol. **2**
 Antibiotic kill or inhibit the growth of microorganism

40. The correct order in aqueous medium of basic strength in case of methyl substituted amines is :
 (1) $\text{Me}_3\text{N} > \text{Me}_2\text{NH} > \text{MeNH}_2 > \text{NH}_3$
 (2) $\text{Me}_2\text{NH} > \text{MeNH}_2 > \text{Me}_3\text{N} > \text{NH}_3$
 (3) $\text{Me}_2\text{NH} > \text{Me}_3\text{N} > \text{MeNH}_2 > \text{NH}_3$
 (4) $\text{NH}_3 > \text{Me}_3\text{N} > \text{MeNH}_2 > \text{Me}_2\text{NH}$

Sol. **2**
 In aqueous medium basic strength is dependent on electron density on nitrogen as well as solvation of cation formed after accepting H^+ . After considering all these factors overall basic strength order is $\text{Me}_2\text{NH} > \text{MeNH}_2 > \text{Me}_3\text{N} > \text{NH}_3$

41. '25 volume' hydrogen peroxide means
 (1) 1 L marketed solution contains 25 g of H_2O_2 .
 (2) 1 L marketed solution contains 75 g of H_2O_2 .
 (3) 1 L marketed solution contains 250 g of H_2O_2 .
 (4) 100 mL marketed solution contains 25 g of H_2O_2 .

Sol. **2**
 25V H_2O_2 means : 1 lit of H_2O_2 on decomposition give 25 lit of $\text{O}_2(\text{g})$ at STP.



$$\text{Mass of } \text{H}_2\text{O}_2 = \frac{2 \times 25}{22.4} \times 34 = 75.89 \text{ gram}.$$

42. The radius of the 2nd orbit of Li^{2+} is x . The expected radius of the 3rd orbit of Be^{3+} is
 (1) $\frac{27}{16}x$ (2) $\frac{4}{9}x$ (3) $\frac{9}{4}x$ (4) $\frac{16}{27}x$

Sol. **1**

$$R = 0.529 \times \frac{n^2}{Z}$$

$$r_{\text{Li}^{2+} \text{ } n=2} = 0.529 \times \frac{(2)^2}{3} = x$$

$$r_{\text{Be}^{3+} \text{ } n=3} = 0.529 \times \frac{(3)^2}{4}$$

$$\frac{r_{\text{Li}^{2+} \text{ } n=2}}{r_{\text{Be}^{3+} \text{ } n=3}} = \frac{\frac{r_0 \times (2)^2}{3}}{\frac{r_0 \times (3)^2}{4}}$$

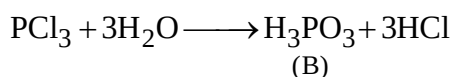
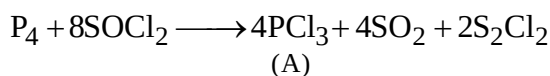
$$\frac{x}{r_{\text{Be}^{3+} \text{ } n=3}} = \frac{16}{27}$$

$$\therefore (r_{\text{Be}^{3+}})_{n=3} = \frac{27x}{16}$$

43. Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a dibasic acid. [A] and [B] are respectively

(1) P_4O_6 and H_3PO_3 (2) PCl_5 and H_3PO_4 (3) $POCl_3$ and H_3PO_4 (4) PCl_3 and H_3PO_3

Sol. 4



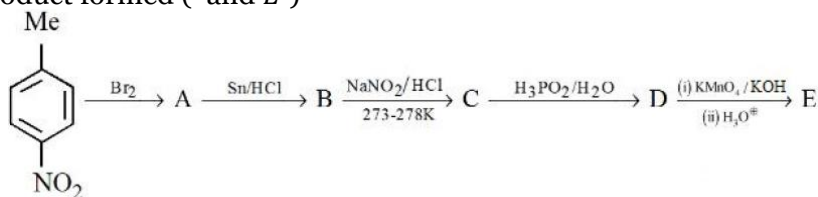
44. Inert gases have positive electron gain enthalpy. Its correct order is

(1) $He < Kr < Xe < Ne$ (2) $He < Xe < Kr < Ne$
 (3) $He < Ne < Kr < Xe$ (4) $Xe < Kr < Ne < He$

Sol. 2

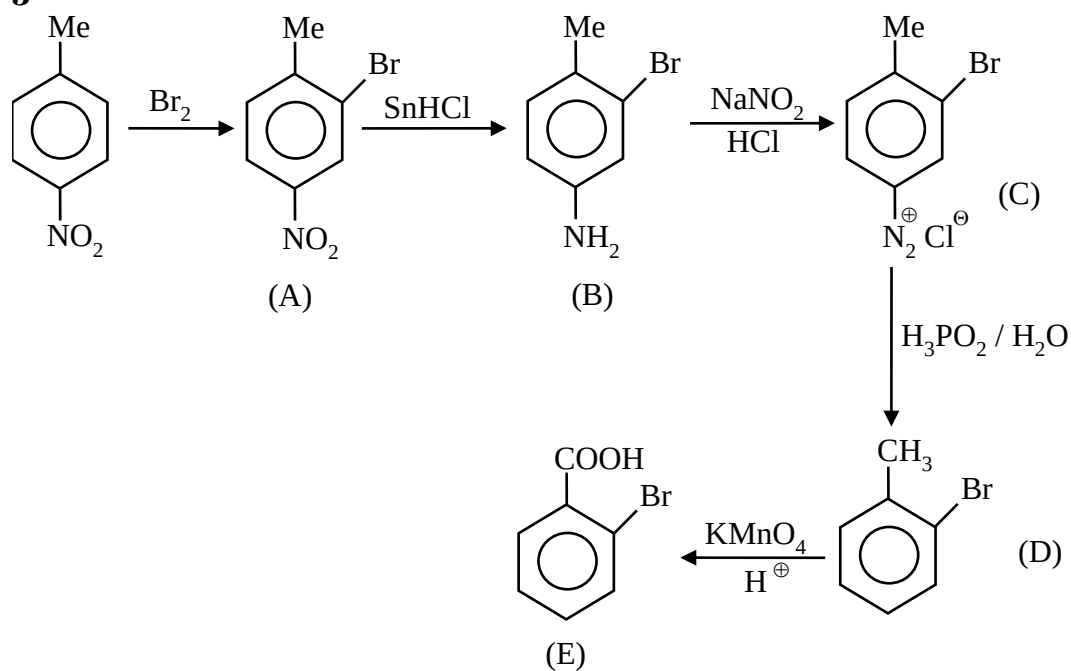
Positive electron gain enthalpy. of inert gas is in order of
 $Ne > Ar > Kr > Xe > He$

45. Identify the product formed (and E)



- (1) $A =$, $E =$
- (2) , $E =$
- (3) , $E =$
- (4) , $E =$

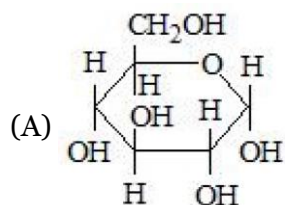
Sol. **3**



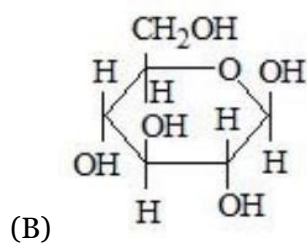
46. Match items of Row I with those of Row II.

Row I

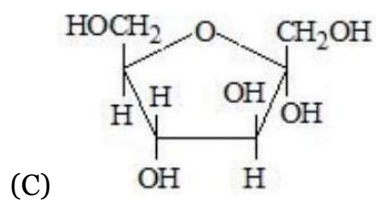
Row II



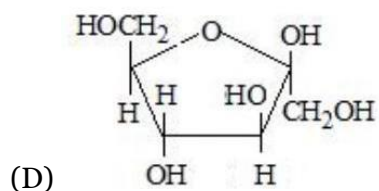
(i) $\alpha - D - (-)$ -Fructofuranose,



(ii) $\beta - D - (-)$ - Fructofuranose



(iii) $\alpha - D - (-)$ Glucopyranose,



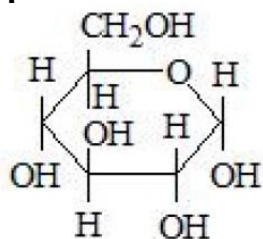
(iv) $\beta - D - (-)$ -Glucopyranose

Correct match is

- (1) A → i, B → ii, C → ii, D → iv
 (3) A → iii, B → iv, C → ii, D → I

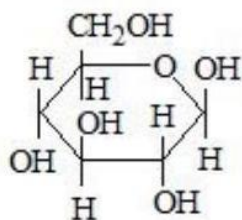
Sol.

4

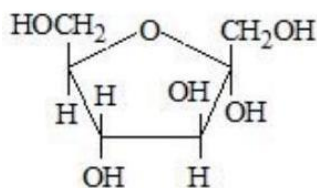


- (2) A → iv, B → iii, C → i, D → ii
 (4) A → iii, B → iv, C → i, D → ii

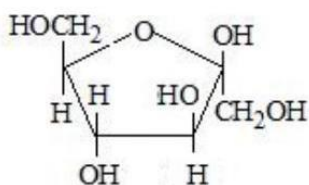
$\alpha - D - (-)$ Glucopyranose



$\beta - D - (-)$ -Glucopyranose



$\alpha - D - (-)$ -Fructofuranose



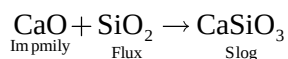
$\beta - D - (-) -$ Fructofuranose

47. Which one of the following reactions does not occur during extraction of copper ?

- (1) $2\text{Cu}_2\text{S} + 3\text{O}_2 \rightarrow 2\text{Cu}_2\text{O} + 2\text{SO}_2$ (2) $\text{FeO} + \text{SiO}_2 \rightarrow \text{FeSiO}_3$
 (3) $2\text{FeS} + 3\text{O}_2 \rightarrow 2\text{FeO} + 2\text{SO}_2$ (4) $\text{CaO} + \text{SiO}_2 \rightarrow \text{CaSiO}_3$

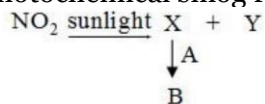
Sol.

4



In metallurgy iron will occur not in metallurgy of Cu.

48. Some reactions of NO_2 relevant to photochemical smog formation are

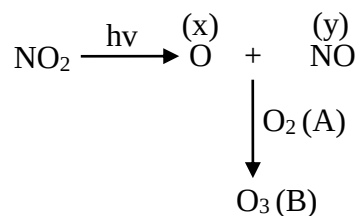


Identify A, B, X and Y

- (1) $\text{X} = \frac{1}{2}\text{O}_2$, $\text{Y} = \text{NO}_2$, $\text{A} = \text{O}_3$, $\text{B} = \text{O}_2$ (2) $\text{X} = [\text{O}]$, $\text{Y} = \text{NO}$, $\text{A} = \text{O}_2$, $\text{B} = \text{O}_3$
 (3) $\text{X} = \text{N}_2\text{O}$, $\text{Y} = [\text{O}]$, $\text{A} = \text{O}_3$, $\text{B} = \text{NO}$ (4) $\text{X} = \text{NO}$, $\text{Y} = [\text{O}]$, $\text{A} = \text{O}_2$, $\text{B} = \text{N}_2\text{O}_3$

Sol.

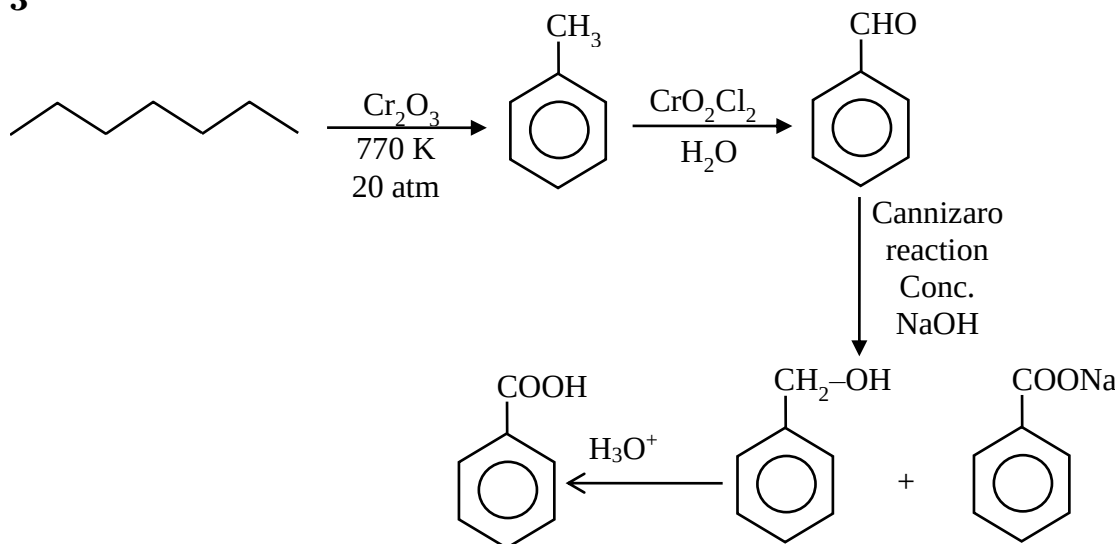
2



$$\text{P} \longrightarrow \text{O} + \text{R}$$

(1) (i) CrO_2Cl_2 , H_3O^+ ; (ii) Cr_2O_3 , 770 K, 20 atm; (iii) NaOH ; (iv) H_3O^+

(3) (i) Cr_2O_3 , 770 K, 20 atm; (ii) CrO_2Cl_2 , H_3O^+ ; (iii) NaOH ; (iv) H_3O^+

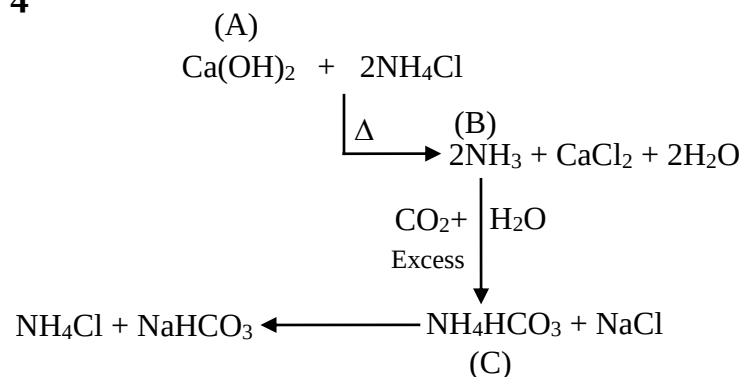
Sol. **3**

Compound A, B and C, are respectively.

(2) $\text{Ca}(\text{OH})_2, \text{NH}_4^{\oplus}, (\text{NH}_4)_2\text{CO}_3$

(4) $\text{Ca}(\text{OH})_2, \text{NH}_3, \text{NH}_4\text{HCO}_3$

Sol. 4



SECTION - B

51. For the first order reaction $A \rightarrow B$, the half life is 30 min. The time taken for 75% completion of the reaction is _____ min. (Nearest integer)

Given : $\log 2 = 0.3010$

$\log 3 = 0.4771$

$\log 5 = 0.6989$

Sol. **60**

$$t_{75\%} = 2t_{1/2} \text{ [For 1}^{\text{st}} \text{ order reaction]}$$

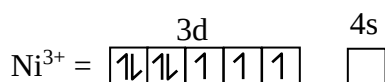
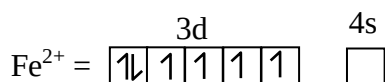
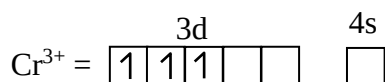
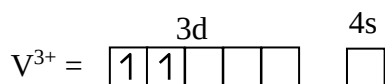
$$t_{75\%} = 2 \times 30 = 60 \text{ min.}$$

52. How many of the following metal ions have similar value of spin only magnetic moment in gaseous state?

(Given: Atomic number : V, 23; Cr, 24; Fe, 26; Ni, 28)

V^{3+} , Cr^{3+} , Fe^{2+} , Ni^{3+}

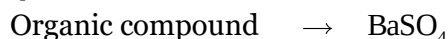
Sol. **2 (Cr^{3+} & Ni^{3+})**



53. In sulphur estimation, 0.471 g of an organic compound gave 1.4439 g of barium sulphate. The percentage of sulphur in the compound is _____ (Nearest Integer)

(Given: Atomic mass Ba: 137u, S: 32u, O: 16u)

Sol. **42**



Weight = 0.417 g

Weight = 1.44 g

$$\text{Moles BaSO}_4 = \frac{1.44}{233} = \text{moles of Sulphur}$$

$$\text{Weight Sulphur} = \frac{1.44}{233} \times 32 \text{ gram}$$

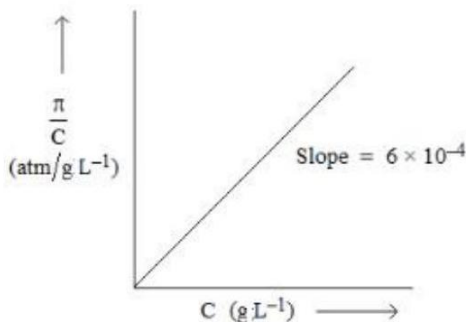
$$\% S = \frac{\text{weight of sulphur}}{\text{weight of organic}} \times 100$$

$$\Rightarrow \frac{1.44 \times 32}{233 \times 0.471} \times 100$$

$$\Rightarrow \frac{46.08}{109.743} \times 100$$

$$\Rightarrow 41.98 \simeq 42$$

54. The osmotic pressure of solutions of PVC in cyclohexanone at 300 K are plotted on the graph. The molar mass of PVC is _____ g mol^{-1} (Nearest integer)



(Given : $R = 0.083 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

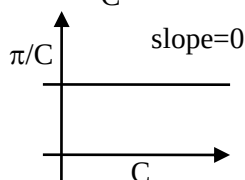
Sol. 41500

$$\pi = M'RT = \left(\frac{W/M}{V} \right) RT$$

$$\Rightarrow \pi = \left(\frac{W}{V} \right) \left(\frac{1}{M} \right) RT = C \left(\frac{RT}{M} \right)$$

$$\Rightarrow \frac{\pi}{C} = \frac{RT}{M} \neq f(c)$$

If we assume graph between $\frac{\pi}{C}$ and C



Assuming π vs C graph

$$\text{Slope} = \frac{RT}{M} = \frac{0.083 \times 300}{M} = 6 \times 10^{-4}$$

$$\therefore M = \frac{0.083 \times 300}{6 \times 10^{-4}} = \frac{830 \times 300}{6}$$

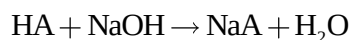
$$= 41,500$$

55. The density of a monobasic strong acid (Molar mass 24.2 g/mol) is 1.21 kg/L. The volume of its solution required for the complete neutralization of 25 mL of 0.24M NaOH is _____ $\times 10^{-2}$ mL (Nearest integer)

Sol. 12

$$\text{Molarity of acid} = \frac{1.2 \times 10^3}{24.2} = \frac{1000}{20} = 50 \text{ M}$$

Neutralization reaction :



$$M_1 V_1 = M_2 V_2$$

$$[50] \times V = [0.24 \times 25]$$

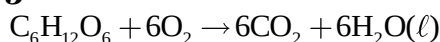
$$V = 0.12 \text{ ml}$$

56. An athlete is given 100 g of glucose ($C_6H_{12}O_6$) for energy. This is equivalent to 1800 kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water he would need to perspire is _____g (Nearest integer)
Assume that there is no other way of consuming stored energy.

Given : The enthalpy of evaporation of water is 45 kJ mol^{-1}

Molar mass of C, H&O are 12,1 and 16 g mol^{-1}

Sol. **360**



$$n = \frac{100}{180}$$

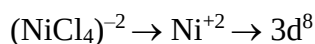
$$\text{Energy needed to perspire water} = 1800 \times \frac{1}{2}$$

$$\text{Moles of water evaporated} = \frac{900}{45} = 20 \text{ moles}$$

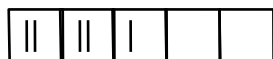
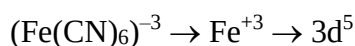
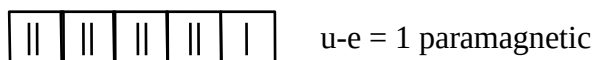
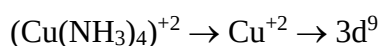
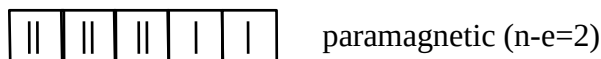
$$\text{Weight of water evaporated} \Rightarrow 20 \times 18 \\ \Rightarrow 360 \text{ gram}$$

57. The number of paramagnetic species from the following is
 $[Ni(CN)_4]^{2-}$, $[Ni(CO)_4]$, $[NiCl_4]^{2-}$
 $[Fe(CN)_6]^{4-}$, $[Cu(NH_3)_4]^{2+}$
 $[Fe(CN)_6]^{3-}$ and $[Fe(H_2O)_6]^{2+}$

Sol. **4**

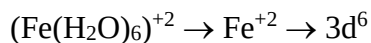


$Cl^- \rightarrow$ weak field layered

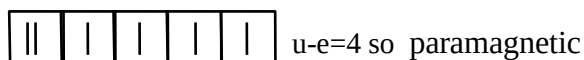


CN^- is strong field ligand so u-e=1

so paramagnetic

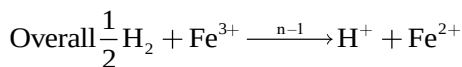
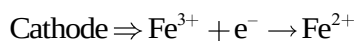
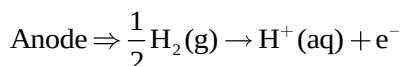


H_2O is weak field ligand



58. Consider the cell
 $Pt(s) | H_2(g) (1 \text{ atm}) | H^+(aq, [H^+] = 1) || Fe^{3+}(aq), Fe^{2+}(aq) | Pt(s)$
 Given $E_{Fe^{3+}/Fe^{2+}}^o = 0.771 \text{ V}$ and $E_{H^+/2H_2}^o = 0 \text{ V}$, $T = 298 \text{ K}$
 If the potential of the cell is 0.712 V , the ratio of concentration of Fe^{2+} to Fe^{3+} is (Nearest integer)

Sol. **10**



$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{0.059}{1} \log \frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} \times \frac{[\text{H}^+]}{[\text{P}_{\text{H}_2}]^{\frac{1}{2}}}$$

$$0.712 = 0.771 - 0.059 \log \frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]}$$

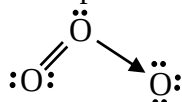
$$\log \frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} = 1$$

$$\text{So } \frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} = 10$$

59. The total number of lone pairs of electrons on oxygen atoms of ozone is

Sol. **6**

Not l.p. e^- in O_3 is = 6



60. A litre of buffer solution contains 0.1 mole of each of NH_3 and NH_4Cl . On the addition of 0.02 mole of HCl by dissolving gaseous HCl , the pH of the solution is found to be $\text{_____} \times 10^{-3}$ (Nearest integer)

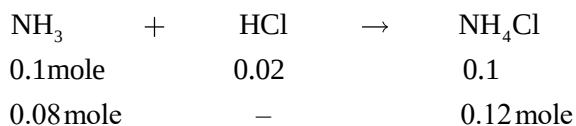
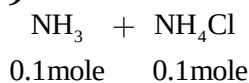
[Given : $\text{pK}_b(\text{NH}_3) = 4.745$

$$\log 2 = 0.301$$

$$\log 3 = 0.477$$

$$T = 298 \text{ K}]$$

Sol. **9**



$$\text{pOH} \Rightarrow \text{pK}_b + \log \frac{[\text{NH}_4\text{Cl}]}{[\text{NH}_3]}$$

$$\Rightarrow 4.745 + \log \left(\frac{0.12}{0.08} \right)$$

$$\Rightarrow 4.745 + \log \left(\frac{3}{2} \right)$$

$$\Rightarrow 4.745 + (0.477 - 0.301)$$

$$\Rightarrow 4.745 + 0.176$$

$$\Rightarrow 4.569$$

$$\text{pH} \Rightarrow 14 - 4.569$$

$$\Rightarrow 9.431 \simeq 9$$

Mathematics

Section A

- 61.** The points of intersection of the line $ax + by = 0$, ($a \neq b$) and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is :

(1) $x^2 + y^2 + 3x + 3y + 4 = 0$

(2) $x^2 + y^2 + 3x + 5y + 8 = 0$

(3) $x^2 + y^2 - 5x - 5y + 12 = 0$

(4) $x^2 + y^2 + 5x + 5y + 12 = 0$

Sol. 4

Only possibilities is $\alpha = 0, \beta = 1$

Equation of circle

$$(x - 0)(x - 1) + (y - 0)(y - 1) = 0$$

$$x^2 + y^2 - x - y = 0$$

Image of circle in line $x + y + 2 = 0$

$$x^2 + y^2 + 5x + 5y + 12 = 0$$

- 62.** The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c$, $m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is :

(1) $\frac{14}{3}$

(2) $5\sqrt{3}$

(3) $\frac{1}{3}$

(4) 5

Sol. 4

$$\left. \begin{array}{l} y^2 = \frac{x}{2} \\ y^2 = x - 1 \end{array} \right\}$$

Tangent to $y^2 = \frac{x}{2}$ is $y = mx + \frac{1}{8m}$... (1)

$$y^2 = x - 1 \text{ is } y = m(x - 1) + \frac{1}{4m}$$

$$y = mx - m + \frac{1}{4m} \quad \dots (2)$$

(1) & (2)

$$\frac{1}{8m} = -m + \frac{1}{4m}$$

$$m = \frac{1}{4m} - \frac{1}{8m}$$

$$m = \frac{1}{8m} \Rightarrow m^2 = \frac{1}{8} \Rightarrow m = \frac{1}{2\sqrt{2}} (m > 0)$$

From (1)

$$y = \frac{1}{2\sqrt{2}}x + \frac{1}{2\sqrt{2}}$$

distance from $(6, -2\sqrt{2})$

$$\frac{\left| \frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{1}{2\sqrt{2}} \right|}{\sqrt{1 + \frac{1}{8}}} = \frac{6 + 8 + 1}{3} = \frac{15}{3} = 5$$

63. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}-\vec{c}}{2}$.

If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to

- (1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Sol. 2

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\vec{b}}{2} - \frac{\vec{c}}{2}$$

$$\vec{a} \cdot \vec{c} = \frac{1}{2}, \vec{a} \cdot \vec{b} = \frac{1}{2}$$

$$\vec{b} \cdot \vec{d} = \frac{1}{2}$$

$$\begin{aligned} (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) &= \vec{a} \cdot [\vec{b} \times (\vec{c} \times \vec{d})] \\ &= \vec{a} \cdot [(\vec{b} \cdot \vec{d})\vec{c} - (\vec{b} \cdot \vec{c})\vec{d}] \\ &= \vec{a} \cdot \left[\frac{\vec{c}}{2} \right] \\ &= \frac{1}{2}(\vec{a} \cdot \vec{c}) \\ &= \frac{1}{4} \end{aligned}$$

64. The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is :

- (1) $2\sqrt{3}$ (2) 1 (3) $3\sqrt{2}$ (4) $\sqrt{6}$

Sol. 3

$$\vec{b} = \lambda\vec{a} + \mu\hat{j}$$

$$= \lambda(-\hat{i} + 2\hat{j} + \hat{k}) + \mu\hat{j}$$

$$\vec{b} = -\lambda\hat{i} + (2\lambda + \mu)\hat{j} + \lambda\hat{k}$$

$$|\vec{a}| = |\vec{b}|$$

$$|\vec{a}|^2 = |\vec{b}|^2 \Rightarrow 6 = \lambda^2 + (2\lambda + \mu)^2 + \lambda^2 \dots\dots(1)$$

$$\because \vec{a} \cdot \vec{b} = 0 \Rightarrow \lambda + 2(2\lambda + \mu) + (1)(\lambda) = 0$$

$$\Rightarrow 6\lambda + 2\mu = 0$$

$$\Rightarrow \mu = -3\lambda \dots\dots\dots(2)$$

from (1) & (2)

$$3\lambda^2 = 6$$

$$\lambda^2 = 2 \Rightarrow \lambda = \pm\sqrt{2}$$

$$\Rightarrow \mu = \pm 3\sqrt{2}$$

$$\begin{aligned} \text{Projection of } 3\vec{a} + 2\vec{b} \text{ on } \vec{c} \text{ is} &= \frac{(3\vec{a} + 2\vec{b}) \cdot \vec{c}}{|\vec{c}|} \\ &= \frac{3\vec{a} \cdot \vec{c} + 2\vec{b} \cdot \vec{c}}{|\vec{c}|} \end{aligned}$$

Sol. 3.96

Let Number of observations is = n

$$\left. \begin{array}{l} \frac{\sum x_i}{n} = 10 \\ \sum x_i = 10n \text{ --- (1)} \end{array} \right| \begin{array}{l} \frac{\sum x_i - 8 + 12}{n} = 10.2 \\ \sum x_i = (10.2)n - 4 \text{ --- (2)} \end{array}$$

$$10n = (10.2)n - 4$$

$$\Rightarrow (.2)n = 4 \Rightarrow \boxed{n = 20}$$

$$\text{Given } \frac{\sum x_i^2}{20} - (10)^2 = 4 \Rightarrow \sum x_i^2 = 2080$$

After Change

$$\begin{aligned} \sum x_i^2 &= 2080 - 8^2 + (12)^2 \\ &= 2160 \end{aligned}$$

$$\begin{aligned} \text{New variance} &= \frac{\sum x_i^2}{20} - (\bar{x})^2 \\ &= \frac{2160}{20} - (10.2)^2 \\ &= 108 - (10.2)^2 \\ &= 3.96 \end{aligned}$$

67. Let S_1 and S_2 be respectively the sets of all $a \in \mathbb{R} - \{0\}$ for which the system of linear equations

$$ax + 2ay - 3az = 1$$

$$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$$

$$(3a + 5)x + (a + 5)y + (a + 2)z = 3$$

has unique solution and infinitely many solutions. Then

(1) S_1 is an infinite set and $n(S_2) = 2$

(2) $S_1 = \Phi$ and $S_2 = \mathbb{R} - \{0\}$

(3) $n(S_1) = 2$ and S_2 is an infinite set

(4) $S_1 = \mathbb{R} - \{0\}$ and $S_2 = \Phi$

Sol. 4

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$$\Delta = a(15a^2 + 31a + 36) = 0$$

$$a = 0$$

$$\Delta \neq 0 \text{ for all } a \in \mathbb{R} - \{0\}$$

$$\therefore S_1 = \mathbb{R} - \{0\}, S_2 = \Phi$$

68. The value of $\lim_{n \rightarrow \infty} \frac{1+2-3+4+5-6+\dots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is :

$$(1) \frac{3}{2}(\sqrt{2} + 1)$$

$$(2) \frac{3}{2\sqrt{2}}$$

$$(3) \frac{\sqrt{2}+1}{2}$$

$$(4) 3(\sqrt{2} + 1)$$

Sol. 1

$$\lim_{n \rightarrow \infty} \frac{(1+2+4+5+\dots+(3n-2)+(3n-1)-3+6+\dots+3n)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\text{Let } N^r = \sum_{n=1}^{\infty} (3n-2) + (3n-1) - 3n$$

$$\begin{aligned}
&= \sum_{n=1}^{\infty} (3n-3) \\
&= \frac{3n(n+1)}{2} - 3n = \frac{3}{2}(n^2 - n) \\
&= \frac{3}{2} \lim_{n \rightarrow \infty} \frac{n^2 \left(1 - \frac{1}{n}\right)}{n^2 \left(\sqrt{2 + \frac{4}{n^3} + \frac{3}{n^4}} - \sqrt{1 + \frac{5}{n^3} + \frac{4}{n^4}}\right)} \\
&= \frac{3}{2(\sqrt{2}-1)} \text{ or } \frac{3}{2}(\sqrt{2}+1) \text{ Ans.}
\end{aligned}$$

69. The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is
 (1) a tautology (2) a contradiction (3) equivalent to $p \vee q$ (4) equivalent to $(\sim p) \vee (\sim q)$

Sol. 1

$$(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$$

P	q	$\sim q$	$p \wedge \sim q$	$p \Rightarrow \sim q$	$(p \wedge \sim q) \Rightarrow (p \Rightarrow \sim q)$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	T	T

Tautology

70. Consider the lines L_1 and L_2 given by

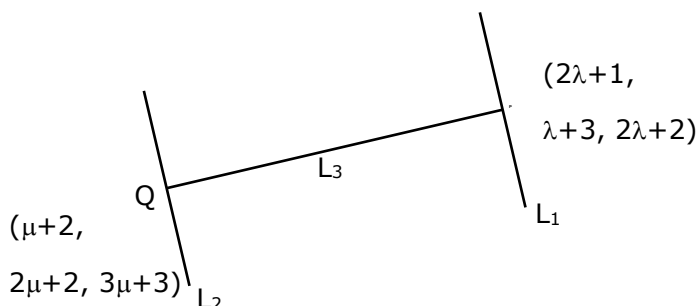
$$L_1: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$$

$$L_2: \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

A line L_3 having direction ratios 1, -1, -2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is

- (1) $3\sqrt{2}$ (2) $4\sqrt{3}$ (3) 4 (4) $2\sqrt{6}$

Sol. 4



D.R's of PQ are = $(2\lambda - \mu - 1, \lambda - 2\mu + 1, 2\lambda - 3\mu - 1)$

given D.R's are = $(1, -1, -2)$

$$\frac{2\lambda - \mu - 1}{1} = \frac{\lambda - 2\mu + 1}{-1} = \frac{2\lambda - 3\mu - 1}{-2}$$

$$\lambda = \mu = 3$$

$$P = (7, 6, 8)$$

$$Q = (5, 8, 12)$$

$$PQ = 2\sqrt{6}$$

71. Let $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)} dx$. If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then $f(4)$ is equal to
- (1) $\log_e 19 - \log_e 20$ (2) $\log_e 17 - \log_e 18$
 (3) $\frac{1}{2}(\log_e 19 - \log_e 17)$ (4) $\frac{1}{2}(\log_e 17 - \log_e 19)$

Sol. 4

$$\text{Let } x^2 = t$$

$$2x dx = dt$$

$$f(x) = \int \frac{dt}{(t+1)(t+3)}$$

$$= \frac{1}{2} \int \left(\frac{1}{t+1} - \frac{1}{t+3} \right) dt$$

$$= \frac{1}{2} \ln \left| \frac{t+1}{t+3} \right| + C$$

$$f(x) = \frac{1}{2} \ln \left| \frac{x^2+1}{x^2+3} \right| + C$$

$$x = 3$$

$$\frac{1}{2} \ln \left(\frac{5}{6} \right) = \frac{1}{2} \ln \left(\frac{5}{6} \right) + C \Rightarrow C = 0$$

$$f(x) = \frac{1}{2} \ln \left(\frac{x^2+1}{x^2+3} \right)$$

$$f(x) = \frac{1}{2} \ln \left(\frac{17}{19} \right)$$

$$= \frac{1}{2} [\ln 17 - \ln 19]$$

72. The minimum value of the function $f(x) = \int_0^2 e^{|x-t|} dt$ is :

- (1) $e(e-1)$ (2) $2(e-1)$ (3) 2 (4) $2e-1$

Sol. 2

Case I $x < 0$

$$f(x) = \int_0^2 e^{-(x-t)} dt$$

$$= e^{-x} \int_0^2 e^t dt = e^{-x} (e^2 - 1)$$

Case II

$$(0 < x < 2) \quad f(x) = \int_0^x e^{x-t} dt + \int_x^2 e^{-(x-t)} dt$$

$$= e^x \left(-e^{-t} \right)_0^x + e^{-x} \left[e^t \right]_x^2$$

$$= e^x \left[-e^{-x} + 1 \right] + e^{-x} \left[e^2 - e^x \right]$$

$$= -1 + e^x + e^{2-x} - 1$$

Case III

$$x \geq 2 \quad f(x) = \int_0^2 e^{(x-t)} dt$$

$$= e^x \left[-e^{-t} \right]_0^2$$

$$= e^x \left[-e^{-2} + 1 \right]$$

$$= e^x (1 - e^{-2})$$

$$f(x) = \begin{cases} e^{-x}(e^2 - 1), & x \leq 0 \rightarrow (e^2 - 1) \\ e^x + e^{2-x} - 2, & 0 \leq x \leq 2 \rightarrow 2(e - 1) \\ ex(1 - e^{-2}), & x \geq 2 \rightarrow (e^2 - 1) \end{cases}$$

Minimum value = $2(e - 1)$

- 73.** Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \{x \in \mathbb{Z} : x(66 - x) \geq \frac{5}{9}M\}$ and the event $A = \{x \in S : x \text{ is a multiple of } 3\}$. Then $P(A)$ is equal to

(1) $\frac{7}{22}$

(2) $\frac{1}{5}$

(3) $\frac{15}{44}$

(4) $\frac{1}{3}$

Sol. 4

Let $a, b \rightarrow 2$ positive number

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$\sqrt{ab} \leq 33$$

$$\boxed{ab \leq (33)^2}$$

$$M = (33)^2$$

$$x(66 - x) \geq \frac{5}{9}(33)^2$$

$$66x - x^2 \geq 605$$

$$0 \geq x^2 - 66x + 605$$

$$(x-11)(x-55) \leq 0$$

$$x \in [11, 55]$$

$$A = \{12, 15, 18, \dots, 54\}$$

Total number in $A = 15$

$$P(A) = \frac{15}{45} = \frac{1}{3} \text{ Ans.}$$

- 74.** Let $x = 2$ be a local minima of the function $f(x) = 2x^4 - 18x^2 + 8x + 12$, $x \in (-4, 4)$. If M is local maximum value of the function f in $(-4, 4)$, then $M =$

(1) $18\sqrt{6} - \frac{31}{2}$

(2) $18\sqrt{6} - \frac{33}{2}$

(3) $12\sqrt{6} - \frac{33}{2}$

(4) $12\sqrt{6} - \frac{31}{2}$

Sol. 3

$$f'(x) = 8x^3 - 36x + 8$$

$$= 4[2x^3 - 9x + 2]$$

$$= 4[(x-2)(2x^2 + 4x - 1)]$$

$$= 4 \left[(x-2) \left(x - \left(-\frac{2-\sqrt{6}}{2} \right) \right) \left(x - \left(\frac{-2+\sqrt{6}}{2} \right) \right) \right]$$

$$\begin{array}{c} - \quad + \quad - \quad + \\ \hline \frac{-2-\sqrt{6}}{2} \quad \frac{-2+\sqrt{6}}{2} \quad 2 \end{array}$$

↘ maximum

$$\begin{aligned} M &= 2 \left(\frac{-2+\sqrt{6}}{2} \right)^4 - 18 \left(\frac{-2+\sqrt{6}}{2} \right)^2 + 8 \left(\frac{-2+\sqrt{6}}{2} \right) + 12 \\ &= 12\sqrt{6} - \frac{33}{2} \end{aligned}$$

75. Let $f: (0,1) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{1}{1-e^{-x}}$, and $g(x) = (f(-x) - f(x))$. Consider two statements

(I) g is an increasing function in $(0,1)$

(II) g is one-one in $(0,1)$

Then,

(1) Both (I) and (II) are true

(2) Neither (I) nor (II) is true

(3) Only (I) is true

(4) Only (II) is true

Sol. 1

$$f(x) = \frac{1}{1-e^{-x}}$$

$$g(x) = (f(-x) - f(x))$$

$$= \frac{1}{1-e^x} - \frac{1}{1-e^{-x}}$$

$$= \frac{1}{1-e^x} - \frac{e^x}{e^x - 1}$$

$$g(x) = \frac{1+e^x}{1-e^x}$$

$$g'(x) = \frac{(1-e^x)(e^x) - (1+e^x)(-e^x)}{(1-e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1-e^x)^2}$$

$$g'(x) = \frac{2e^x}{(1-e^x)^2}$$

$$g'(x) > 0 \Rightarrow g(x) \uparrow$$

$g(x)$ is one-one

76. Let $y(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$. Then $y' - y''$ at $x = -1$ is equal to :

(1) 976

(2) 944

(3) 464

(4) 496

Sol. 4

$$f(x) = y = \frac{(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})}{(1-x)}$$

$$f(x) = y = \frac{(1-x^{32})}{1-x} \Rightarrow f(-1) = 0$$

$$(1-x)y = 1 - x^{32}$$

differentiate both side

$$(1-x)y' + y(-1) = -32x^{31} \quad \boxed{x = -1 \Rightarrow y' = 16}$$

differentiate both side

$$(1-x)y' + y'(-1) - y' = -(32)(31) \times 30$$

Put $x = -1$

$$2y'' - 2y' = -(32)(31)$$

$$y'' - y' = -(16)(31)$$

$$\boxed{y' - y'' = 496}$$

77. The distance of the point $P(4, 6, -2)$ from the line passing through the point $(-3, 2, 3)$ and parallel to a line with direction ratios $3, 3, -1$ is equal to :

(1) $\sqrt{14}$ (2) 3 (3) $\sqrt{6}$ (4) $2\sqrt{3}$

Sol. 1

equation of line

$$\vec{r} = (-3, 2, 3) + \lambda(3, 3, -1)$$

$$\overline{PM} \cdot (3, 3, -1) = 0$$

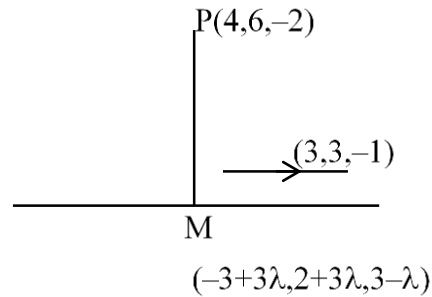
$$(3\lambda - 7, 3\lambda - 4, 5 - \lambda) \cdot (3, 3, -1) = 0$$

$$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(5 - \lambda) = 0$$

$$\Rightarrow 19\lambda = 38 \Rightarrow \lambda = 2$$

$$M = (3, 8, 1)$$

$$PM = \sqrt{1+4+9} = \sqrt{14}$$



78. Let $x, y, z > 1$ and $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$. Then $|\text{adj}(\text{adj} A^2)|$ is equal to
- (1) 2^8 (2) 4^8 (3) 6^4 (4) 2^4

Sol. 1

$$|\text{adj}(\text{adj} A^2)| = |A^2|^{(3-1)^2} = |A|^8$$

$$|A| = \begin{vmatrix} 1 & \frac{\ln y}{\ln x} & \frac{\ln z}{\ln x} \\ \frac{\ln x}{\ln y} & 2 & \frac{\ln z}{\ln y} \\ \frac{\ln x}{\ln z} & \frac{\ln y}{\ln z} & 3 \end{vmatrix}$$

$$= \frac{1}{\ln x \ln y \ln z} \begin{vmatrix} \ln x & \ln y & \ln z \\ \ln x & 2 \ln y & \ln z \\ \ln x & \ln y & 3 \ln z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$$|A| = 2$$

$$\therefore |\text{adj}(\text{adj} A)| = 2^8 \text{ Ans.}$$

79. If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^{10}$, then

$$\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}} \right)^2 \text{ is equal to}$$

(1) 5445 (2) 3025 (3) 4895 (4) 1210

Sol. 4

$$\sum_{r=1}^{10} r^3 \left[\frac{a_r}{a_{r-1}} \right]^2$$

$$\therefore \frac{a_r}{a_{r-1}} = \frac{10-r+1}{r} = \frac{11-r}{r}$$

$$\therefore (1+x)^{10} \Rightarrow {}^{10}C_r x^r$$

$$\Rightarrow {}^{10}C_{10-r} x^{10-r}$$

$$= {}^{10}C_{10-r} \text{ or } = {}^{10}C_r x^{10-r}$$

$$a_r = {}^{10}C_r$$

$$\begin{aligned}
& \sum_{r=1}^{10} r^3 \left[\frac{11-r}{r} \right]^2 \\
& \sum_{r=1}^{10} r(11-r)^2 \\
& \sum_{r=1}^{10} [r^3 - 22r^2 + 121r] \\
& = \left(\frac{(10)(11)}{2} \right)^2 - 22 \left(\frac{(10)(11)(21)}{6} \right) + \left(\frac{(10)(11)}{2} \right) (121) \\
& = 1210 \text{ Ans.}
\end{aligned}$$

80. Let $y = y(x)$ be the solution curve of the differential equation

$$\frac{dy}{dx} = \frac{y}{x} (1 + xy^2(1 + \log_e x)), x > 0, y(1) = 3. \text{ Then } \frac{y^2(x)}{9} \text{ is equal to :}$$

- (1) $\frac{x^2}{2x^3(2+\log_e x^3)-3}$ (2) $\frac{x^2}{3x^3(1+\log_e x^2)-2}$
 (3) $\frac{x^2}{7-3x^3(2+\log_e x^2)}$ (4) $\frac{x^2}{5-2x^3(2+\log_e x^3)}$

Sol. (4)

$$\frac{dy}{dx} = \frac{y}{x} [1 + xy^2(1 + \ln x)]$$

$$\frac{dy}{dx} - \frac{y}{x} = y^3(1 + \ln x)$$

$$\frac{1}{y^3} \frac{dy}{dx} - \frac{1}{x} \cdot \frac{1}{y^2} = 1 + \ln x \quad \dots(1)$$

$$-\frac{1}{y^2} = t \Rightarrow \frac{2}{y^3} \frac{dy}{dx} = \frac{dt}{dx}$$

From (1)

$$\frac{1}{2} \frac{dy}{dx} + t \left(\frac{1}{x} \right) = 1 + \ln x$$

$$\frac{dy}{dx} + t \left(\frac{2}{x} \right) = 2(1 + \ln x)$$

$$\text{I.F} = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = x^2$$

$$t(x^2) = \int [2(1 + \ln x) \cdot x^2] dx$$

$$\Rightarrow t \cdot x^2 = \frac{2x^3}{3} + 2 \int x^2 \ln x dx$$

$$\Rightarrow \frac{-x^2}{y^2} - \frac{2x^3}{3} + 2 \left[\ln x \cdot \frac{x^3}{3} - \frac{x^2}{9} \right] + C$$

$$x = 1, y = 3 \Rightarrow C = \frac{-5}{9}$$

$$\frac{-x^2}{y^2} = \frac{2x^3}{3} + 2 \left[\ln x \cdot \frac{x^3}{3} - \frac{x^2}{9} \right] - \frac{5}{9}$$

Section B

81. The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is

Sol. 1080

$$\begin{aligned}\text{General term} &= \frac{5!}{r_1!r_2!r_3!} (2x)^{r_1} \left(\frac{1}{x}\right)^{r_2} (3x^2)^{r_3} \\ &= \frac{5!}{r_1!r_2!r_3!} 2^{r_1} \cdot 3^{r_3} \left[x^{r_1-7r_2+2r_3} \right] \\ &\quad \left[\begin{array}{l} r_1 - 7r_2 + 2r_3 = 0 \\ r_1 + r_2 + r_3 = 5 \end{array} \right] \\ &\quad r_1 = 1, r_2 = 1, r_3 = 3\end{aligned}$$

Constant term = 1080

82. For some $a, b, c \in \mathbb{N}$, let $f(x) = ax - 3$ and $g(x) = x^b + c, x \in \mathbb{R}$. If $(f \circ g)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$, then $(f \circ g)(ac) + (g \circ f)(b)$ is equal to

Sol. 2039

$$\begin{aligned}\text{Let } f(g(x)) &= h(x) \\ f(g(x)) &= 2x^3 + 7 \\ a(x^b + c) - 3 &= 2x^3 + 7 \\ a = 2, b = 3, ac = 10 \\ c &= 5 \\ g(f(x))(3) &= 32 \\ f(g(10)) &= 2007 \\ \text{Sum} &= 2039\end{aligned}$$

83. Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is

Sol. 43

$$\begin{aligned}\text{No. of element 1} &= \{3\} \\ \text{No. of element 2} &= \{(3K + 1), (3K + 2)\} \\ &\quad (3) (3) = 9 \\ \text{No. of element 3} &= \{3k, 3k+1, 3K + 2\} = (1) (3) (3) = 9 \\ &= \{(3k + 1), (3k + 1), (3k + 1)\} = 1 \\ &= \{(3K + 2), (3k + 2), (3k + 2)\} = \frac{1}{11} \\ \text{No. of element 4} &= \{3k, 3k+1, 3k+1, 3k+1\} \rightarrow 1 \\ &= \{3k, 3k + 2, 3k+2, 3k+2\} \rightarrow 1 \\ &= \{3k+1, 3k+2, 3k+2, 3k+1\} \rightarrow {}^3C_2 \times {}^3C_2 = 9 \\ \text{No. of element 5} &= 9, \text{ no. of element 6} = 1, \text{ no. of element 7} = 1 \\ \text{Total} &= 43.\end{aligned}$$

84. Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is

Sol. Equation of plane is
 $(x - 2y - z - 5) + b(x + y + 3z - 5) = 0$

$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

 $\Rightarrow b = 12$
 Plane is $13x + 10y + 35z = 65$
 Distance From given point is $= 9$

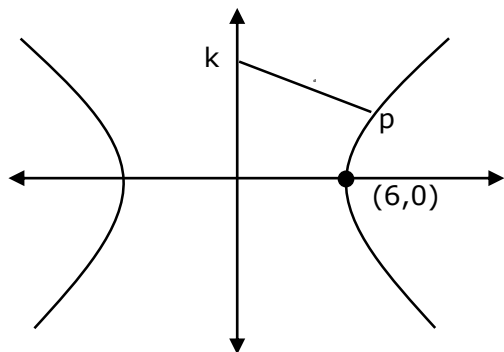
85. If the sum of all the solutions of $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}$, $-1 < x < 1$, $x \neq 0$, is $\alpha - \frac{4}{\sqrt{3}}$, then α is equal to

Sol. $\alpha = 2$

$$\begin{aligned} x \in (-1, 1) \quad \tan^{-1}\left(\frac{2x}{1-x^2}\right) &= 2\tan^{-1}x \\ x \in (0, 1) \quad \cot^{-1}\left(\frac{1-x^2}{2x}\right) &= \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x \\ x \in (-1, 0) \quad \cot^{-1}\left(\frac{1-x^2}{2x}\right) &= \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \pi + 2\tan^{-1}x \\ x \in (0, 1) \quad 2\tan^{-1}x + 2\tan^{-1}x &= \frac{\pi}{3} \\ \tan^{-1}x &= \frac{\pi}{12} \\ \boxed{x = 2 - \sqrt{3}} \\ x \in (-1, 0) \quad 2\tan^{-1}x + \pi + 2\tan^{-1}x &= \frac{\pi}{3} \\ 4\tan^{-1}x &= \frac{-2\pi}{3} \\ \tan^{-1}x &= \frac{-\pi}{6} \\ \boxed{x = -\frac{1}{\sqrt{3}}} \\ (2 - \sqrt{3}) + \left(-\frac{1}{\sqrt{3}}\right) &= \alpha - \frac{4}{\sqrt{3}} \\ 2 - \frac{4}{\sqrt{3}} &= \alpha - \frac{4}{\sqrt{3}} \\ \boxed{\alpha = 2} \end{aligned}$$

86. The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y -axis then d^2 is equal to

Sol. 216



$$H : \frac{x^2}{36} - \frac{y^2}{9} = 1$$

Equation of normal is $6x \cos\theta + 3y \cot\theta = 45$

$$M = -2\sin\theta = -\sqrt{2}$$

$$\theta = \pi/4$$

Equation of normal is $\sqrt{2}x + y = 15$

$P(\text{asec}\theta, b\tan\theta)$

$P(6\sqrt{2}, 3), K(0, 15)$

$$d^2 = 216$$

87. Let x and y be distinct integers where $1 \leq x \leq 25$ and $1 \leq y \leq 25$. Then, the number of ways of choosing x and y , such that $x + y$ is divisible by 5, is

Sol.

$$x + y = 5\lambda$$

X	y	No. of ways
5λ	5λ	20
$5\lambda + 1$	$5\lambda + 4$	25
$5\lambda + 2$	$5\lambda + 3$	25
$5\lambda + 3$	$5\lambda + 2$	25
$5\lambda + 4$	$5\lambda + 1$	25
		1200

Total Ways = 120

88. Let $S = \left\{ \alpha : \log_2(9^{2\alpha-4} + 13) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2 \right\}$. Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{\alpha \in S} \alpha\right)^2 x + \sum_{\alpha \in S} (\alpha+1)^2 \beta = 0$ has real roots, is

Sol. 25

$$\log_2 \left[\frac{9^{2\alpha-4} + 13}{3^{2\alpha-4} \cdot \frac{5}{2} + 1} \right] = 2$$

$$= \frac{9^{2\alpha-4} + 13}{3^{2\alpha-4} \cdot \frac{5}{2} + 1} = 4$$

$$= 9^{2\alpha-4} + 13 = 10 \cdot 3^{2\alpha-4} + 4$$

$$t^2 - 10t + 9 = 0$$

$$t = 1, 9$$

$$3^{2\alpha-4} = 3^0, 3^2$$

$$2\alpha - 4 = 0, 2$$

$$\alpha = 2, 3$$

$$x^2 - 2(25)x + 25\beta = 0$$

$$D \geq 0$$

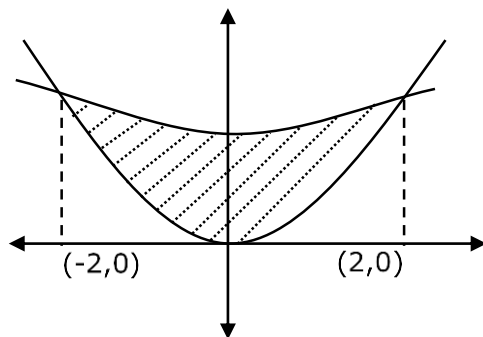
$$(2)^2 (25)^2 - 4(25)(\beta) \geq 0$$

$$\beta \leq 25$$

$$\beta_{\max} = 25$$

89. If the area enclosed by the parabolas $P_1: 2y = 5x^2$ and $P_2: x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = \alpha x$, $\alpha > 0$, then α^3 is equal to

Sol. **600**



$$y = \frac{5x^2}{2}, \quad y = x^2 + 6$$

$$\frac{5x^2}{2} = x^2 + 6$$

$$3x^2 = 12 \Rightarrow x^2 = 4$$

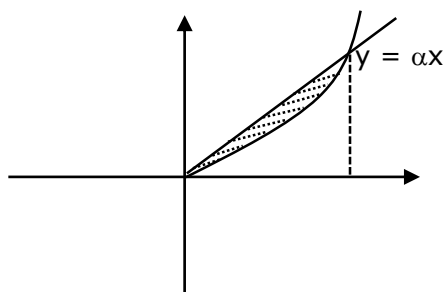
$$\boxed{x = \pm 2}$$

$$A_1 = 2 \int_0^2 \left(x^2 + 6 - \frac{5}{2}x^2 \right) dx$$

$$= 2 \int_0^2 \left(6 - \frac{3x^2}{2} \right) dx$$

$$= 2 \left[6x - \frac{x^3}{2} \right]_0^2 = 2[12 - 4]$$

$$= 16$$



$$y = \frac{5}{2}x^2, y = \alpha x (\alpha > 0)$$

$$\text{area} = \frac{8}{3} [a^2 m^3]$$

$$= \frac{8}{3} \left[\frac{1}{10} \right]^2 \cdot \alpha^3$$

$$= \frac{8}{300} - \alpha^3 = \frac{2}{75} \alpha^3$$

$$\therefore \frac{2}{75} - \alpha^3 = 16 \quad \Rightarrow \alpha^3 = 8 \times 75$$

$$\boxed{\alpha^3 = 600}$$

- 90.** Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A+1, A+2$, respectively. Let a, b, c be the $7^{\text{th}}, 9^{\text{th}}, 17^{\text{th}}$ terms of A_1, A_2, A_3 , respectively such that

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0.$$

If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal to

Sol.
$$\begin{vmatrix} A+6d & 7 & 1 \\ 21(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix} + 70 = 0$$

$$A = -7, d = 6$$

$$\therefore c - a - b = 20$$

$$\therefore S_{20} = 495$$