

JEE-Main-30-01-2023 (Morning shift)
[MORNING SHIFT]

Physics

Question: If the height of capillary rise is 5 cm for a liquid. What is the rise in height of the surface tension and density is doubled

Options:

- (a) 10 cm
- (b) 5 cm
- (c) 2.5 cm
- (d) 20 cm

Answer: (b)

Solution:

$$h = \frac{2T \cos \theta}{\rho g r} \Rightarrow h \propto \frac{T}{\rho}$$

h will remain same.

$$h = 5 \text{ cm}$$

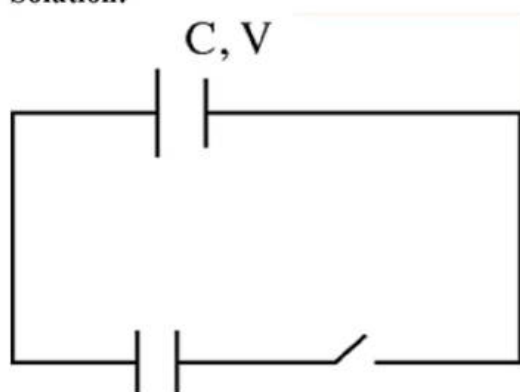
Question: Capacitor of $400 \mu F$ is connected to a 100 V battery. Now battery is removed and identical capacitor is connected. Find change in P.E.

Options:

- (a) 1 J
- (b) 2 J
- (c) 3 J
- (d) 4 J

Answer: (a)

Solution:



$$V_{\text{common}} = \frac{CV}{C+C} = \frac{V}{2}$$

$$\Delta P.E = \frac{1}{2}C\left(\frac{V}{2}\right)^2 \times 2 - \frac{1}{2}CV^2 = -\frac{1}{4}CV^2$$

Question: What is the correct relation between Young's Modulus (Y), modulus of rigidity (η), and Poisson ratio (σ) ?

Options:

- (a) $Y = 2\eta(1 + \sigma)$
- (b) $Y = \eta(1 - 2\sigma)$
- (c) $Y = 2\eta(1 + 2\sigma)$
- (d) $Y = 2\eta(1 - \sigma)$

Answer: (a)

Solution:

$$Y = 2\eta(1 + \sigma)$$

Question: The maximum and minimum voltage of an amplitude modulated signal are 120 V and 8V respectively. Find the amplitude of the side band.

Options:

- (a) 10 V
- (b) 20 V
- (c) 30 V
- (d) 60 V

Answer: (a)

Solution:

$$\mu = \frac{A_m}{A_c}$$

$$\mu = 0.2$$

$$A_{\max} = 120V = A_c + A_m$$

$$A_{\min} = 80V = A_c - A_m$$

$$\Rightarrow \mu \frac{AC}{2} = 0.2 \times \frac{100}{2} = 10V$$

Question: If in an isothermal process heat is given to a gas then (1) Work is positive (2) Work is negative (3) ΔU negative (5) $\Delta U = 0$. Choose the correct statement/s

Options:

- (a) Only 1 is correct
- (b) 1 and 5 are correct
- (c) 1, 3, and 5 are correct
- (d) None is correct

Answer: (b)

Solution:

$$\Delta Q = \Delta U + \Delta W$$

$$W = +ve$$

Hence, option b is correct

Question: Two coils of N_A and N_B number of turns carrying currents I_A and I_B respectively are having the radius as $r_A = 10\text{cm}$, $r_B = 20\text{cm}$. If their magnetic moments are same then

Options:

(a) $N_A I_A = 4 N_B I_B$

(b) $4 N_A I_A = N_B I_B$

(c) $N_A I_A = 2 N_B I_B$

(d) $2 N_A I_A = N_B I_B$

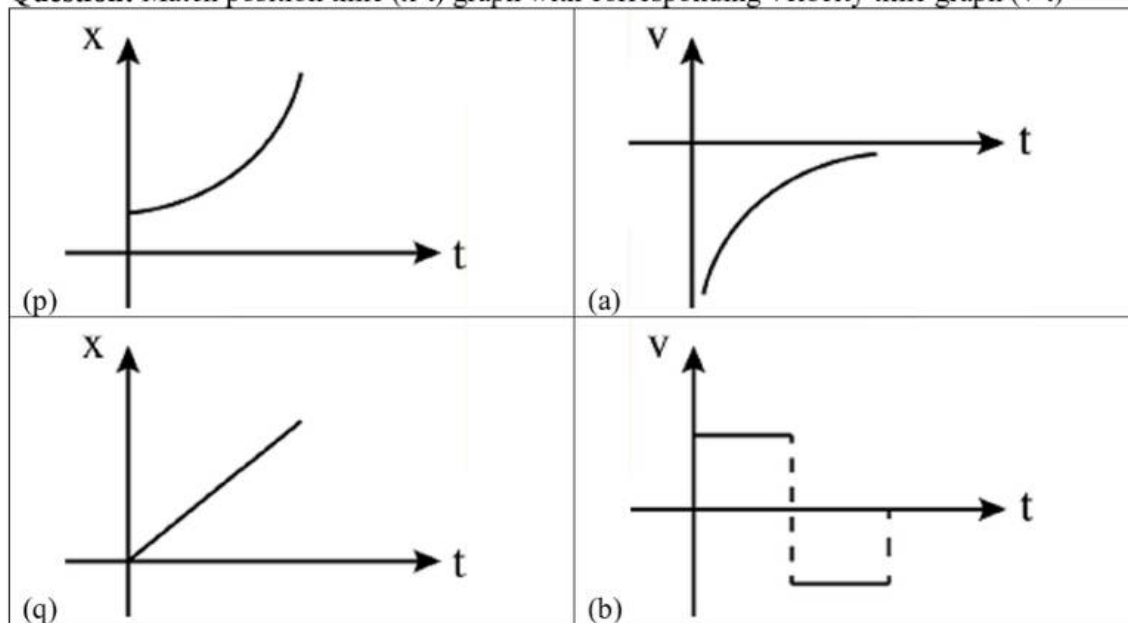
Answer: (a)

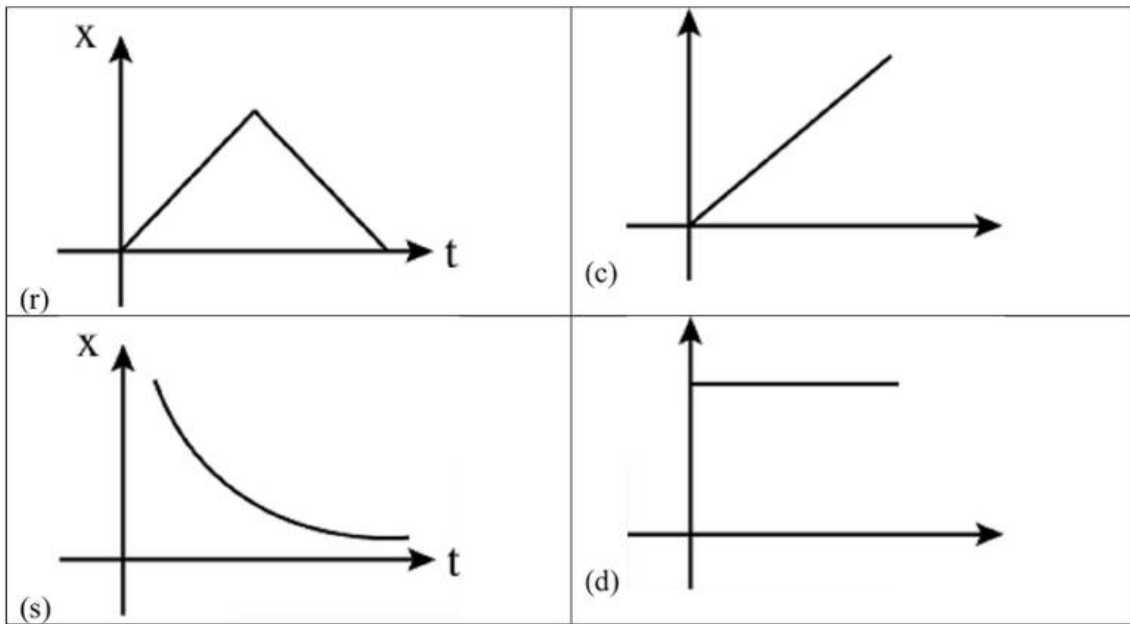
Solution: $m = niA \Rightarrow m_A = m_B$

$$N_A I_A (\pi r_A^2) = N_B I_B (\pi r_B^2)$$

$$\Rightarrow N_A I_A = 4 N_B I_B$$

Question: Match position time (x-t) graph with corresponding velocity time graph (v-t)

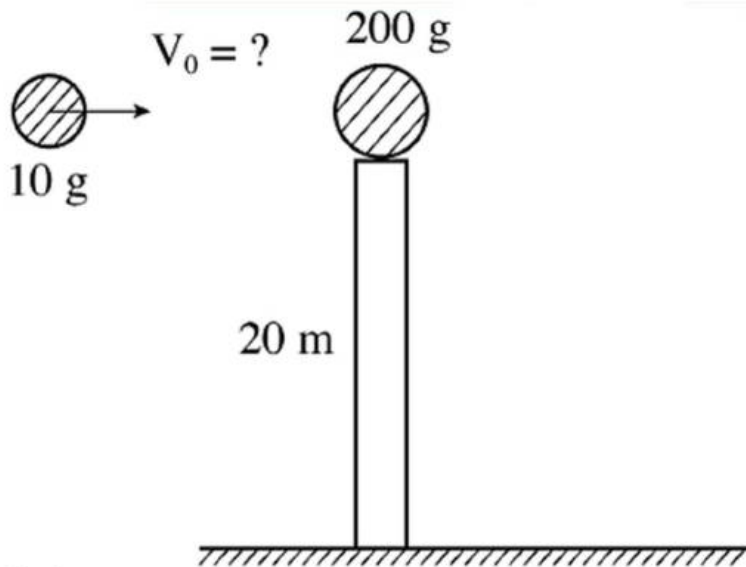




Solution:

$$p \rightarrow c, r \rightarrow v, q \rightarrow d, s \rightarrow a$$

Question: A bullet of mass 10 g strikes a ball of mass 200 g placed on a tower as shown. After collision bullet falls at 120 m from base of tower & ball falls at 30 m from the base of the tower. Find V_0 ?

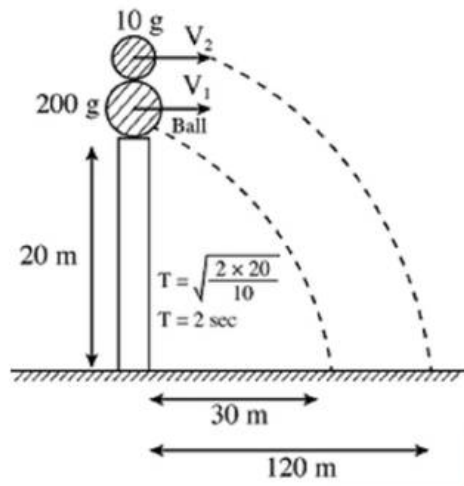


Options:

- (a) $360ms^{-1}$
- (b) $60ms^{-1}$
- (c) $400ms^{-1}$
- (d) $100ms^{-1}$

Answer: (a)

Solution:



$$\frac{10}{1000} \times V_0 = 0.2 \times 15 + 0.01 \times 60$$

$$V_0 = 360 \text{ ms}^{-1}$$

$$R = u \sqrt{\frac{2H}{g}}$$

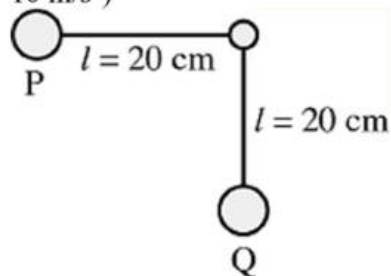
$$30 = V_1 (2)$$

$$V_1 = 15$$

$$120 = V_2 (2)$$

$$V_2 = 60$$

Question: Bob P is released from the position of rest at the moment shown. If it collides elastically with an identical bob Q hanging freely then velocity of Q just after collision is ($g = 10 \text{ m/s}^2$)

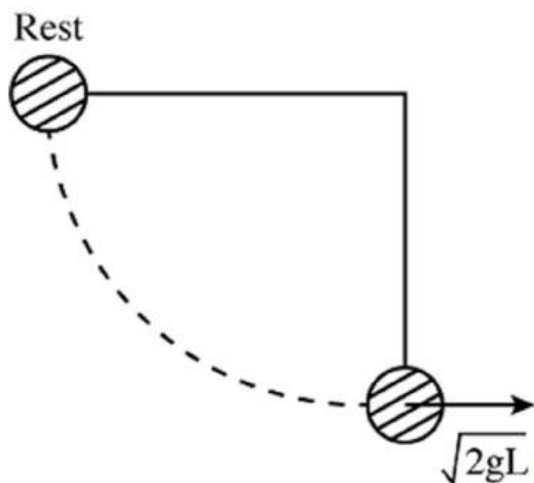


Options:

- (a) 1 m/s
- (b) 4 m/s
- (c) 2 m/s
- (d) 8 m/s

Answer: (c)

Solution:



$$L = \frac{1}{2}MV^2$$

$$V = \sqrt{2gL}$$

$$= \sqrt{2 \times 10^2 \times \frac{1}{5}}$$

$$\gamma = 2ms^{-1}$$

Question: The heat passing through the cross-section of a conductor, varies with time 't' as $Q(t) = \alpha t - \beta t^2 + \gamma t^3$ (α, β and γ are positive constants). The minimum heat current through the conductor is

Options:

(a) $\frac{\alpha - \beta^2}{2\gamma}$

(b) $\frac{\alpha - \beta^2}{3\gamma}$

(c) $\frac{\alpha - \beta^2}{\gamma}$

(d) $\frac{\alpha - 3\beta^2}{\gamma}$

Answer: (b)

Solution: $q = \alpha t - \beta t^2 + \gamma t^3$

$$I = \frac{dq}{dt} = \alpha - 2\beta t + 3\gamma t^2$$

Minima $I = \alpha - 2\beta t + 3\gamma t^2$

$$\frac{dI}{dt} = \alpha - 2\beta(1) + 3\gamma(2t) = 0$$

$$t = \frac{\beta}{3r}$$

$$I = \alpha - 2\beta \left[\frac{\beta}{3r} \right] + 3r \left[\frac{\beta^2}{9r^2} \right]$$

$$I = \alpha - \frac{2\beta^2}{3r} + \frac{\beta^2}{3r} = \alpha - \frac{\beta^2}{3r}$$

Question: In SHM $x = 20 \sin(\omega t)$. The slope of potential energy Vs time graph is maximum

at time $t = \frac{T}{\beta}$. Find β

Options:

- (a) 2
- (b) 4
- (c) 8
- (d) 16

Answer: (c)

Solution: $x = 20 \sin(\omega t)$

$$U = \frac{1}{2} kx^2$$

$$U = \frac{k}{2} \times 400 \sin^2(\omega t)$$

$$U = U_0 \sin^2(\omega t)$$

$$\text{Slope } \frac{dU}{dt} = U_0 2 \sin(\omega t) + \cos(\omega t) \omega$$

$$\text{Slope } \frac{dU}{dt} = [V_0 \omega] \sin[2\omega t]$$

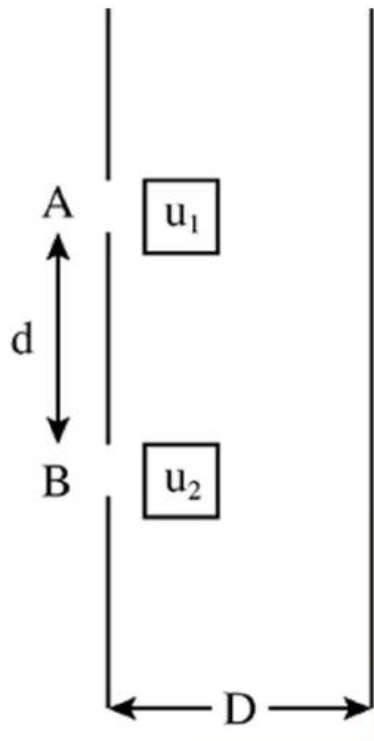
$$\sin(2\omega t) = 1$$

$$2\omega t = \frac{\pi}{2}$$

$$2 \times \frac{2\pi}{T} \times t = \frac{\pi}{2}$$

$$t = \frac{T}{8}$$

Question: In YDSE, with slits separation d and D is distance between slits and screen two slabs of thickness 't' each of $u_1 = 1.5$ and $u_2 = 2$ are introduced in front of slits. Find number of fringes that will surface after introducing slabs. Wavelength ' λ ' is used.



Options:

(a) $\mu_1 = 1.51$

(b) $\mu_2 = 1.55$

(c) $\lambda = 4000 \text{ \AA}$

(d) $t = 0.5 \text{ mm}$

Answer: (d)

Solution: $n\beta = (u_1 - 1)t - (u_2 - 1)t$

$$n \frac{\lambda D}{d} = 1(u_1 - u_2)t = .5t$$

$$n = \frac{td}{2\lambda d}$$

Question: Linear momentum vs time is shown $[t_1 > (t_2 - t_3)]$, Find the region of maximum and minimum force.

Options:

(a) Only a

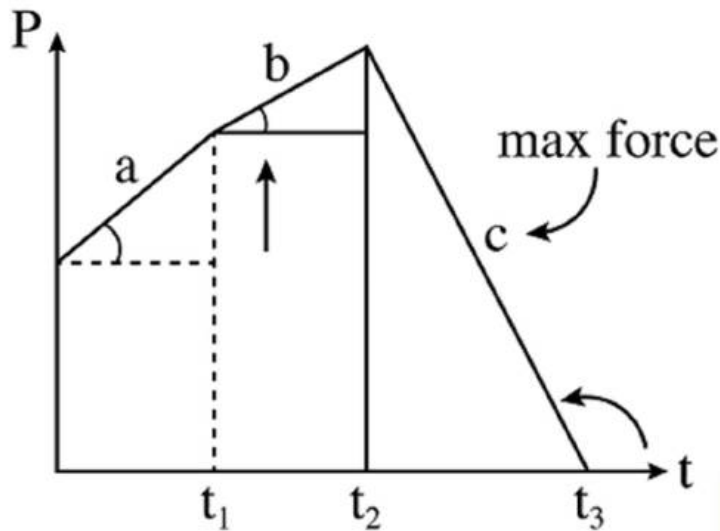
(b) a, b

(c) c, d

(d) None of these

Answer: (c)

Solution:



$$F = \frac{dp}{dt}$$

F = Slope of $p - t$

Question: If $\vec{E} = \frac{\alpha}{x^2} \hat{i} + \frac{\beta}{y^2} \hat{j}$; x and y are distances (in m) find SI units of α and β

Options:

(a) $\frac{Nm^2}{C}, \frac{Nm^3}{C}$

(b) $Nm^2, \frac{Nm^3}{C}$

(c) $\frac{Nm^2}{C}, Nm^3$

(d) Nm^2, Nm^3

Answer: (a)

Solution: $\vec{E} = \left[\frac{\alpha}{x^2} + \frac{\beta}{y^3} \right]$

$$\alpha \Rightarrow Ex^2 \Rightarrow NC^{-1}m^2$$

$$\beta \Rightarrow Ey^3 \Rightarrow NC^{-1}m^3$$

Question: Two spheres of radius ' r ' and ' $2r$ ' having same charge density u_0 are connected by a wire.

The new charge density is u' . Find $\frac{u'}{u_0}$ for each sphere.

Options:

(a) $\frac{5}{6}, \frac{5}{3}$

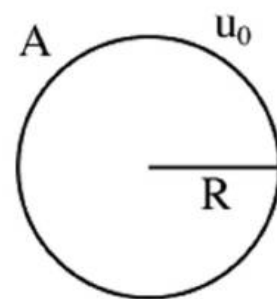
(b) $\frac{10}{3}, \frac{5}{6}$

(c) $\frac{5}{3}, \frac{5}{6}$

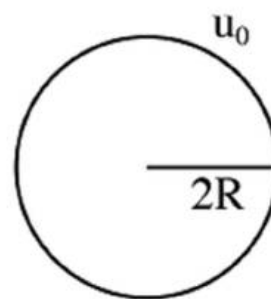
(d) $\frac{5}{6}, \frac{10}{3}$

Answer: (c)

Solution:

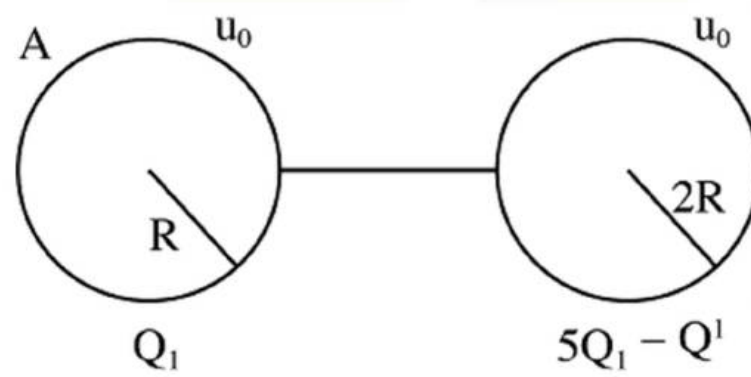


$$Q_1 = 4\pi R^2 u_0$$



$$Q_2 = 4\pi(2R)^2 u_0 = 4Q_1$$

$$Q_{total} = 5Q_1$$

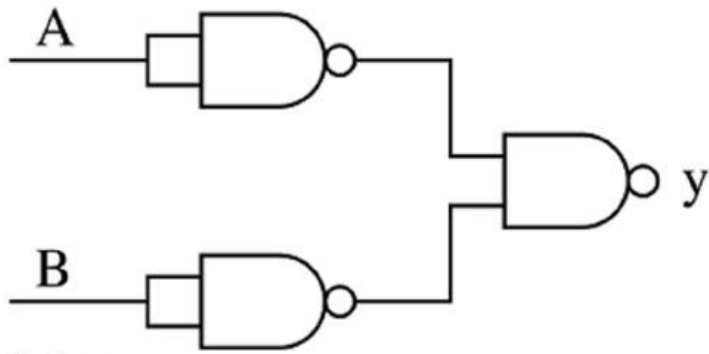


$$\frac{kQ'}{R} = \frac{k(5Q_1 - Q')}{2R}$$

$$\Rightarrow Q' = \frac{5Q_1 - Q'}{2} \Rightarrow Q' = \frac{5Q_1}{3}$$

$$\left(\frac{u'}{u_0}\right)_A = \frac{5}{3}; \quad \left(\frac{u'}{u}\right)_B = \frac{5}{6}$$

Question: Which gate is this



Options:

- (a) OR
- (b) AND
- (c) NAND
- (d) NOR

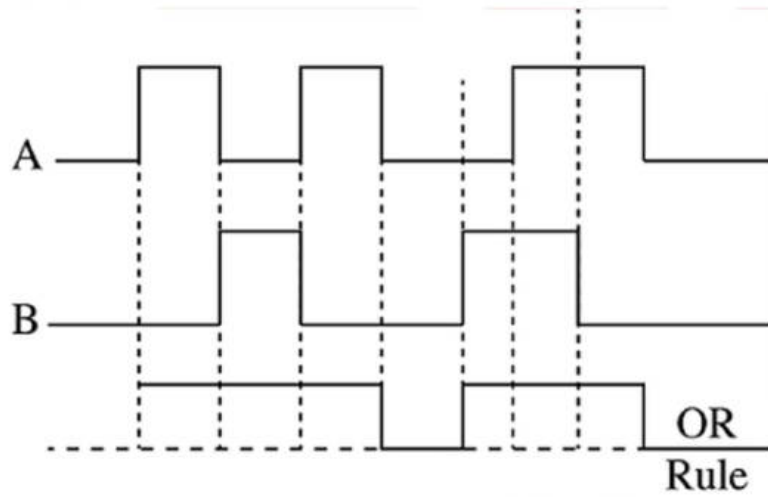
Answer: (a)

Solution:

$$y = [A'.B'] = A + B$$

$$(A + B)' = A'B'$$

$$(AB)' = A' + B'$$



JEE-Main-30-01-2023 (Memory Based)
[Morning Shift]

Chemistry

Question: Which of the following is antacid?

Options:

- (a) Sodium bicarbonate
- (b) Magnesium hydroxide
- (c) Magnesium carbonate
- (d) All of the above

Answer: (d)

Solution: Examples of antacid include sodium bicarbonate, magnesium hydroxide, magnesium carbonate and aluminium hydroxide, as they are all basic in nature.

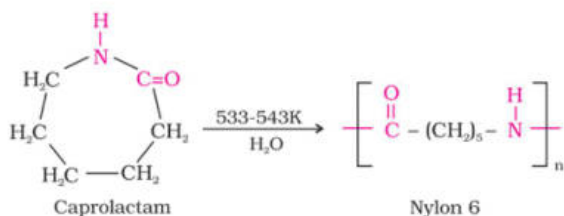
Question: Which of the following is formed on heating Caprolactum?

Options:

- (a) Nylon 6
- (b) Nylon 6,6
- (c) Nylon 2,6
- (d) None of these

Answer: (a)

Solution:



Question: $\text{NO}_2 + \text{sunlight} \rightarrow \text{A} + \text{B}$

$\text{B} + \text{O}_2 \rightarrow \text{O}_3$

$\text{NO} + \text{O}_3 \rightarrow \text{C} + \text{O}_2$

What is A, B and C?

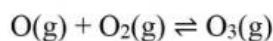
Options:

- (a) NO, O, NO_2
- (b) NO_2 , O, NO
- (c) NO_2 , NO, O
- (d) O, NO_2 , NO

Answer: (a)

Solution: $\text{NO}_2(\text{g}) \xrightarrow{h\nu} \text{NO}(\text{g}) + \text{O}(\text{g})$

Oxygen atoms are very reactive and combine with the O_2 in air to produce ozone.



The ozone formed in the above reaction (ii) reacts rapidly with the NO(g) formed in the reaction (i) to regenerate NO₂. NO₂ is a brown gas and at sufficiently high levels can contribute to haze.



Question: Which of the following is correct about OF₂?

Options:

- (a) Oxidation state of O is +2
- (b) Tetrahedral
- (c) V shaped
- (d) Bond angle is less than 104.5°

Answer: (a)

Solution:

$$\text{OF}_2 = x - 2 = 0$$

$$x = +2$$

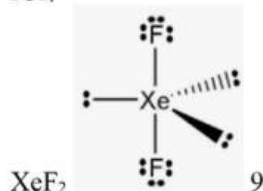
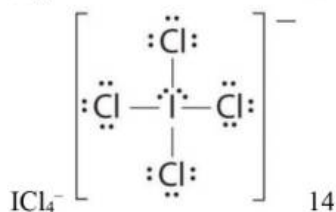
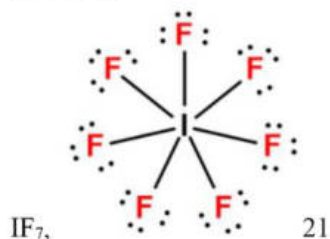
Question: Number of lone pairs in IF₇, ICl₄⁻, XeF₂, XeF₆, ICl

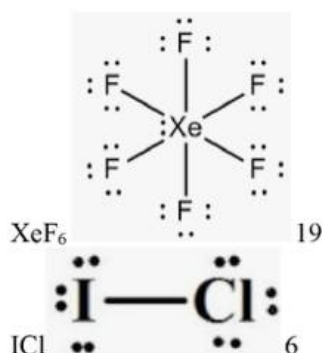
Options:

- (a) 21, 14, 9, 6, 19
- (b) 14, 21, 9, 6, 19
- (c) 19, 9, 21, 6, 14
- (d) 21, 14, 9, 19, 6

Answer: (d)

Solution:





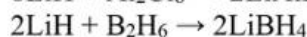
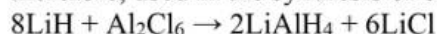
Question: Which of the following reaction can be used to prepared LiAlH₄?

Options:

- (a) LiCl + AlCl₃
- (b) LiH + Al(OH)₃
- (c) LiH + Al₂Cl₆
- (d) None of these

Answer: (c)

Solution: Lithium hydride is rather unreactive at moderate temperatures with O₂ or Cl₂. It is, therefore, used in the synthesis of other useful hydrides, e.g.,



Question: Which coordination compound is used for the treatment of cancer?

Options:

- (a) Potassium sulphocyanide
- (b) Cis-diamine dichloro platinum (II)
- (c) Trans-dichlorodiammine platinum (II)
- (d) [Ag(NH₃)₂]NO₃

Answer: (b)

Solution: Cisplatin {cis-[Pt(NH₃)₂Cl₂]} is used in the treatment of cancer.

Question: Arrange the following in increasing order of Strength

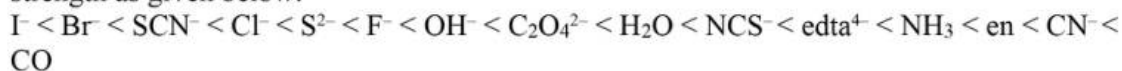
S²⁻, Oxalate, CO, ethylenediamine

Options:

- (a) S²⁻ < Oxalate < ethylenediamine < CO
- (b) S²⁻ < CO < ethylenediamine < Oxalate
- (c) ethylenediamine < CO < S²⁻ < Oxalate
- (d) ethylenediamine < Oxalate < S²⁻ < CO

Answer: (a)

Solution: In general, ligands can be arranged in a series in the order of increasing field strength as given below:



Question: Permanganate $\xrightarrow{\text{Acidic}}$ Manganese oxide

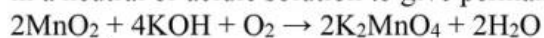
Change in oxidation number of Mn

Options:

- (a) +6 to +4
- (b) +4 to +6
- (c) +4 to +5
- (d) +7 to +4

Answer: (d)**Solution:** Potassium permanganate KMnO_4

Potassium permanganate is prepared by fusion of MnO_2 with an alkali metal hydroxide and an oxidising agent like KNO_3 . This produces the dark green K_2MnO_4 which disproportionates in a neutral or acidic solution to give permanganate.

**Question:** Frequency = 2×10^{12} Hertz

Calculate energy for one mole

Options:

- (a) 737.04
- (b) 797.04
- (c) 812.04
- (d) 997.14

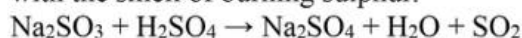
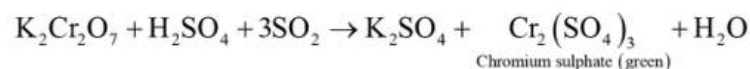
Answer: (b)**Solution:** The energy of one photon (E) = $h\nu$ Here, $h = 6.626 \times 10^{-34}$ js $\nu = 2 \times 10^{12}$ Hertz

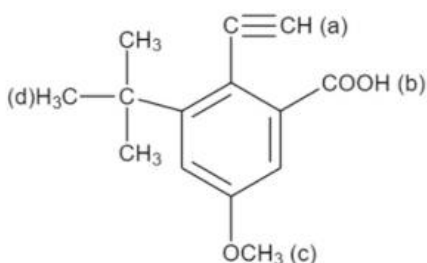
$$E = 6.626 \times 10^{-34} \times 2 \times 10^{12} \times 6.02 \times 10^{23}$$

$$= 797.04$$

Question: During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4 , SO_2 gas evolved which turns $\text{K}_2\text{Cr}_2\text{O}_7$ solution**Options:**

- (a) Green
- (b) Black
- (c) Blue
- (d) Red

Answer: (a)**Solution:** On treating sulphite with warm dil. H_2SO_4 , SO_2 gas is evolved which is suffocating with the smell of burning sulphur.The gas turns potassium dichromate paper acidified with dil. H_2SO_4 , green.**Question:**



Correct order of acidic strength of H_a , H_b , H_c , and H_d

Options:

(a) $H_b > H_a > H_c > H_d$

(b) $H_d > H_a > H_c > H_b$

(c) $H_c > H_a > H_d > H_b$

(d) $H_a > H_d > H_b > H_c$

Answer: (a)

Solution: $H_b > H_a > H_c > H_d$

Question: Which of the following is water soluble?

a) $BeSO_4$, b) $MgSO_4$, c) $CaSO_4$, d) $SrSO_4$, e) $BaSO_4$

Options:

(a) Only a and b

(b) Only a, b and c

(c) Only d and e

(d) Only a and e

Answer: (a)

Solution: Sulphates: The sulphates of the alkaline earth metals are all white solids and stable to heat. $BeSO_4$, and $MgSO_4$ are readily soluble in water; the solubility decreases from $CaSO_4$ to $BaSO_4$. The greater hydration enthalpies of Be^{2+} and Mg^{2+} ions overcome the lattice enthalpy factor and therefore their sulphates are soluble in water.

Question: Molarity of CO_2 in soft drink is 0.01 M. The volume of soft drink is 300 mL. Mass of CO_2 in soft drink is:

Options:

(a) 0.132 g

(b) 0.481 g

(c) 0.312 g

(d) 0.190 g

Answer: (a)

Solution:

0.01 mole in 1000 mL of solution

In 300 mL CO_2 will be 0.003 mole

Mass of CO_2 in 0.003 mole = $0.003 \times 44 = 0.132$ g

Question: Match the following.

Atomic no (Column-I)	(Column-II)
(i) 52	(p) s block

(ii) 37	(q) p block
(iii) 64	(r) d block
(iv) 78	(s) f block

Options:

- (a) (i) - q, (ii) - p, (iii) - s, (iv) - r
 (b) (i) - p, (ii) - q, (iii) - s, (iv) - r
 (c) (i) - s, (ii) - r, (iii) - p, (iv) - q
 (d) (i) - p, (ii) - r, (iii) - q, (iv) - s

Answer: (a)

Solution:

s block	37
p block	52
d block	78
f block	64

JEE-Main-30-01-2023 (Memory Based)
[Morning Shift]

Mathematics

Question: Let $S = \{1, 2, 3, 4, 5\}$. Find the number of one-one functions from S to $P(S)$.

Answer: ${}^{32}C_5 \times 5!$

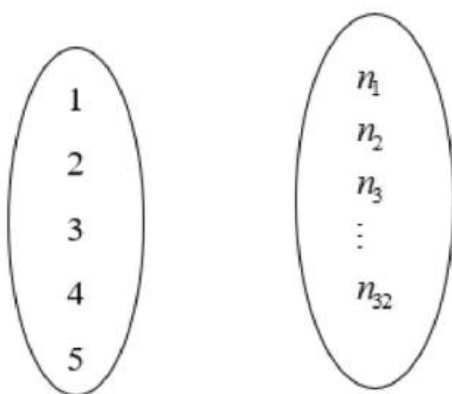
Solution:

$$f: S \rightarrow P(S)$$

$$S = \{1, 2, 3, 4, 5\}$$

$$n(S) = 5$$

$$n(P(S)) = 2^5 = 32$$



Thus, number of one-one function = ${}^{32}C_5 \times 5!$

Question: If $z = 1 + i$ and $z_1 = \frac{i + \bar{z}(1-i)}{\bar{z}(1-z)}$, then $\frac{12}{\pi} \arg(z_1) = ?$

Answer: 3.00

Solution:

We have, $z = 1 + i$

$$\text{And } z_1 = \frac{i + \bar{z}(1-i)}{\bar{z}(1-z)}$$

$$= \frac{i + (1-i)(1-i)}{(1-i)(1-1-i)}$$

$$\begin{aligned}
&= \frac{i + (1-i)^2}{(1-i)(-i)} \\
&= \frac{-i}{-i(1-i)} \\
&= \frac{1}{1-i} \\
&= \frac{i+1}{2}
\end{aligned}$$

$$\therefore \arg(z_1) = \arg\left(\frac{1}{2} + \frac{1}{2}i\right) = \frac{\pi}{4}$$

$$\therefore \frac{12}{\pi} \arg(z_1) = \frac{12}{\pi} \times \frac{\pi}{4} = 3$$

Question: Find the number of 4 digits numbers divisible by 15 using 1, 2, 3, 5, given that repetition is allowed.

Answer: 21.00

Solution:

Since required number is divisible by 15, so last digit will be 5.

$$\underline{a} \quad \underline{b} \quad \underline{c} \quad \underline{5}$$

The number should also be divisible by 3.

$$\text{So, } a + b + c + 5 = 3k$$

$$\Rightarrow a + b + c = 3t + 1$$

$$\text{Case-1: } a + b + c = 4$$

The digits can be filled by numbers (1, 1, 2) in 3 ways

$$\text{Case-2: } a + b + c = 7$$

The digits can be filled by numbers

(5, 1, 1) in 3 ways

(3, 3, 1) in 3 ways

(3, 2, 2) in 3 ways

$$\text{Case-3: } a + b + c = 10$$

The digits can be filled by numbers (5, 3, 2) in 6 ways.

$$\text{Case-4: } a + b + c = 13$$

The digits can be filled by numbers (5, 5, 3) in 3 ways

$$\therefore \text{Total number of ways} = 3 + 3 + 3 + 3 + 6 + 3 = 21 \text{ ways}$$

Question: The mean and variance of seven observations are 8 and 16 respectively. If observation 14 is omitted, the new mean and variance are 'a' and 'b'. Find $a + 3b - 5$.

Answer: 27

Solution:

Mean = 8, Variance = 16

$$\text{Thus, } \frac{\sum_{i=1}^6 x_i + 14}{7} = 8$$

$$\Rightarrow \sum_{i=1}^6 x_i = 56 - 14 = 42$$

$$\text{Now, new mean, } a = \frac{\sum x_i}{6} = \frac{42}{6} = 7$$

$$\text{Also, } \frac{\sum_{i=1}^6 x_i^2 + 14^2}{7} - 8^2 = 16$$

$$\Rightarrow \sum_{i=1}^6 x_i^2 = 560 - 196 = 364$$

$$\text{New Variance } = b = \frac{\sum x_i^2}{6} - a^2$$

$$= \frac{364}{6} - 7^2 = \frac{25}{3}$$

$$\therefore a + 3b - 5 = 7 + 3 \times \frac{25}{3} - 5$$

$$= 32 - 5 = 27$$

Question: $\lim_{x \rightarrow 0} \frac{48}{x^4} \int_0^x \frac{t^3}{t^6 + 1} dt = ?$

Answer: 12.00

Solution:

$$\lim_{x \rightarrow 0} \frac{48}{x^4} \int_0^x \frac{t^3}{t^6 + 1} dt$$

$$= \lim_{x \rightarrow 0} \frac{48 \int_0^x \frac{t^3}{t^6 + 1} dt}{x^4}$$

$$\lim_{x \rightarrow 0} \frac{48 \times \frac{x^3}{(x^6 + 1)}}{4x^3}$$

$$= \lim_{x \rightarrow 0} \frac{12}{x^6 + 1}$$

$$= 12$$

Question: Coefficient of x^{15} in $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ and coefficient of x^{-15} in $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$ are

equal, find relation between a and b .

Answer: $(ab)^3 = 1$

Solution:

General term of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is

$$T_{k+1} = {}^{15}C_k (ax^3)^{15-k} \left(\frac{1}{bx^{\frac{1}{3}}}\right)^k$$

$$= {}^{15}C_k \cdot a^{15-k} \cdot b^{-k} \cdot x^{45-3k-\frac{k}{3}}$$

For coefficient of x^{15} , we have

$$45 - 3k - \frac{k}{3} = 15$$

$$\Rightarrow \frac{10k}{3} = 30$$

$$\Rightarrow k = 9$$

General term of $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$ is

$$T_{k+1} = {}^{15}C_k \left(ax^{\frac{1}{3}}\right)^{15-k} \left(\frac{-1}{bx^3}\right)^k$$

For coefficient of x^{-15} , we have

$$5 - \frac{k}{3} - 3k = -15$$

$$\Rightarrow \frac{10k}{3} = 20$$

$$\Rightarrow k = 6$$

According to Question

$${}^{15}C_9 \frac{a^6}{b^9} = {}^{15}C_6 \cdot \frac{a^9}{b^6}$$

$$\Rightarrow (ab)^6 = (ab)^9$$

$$\Rightarrow (ab)^3 = 1$$

Question: A dice with numbers -2, -1, 0, 1, 2, 3, written on its faces is rolled 5 times. What is the probability that product of the numbers obtained is positive?

Answer: $\frac{521}{2592}$

Solution:

We have numbers -2, -1, 0, 1, 2, 3 on the dice.

$$\therefore P(\text{positive numbers}) = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{negative numbers}) = \frac{2}{6} = \frac{1}{3}$$

Now, for the product of numbers to be positive, we may obtain 0 negative numbers, 2 negative numbers or 4 negative numbers.

Let X be number of times negative number is obtained.

$$\therefore P(X=0) = {}^5C_0 \left(\frac{1}{3}\right)^0 \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$P(X=2) = {}^5C_2 \left(\frac{1}{3}\right)^2 \left(\frac{1}{2}\right)^3 = \frac{5}{36}$$

$$P(X=4) = {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{1}{2}\right) = \frac{5}{162}$$

$$\text{Required probability} = P(X=0) + P(X=2) + P(X=4)$$

$$= \frac{1}{32} + \frac{5}{36} + \frac{5}{162} = \frac{521}{2592}$$

Question: For a sequence, if $a_n = \frac{-2}{4n^2 - 16n + 15}$, then $a_1 + a_2 + \dots + a_{25} = \underline{\hspace{2cm}}$.

Answer: $\frac{50}{141}$

Solution:

$$a_n = \frac{-2}{4n^2 - 6n + 15}$$

$$\Rightarrow a_n = \frac{-2}{(2n-3)(2n-5)}$$

$$\Rightarrow a_n = \frac{(2n-5) - (2n-3)}{(2n-3)(2n-5)}$$

$$\Rightarrow a_n = \frac{1}{2n-3} - \frac{1}{2n-5}$$

$$\therefore a_1 = \frac{1}{-1} - \frac{1}{-3}$$

$$a_2 = \frac{1}{1} - \frac{1}{-1}$$

$$a_6 = \frac{1}{3} - \frac{1}{1}$$

$\vdots$

$$a_{25} = \frac{1}{47} - \frac{1}{45}$$

$$\therefore a_1 + a_2 + \dots + a_{25} = \frac{1}{3} + \frac{1}{47} = \frac{50}{141}$$

Question: The shortest distance between the line $\frac{x+4}{2} = \frac{y+6}{-1} = \frac{z}{2}$ and the line passing through $(2, 6, 2)$ and perpendicular to the plane $2x - 3y + z = 0$.

Answer: $\frac{46}{\sqrt{45}}$

Solution:

We have,

$$L_1: \frac{x+4}{2} = \frac{y+6}{-1} = \frac{z}{2}$$

L_2 : Line passing through $(2, 6, 2)$ and perpendicular to $2x - 3y + z = 0$

$\therefore$ Shortest distance between L_1 & L_2 is $\frac{a}{b}$.

$$\text{Where } a = \begin{vmatrix} 6 & 12 & 2 \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = 6(-1+6) - 12(2-4) + 2(-6+2)$$

$$= 6(5) - 12(-2) + 2(-4)$$

$$= 30 + 24 - 8$$

$$= 46$$

And $b = \text{magnitude of } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = \text{magnitude of } (5\hat{i} + 2\hat{j} + 4\hat{k})$

$$= \sqrt{25 + 4 + 16} = \sqrt{45}$$

$$\therefore \text{S.D.} = \frac{46}{\sqrt{45}}$$

Question: $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero vectors such that $\hat{n} \perp \vec{c}$, $\vec{a} = \alpha\vec{b} - \hat{n}$; $\alpha \neq 0$ and $\vec{b} \cdot \vec{c} = 12$, then $|\vec{c} \times (\vec{a} \times \vec{b})| = ?$

Options:

- (a) 9
- (b) 6
- (c) 12
- (d) 5

Answer: (c)

Solution:

We have,

$$\vec{a} = \alpha\vec{b} - \hat{n}$$

$$\Rightarrow \vec{a} + \hat{n} = \alpha\vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{c} + \hat{n} \cdot \vec{c} = \alpha\vec{b} \cdot \vec{c}$$

$$\Rightarrow \vec{a} \cdot \vec{c} = 12\alpha$$

$$\text{Now } |\vec{c} \times (\vec{a} \times \vec{b})|$$

$$= |(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}|$$

$$= |12\vec{a} - 12\alpha\vec{b}|$$

$$= 12|\vec{a} - \alpha\vec{b}|$$

$$= 12|-\hat{n}|$$

$$= 12$$

Question: The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \dots + x^{500}$ is

Answer: $^{501}C_{301}$

Solution:

$$\text{Let } S = (1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \dots + x^{500}$$

This is a GP with $a = (1+x)^{500}$ and $r = \left(\frac{x}{1+x}\right)$

$$\begin{aligned}\therefore S &= (1+x)^{500} \left[\frac{1 - \left(\frac{x}{1+x}\right)^{501}}{1 - \left(\frac{x}{1+x}\right)} \right] \\ &= \frac{(1+x)^{500} \left[(1+x)^{501} - x^{501} \right]}{(1+x)^{501}} (1+x) \\ &= (1+x)^{501} - x^{501}\end{aligned}$$

Thus, the coefficient of x^{301} is given by $^{501}C_{301}$

Question: If $\sum_{n=0}^{\infty} \frac{n^3 \{(2n)!\} + (2n-1)n!}{n! \times (2n)!} = ae + \frac{b}{c} + c$, where $e = \sum_{n=0}^{\infty} \frac{1}{n!}$, then find $a^2 - b + c$.

Answer: 26.00

Solution:

$$\sum_{n=0}^{\infty} \frac{n^3 (2n)!}{n! (2n)!} + \sum_{n=0}^{\infty} \frac{(2n-1)n!}{n! (2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(2n-1)n!}{n! (2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(2n) \times n!}{n! (2n)!} - \frac{n!}{n! (2n)!}$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

$$\left. \frac{e^x - e^{-x}}{2} \right|_{x=1} - \left. \frac{e^x + e^{-x}}{2} \right|_{x=1}$$

$$\sum_{n=0}^{\infty} \frac{n^3 (2n)!}{n! (2n)!} = \sum_{n=0}^{\infty} \frac{n^3}{n!}$$

$$n^3 = an(n-1)(n-2) + b(n)(n-1) + cn + d$$

$$n=1 \Rightarrow 1 = C$$

$$n=2 \Rightarrow 8 = 2b + 2 \Rightarrow b = 3$$

$$\sum_{n=0}^{\infty} \frac{n(n-1)(n-2) + 3(n)(n-1) + n}{n!}$$

$$\sum_{n=3}^{\infty} \frac{1}{(n-3)!} + 3 \sum_{n=2}^{\infty} \frac{1}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$$

$$\sum_{n=0}^{\infty} \frac{n^3 (2n)!}{n! (2n)!} + \sum_{n=0}^{\infty} \frac{(2n-1)n!}{n! (2n)!}$$

$$= 5e + \frac{e}{2} - \frac{1}{\frac{e}{2}} - \frac{e}{2} + \frac{1}{\frac{e}{2}}$$

$$= 5e - \frac{1}{e}$$

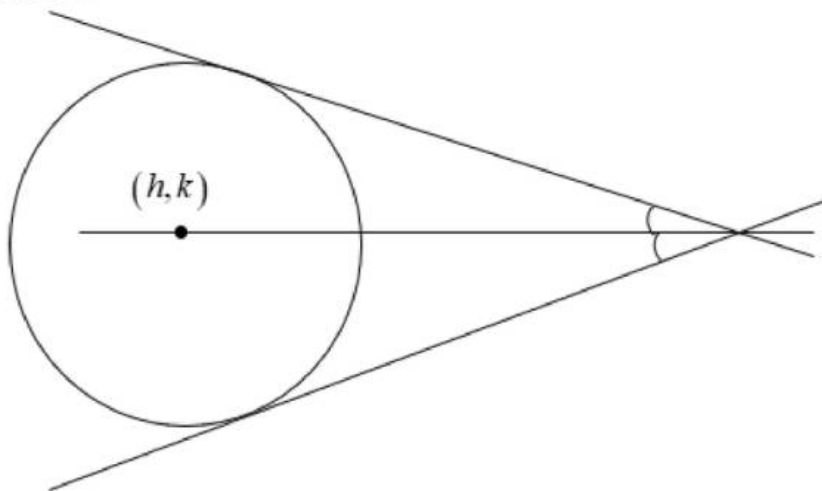
$$a = 5, b = -1, c = 0$$

$$\text{Now, } a^2 - b + c = 25 - (-1 + 0) = 26$$

Question: If $y = x + 1, 3y = 4x + 3, 4y = 3x + 6$ are tangents of the circle $(x-h)^2 + (y-k)^2 = r^2$, then find $(h+k)$.

Answer: 3.00

Solution:



$$4x - 3y + 3 = 0; 2x - 4y + 6 = 0$$

Angle bisectors:

$$\frac{4x - 3y + 3}{5} = \pm \left(\frac{2x - 4y + 6}{5} \right) \quad \dots (i)$$

Taking '+' on RHS we get

$$20x - 15y + 15 = 15x - 20y + 30$$

$$\Rightarrow 5x + 5y - 15 = 0$$

$$\Rightarrow x + y - 3 = 0$$

Now, this pass through centre (h, k)

$$\therefore h + k - 3 = 0$$

$$\Rightarrow h + k = 3$$

On taking '-' on RHS of (i), we get

$$20x - 15y + 15 = -15x + 20y - 30$$

$$\Rightarrow 35x - 35y + 45 = 0$$

Slope of above line is equal to the slope of third tangent, $y = x + 2$

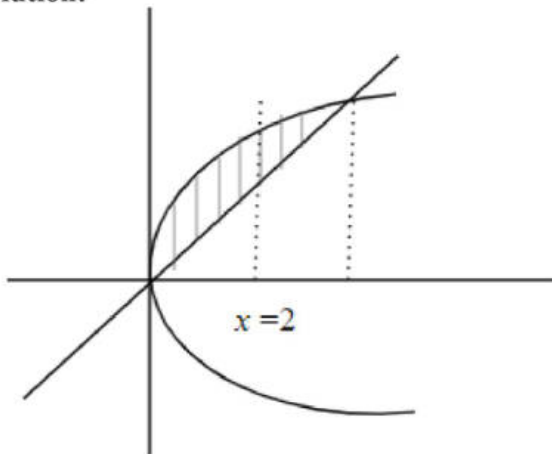
Thus, this forms external angle bisector

So, we reject this case.

Question: If the bigger area in first quadrant bounded by the curve $y^2 = 8x$, and the lines $y = x$, and $x = 2$ is α , then the value of 3α is

Answer: 22.00

Solution:



On solving $y^2 = 8x$ and $y = x$, we get

$$x = 0, 8$$

$$\text{Now, Shaded area} = \int_2^8 (2\sqrt{2} \cdot \sqrt{x} - x) dx$$

$$= 2\sqrt{2} \int_2^8 \sqrt{x} dx - \int_2^8 x dx$$

$$= \frac{4\sqrt{2}}{3} \left[x^{\frac{3}{2}} \right]_2^8 - \frac{1}{2} \left[x^2 \right]_2^8$$

$$\begin{aligned}
&= \frac{4\sqrt{2}}{3} \left[8^{\frac{3}{2}} - 2^{\frac{3}{2}} \right] - \frac{1}{2} [8^2 - 2^2] \\
&= \frac{4\sqrt{2} \times 2\sqrt{2}}{3} [8 - 1] - \frac{1}{2} \times 60 \\
&= \frac{8 \times 2 \times 7}{3} - 30 = \frac{22}{3}
\end{aligned}$$

Given that, shaded area = $\alpha = \frac{22}{3}$

$$\therefore 3\alpha = 22$$

Question: If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then $a + \frac{1}{a}$ is equal to

Answer:

Solution:

$$\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$$

$$\Rightarrow \tan 15^\circ + \frac{1}{\cot 15^\circ} + \frac{1}{(-\cot 15^\circ)} + \tan 15^\circ = 2a$$

$$\Rightarrow 2 \tan 15^\circ = 2a$$

$$\Rightarrow \tan 15^\circ = a$$

$$\text{Now, } a + \frac{1}{a} = \tan 15^\circ + \frac{1}{\tan 15^\circ}$$

$$= 2 - \sqrt{3} + \frac{1}{2 - \sqrt{3}}$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3}$$

$$= 4$$

Question: The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c)\}$ defined on the set $\{a, b, c\}$ to make it symmetric and transitive is

Answer: 7.00

Solution:

Taking symmetric, transitive elements

$$\{(a, b), (b, c), (b, a), (c, b), (a, c), (a, a), (b, b), (c, c), (c, a)\}$$

We have added 7 new elements

Question: If $5f(x+y) = f(x) \cdot f(y)$ and $f(2) = 3$, then $\sum_{n=0}^5 f(n) = ?$

Answer: 6825.00

Solution:

$$5f(x+y) = f(x) \cdot f(y) \quad \dots(1)$$

Put $x=1, y=2$ in (1)

$$5f(3) = f(1) \cdot f(2)$$

$$\Rightarrow f(1) \cdot f(2) = 5 \times 320 = 1600 \quad \dots(2)$$

Put $x=y=1$ in (1)

$$f(2) = \frac{(f(1))^2}{5} \quad \dots(3)$$

Using (2) and (3)

$$f(1) \cdot \frac{f(1)^2}{5} = 16000$$

$$(f(1))^3 = 80000$$

$$f(1) = 20$$

$$x=1, y=1$$

$$5f(2) = (20)^2$$

$$f(2) = 20 \times 4 = 80$$

$$x=1, y=2$$

$$5f(3) = f(1) \times f(2)$$

$$f(3) = \frac{20 \times 80}{5} = 320$$

$$x=1, y=3$$

$$5f(4) = 20 \times 320$$

$$f(4) = 1280$$

$$x=1, y=4$$

$$5f(5) = 20 \times 1280$$

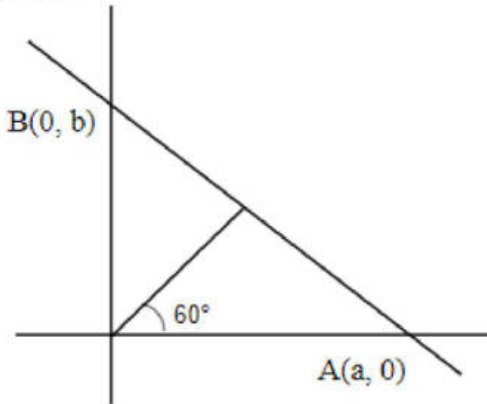
$$f(5) = 4 \times 1280 = 5120$$

So, total = $5 + 20 + 80 + 320 + 1280 + 5120 = 6825$

Question: A line intercepts x and y-axes at $A(a, 0)$ and $B(0, b)$. Area of triangle OAB is $\frac{98}{\sqrt{3}}$ and normal to line from origin makes angle 30° with y-axis. Find $a^2 - b^2$.

Answer: $\frac{392}{3}$

Solution:



$$\frac{1}{2} a \times b = \frac{98}{\sqrt{3}}$$

$$\text{Slope} = \frac{-b}{a} = -\frac{1}{\sqrt{3}}$$

$$a = \sqrt{3}b$$

$$\frac{1}{2} \sqrt{3}b^2 = \frac{98}{\sqrt{3}}$$

$$b^2 = \frac{196}{3}$$

$$a^2 = 3b^2$$

$$a^2 - b^2 = 3b^2 - b^2 = 2b^2$$

$$2b^2 = 2 \times \frac{196}{3} = \frac{392}{3}$$

Question: A line has direction ratios $(\cos \alpha, \cos \beta, \cos \gamma)$, $\beta \in \left(0, \frac{\pi}{2}\right)$. If this line is perpendicular to $2x - 3y + z = 10$, then α and γ belongs to ____.

Answer: Second Quadrant

Solution:

Given line has direction ratios as $\langle \cos \alpha, \cos \beta, \cos \gamma \rangle$

This line is perpendicular to $2x - 3y + z = 10$

Let $l = \cos \alpha$, $m = \cos \beta$, $n = \cos \gamma$

Then, $\hat{l}\hat{i} + m\hat{j} + n\hat{k}$ is parallel to $2\hat{i} - 3\hat{j} + \hat{k}$

$$\text{i.e., } \hat{l}\hat{i} + m\hat{j} + n\hat{k} = \frac{\pm(2\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{14}}$$

$$\text{Now, } \beta \left(0, \frac{\pi}{2} \right) \Rightarrow m > 0$$

$$\therefore \hat{l}\hat{i} + m\hat{j} + n\hat{k} = \frac{-2\hat{i}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$$

$$\Rightarrow \cos \alpha = \frac{-2}{\sqrt{14}}, \cos \gamma = \frac{-1}{\sqrt{14}}$$

$\Rightarrow \alpha$ & γ belongs to IInd quadrant.

Question: Evaluate: $I = 3 \left(\frac{e-1}{e} \right) \int_1^2 x^2 \cdot e^{[x] + [x^3]} dx$

Answer: $e(e^7 - 1)$

Solution:

$$\int_1^2 x^2 e^{1+[x^3]} dx$$

$$e \int_1^2 x^2 \times e^{[x^3]} dx$$

Put $x^3 = t$

$$3x^2 dx = dt$$

$$\frac{e}{3} \int_1^8 e^{[t]} dt$$

$$\frac{e}{3} \left[\int_1^2 e^1 + \int_2^3 e^2 + \int_3^4 e^3 + \dots + \int_7^8 e^7 \right]$$

$$\frac{e}{3} [e + e^2 + \dots + e^7]$$

$$= \frac{e}{3} \times e \frac{(e^7 - 1)}{(e - 1)}$$

$$= e(e^7 - 1)$$

Question: A line with Direction ratios $(1, 4, 3)$ is perpendicular to the plane $ax + by + cz = 1$. If the point $(1, 1, 2)$ lies in the plane, then find $a - b + c$.

Answer: 0.00

Solution:

Given $(1, 4, 3)$

$$a + b + 2c = 1$$

$$a, b, c \propto (1, 4, 3)$$

$$a, b, c = t, 4t, 3t$$

$$t + 4t + 6t = 1$$

$$11t = 1$$

$$t = \frac{1}{11}$$

$$(a, b, c) = \left(\frac{1}{11}, \frac{4}{11}, \frac{3}{11} \right)$$

$$a - b + c = 0$$