

26th Feb. 2021 | Shift - 2 PHYSICS

SECTION – A

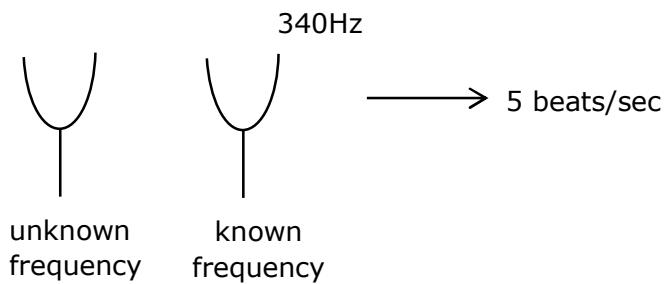
1. A tuning fork A of unknown frequency produces 5beats/s with a fork of known frequency 340 HZ. When fork A filed, the beat frequency decreases to 2beats/s. What is the frequency of fork A?

- (1) 342 Hz
- (2) 335 Hz
- (3) 338 Hz
- (4) 345 Hz

Sol. (2)

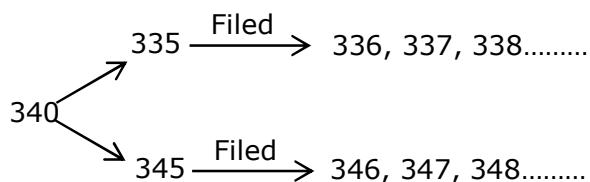
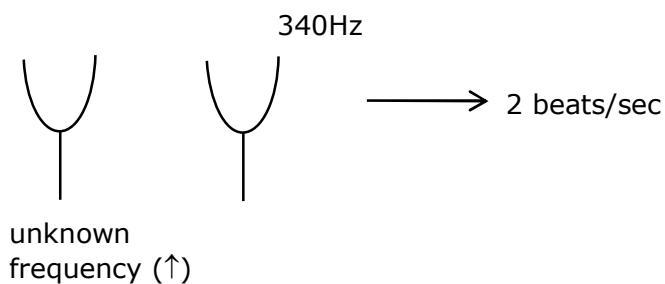
Given

Before Filed:



So answer should be 335 Hz or 345 Hz.

After Filed :



After filed beat/sec decreases only in case of 335 Hz.

2. The trajectory a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distance of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by:

(1) $\tan^{-1} \alpha, \frac{\alpha^2}{4\beta}$

(2) $\tan^{-1} \beta, \frac{\alpha^2}{2\beta}$

(3) $\tan^{-1} \left(\frac{\beta}{\alpha} \right), \frac{\alpha^2}{\beta}$

(4) $\tan^{-1} \alpha, \frac{4\alpha^2}{\beta}$

Sol. (1)

Given :

$$y = \alpha x - \beta x^2 \quad \dots(1)$$

for maximum height, we should find out maximum value of y from equation (1)

so, for maximum value of y

$$\frac{dy}{dx} = 0 \Rightarrow \alpha - 2\beta x = 0$$

$$x = \frac{\alpha}{2\beta} \quad \dots(2)$$

Now, put value of x from equation (2) in equation (1)

$$y = \alpha \left(\frac{\alpha}{2\beta} \right) - \beta \left(\frac{\alpha^2}{4\beta^2} \right)$$

$$\Rightarrow \left(\frac{\alpha^2}{2\beta} \right) - \left(\frac{\alpha^2}{4\beta} \right) \Rightarrow \frac{\alpha^2}{4\beta}$$

$$\text{So, } H_{\max} = \frac{\alpha^2}{4\beta} \quad \dots(3)$$

$$\text{As we know maximum height } H_{\max} = \frac{u^2 \sin^2 \theta}{2g} \quad \dots(4)$$

$$\text{from (3) and (4) } u^2 = \left(\frac{\alpha^2}{4\beta} \right) \left(\frac{2g}{\sin^2 \theta} \right)$$

$$\text{and range (R) = } 2x = \frac{u^2 \times 2 \sin \theta \cos \theta}{g}$$

$$2 \left(\frac{\alpha}{2\beta} \right) = \frac{\left(\frac{\alpha^2}{4\beta} \right) \left(\frac{2g}{\sin^2 \theta} \right) \times 2 \sin \theta \cos \theta}{g}$$

$$\tan \theta = \alpha \Rightarrow \theta = \tan^{-1} (\alpha)$$

3. A cord is wound round the circumference of wheel of radius r . The axis of the wheel is horizontal and the moment of inertia about it is I . A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance ' h ', the square of angular velocity of wheel will be:

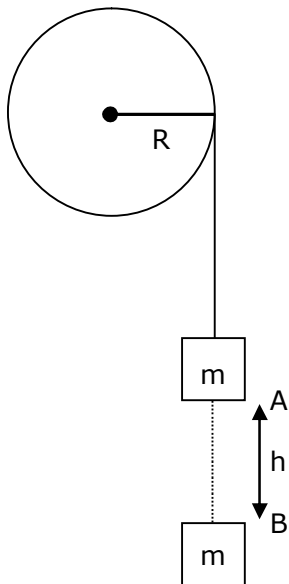
(1) $\frac{2gh}{I + mr^2}$

(2) $2gh$

(3) $\frac{2mgh}{I + 2mr^2}$

(4) $\frac{2mgh}{I + mr^2}$

Sol. (4)



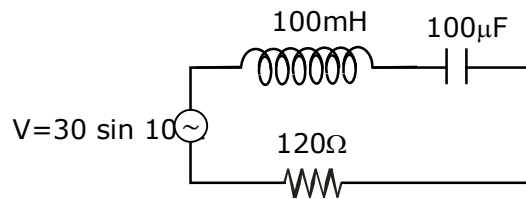
using energy conservation between A and B point

$$mgh = \frac{1}{2} m (wR)^2 + \frac{1}{2} I \omega^2$$

$$2mgh = (MR^2 + I) \omega^2$$

$$\omega^2 = \frac{2mgh}{I + MR^2}$$

4. Find the peak current and resonant frequency of the following circuit (as shown in figure)



- (1) 0.2 A and 100 Hz
 (2) 2 A and 50 Hz
 (3) 2 A and 100 Hz
 (4) 0.2 A and 50 Hz

Sol. (4)

Peak current in series LCR CKT

$$i = \frac{v_0}{z} \Rightarrow \frac{30}{\sqrt{(x_L - x_C)^2 + R^2}}$$

$$i = \frac{30}{\sqrt{(10 - 100)^2 + (120)^2}}$$

$$i \Rightarrow \frac{30}{150} \Rightarrow \frac{1}{5} \Rightarrow 0.2 \text{ Amp.}$$

$$\therefore X_L = \omega \times L$$

$$\Rightarrow (100) (100 \times 10^{-3}) \Rightarrow 10$$

$$X_L = \frac{1}{\omega \times c} \Rightarrow \frac{1}{100 \times 100 \times 10^{-6}}$$

$$\Rightarrow \frac{10^6}{10^4} \Rightarrow 100$$

$$\text{Resonance frequency } \omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}} \Rightarrow \frac{1}{\sqrt{10^{-5}}}$$

$$\therefore \omega = 2\pi F$$

$$F = \frac{1}{2\pi} \times \frac{1}{\sqrt{10^{-5}}}$$

$$\Rightarrow \frac{1}{2\pi} \sqrt{10^5}$$

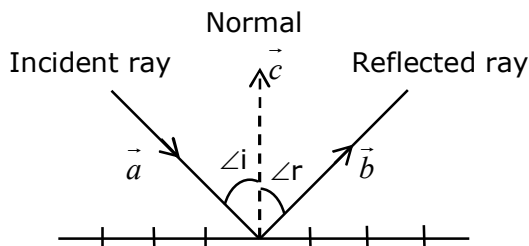
$$\Rightarrow \frac{100}{2\pi} \sqrt{10}$$

$$\Rightarrow 50 \text{ Hz}$$

5. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

- (1) $\vec{b} = 2\vec{a} + \vec{c}$
 (2) $\vec{b} = \vec{a} - \vec{c}$
 (3) $\vec{b} = \vec{a} + 2\vec{c}$
 (4) $\vec{b} = \vec{a} - 2(\vec{a} \cdot \vec{c})\vec{c}$

Sol. (4)



We see from the diagram that because of the law of reflection, the component of the unit vector $\vec{a}$ along $\vec{b}$ changes sign on reflection while the component parallel to the mirror remain unchanged.

$$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$$

$$\text{and } \vec{a}_{\perp} = \vec{c}(\vec{a} \cdot \vec{c})$$

we see that the reflected unit vector is

$$\vec{b} = \vec{a}_{\parallel} - \vec{a}_{\perp} \Rightarrow \vec{b} = \vec{a} - 2(\vec{a} \cdot \vec{c})\vec{c}$$

6. A radioactive sample is undergoing α decay. At any time t_1 , its activity is A and another time t_2 , the activity is $\frac{A}{5}$. What is the average life time for the sample?

- (1) $\frac{t_2 - t_1}{\ln 5}$
 (2) $\frac{\ln(t_2 + t_1)}{2}$
 (3) $\frac{t_1 - t_2}{\ln 5}$
 (4) $\frac{\ln 5}{t_2 - t_1}$

Sol. (1)

For activity of radioactive sample

$$A = A_0 e^{-\lambda t_1} \quad \dots(1)$$

$$\frac{A}{5} = A_0 e^{-\lambda t_2} \quad \dots(2)$$

From (1)/(2)

$$5 = e^{-\lambda(t_1 - t_2)}$$

$$\ln(5) = (t_2 - t_1) \lambda \Rightarrow \lambda = \frac{\ln(5)}{t_2 - t_1}$$

$$\text{avg. life} = \frac{1}{\lambda} \Rightarrow \frac{t_2 - t_1}{\ln(5)}$$

7. A particle executes S.H.M., the graph of velocity as a function of displacement is:

- (1) a circle
- (2) a parabola
- (3) an ellipse
- (4) a helix

Sol. (3)

For a body performing SHM, relation between velocity and displacement

$$v = \omega \sqrt{A^2 - x^2}$$

now, square both side

$$v^2 = \omega^2 (A^2 - x^2)$$

$$\Rightarrow v^2 = \omega^2 A^2 - \omega^2 x^2$$

$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

divide whole equation by $\omega^2 A^2$

$$\frac{v^2}{\omega^2 A^2} + \frac{\omega^2 x^2}{\omega^2 A^2} = \frac{\omega^2 A^2}{\omega^2 A^2}$$

$$\frac{v^2}{(\omega A)^2} + \frac{x^2}{A^2} = 1$$

above equation is similar as standard equation of ellipses, so graph between velocity and displacement will be ellipses.

8. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be:

(1) $\frac{a_1 + a_2}{a_2}$

(2) $\frac{a_2}{a_1}$

(3) $\frac{a_1 + a_2}{a_1}$

(4) $\frac{a_1}{a_2}$

Sol. (2)

From given information:

For 1st interval

$$a_1 = \frac{v_0}{t_1}$$

$$v_0 = a_1 t_1 \quad \dots(1)$$

For 2nd interval

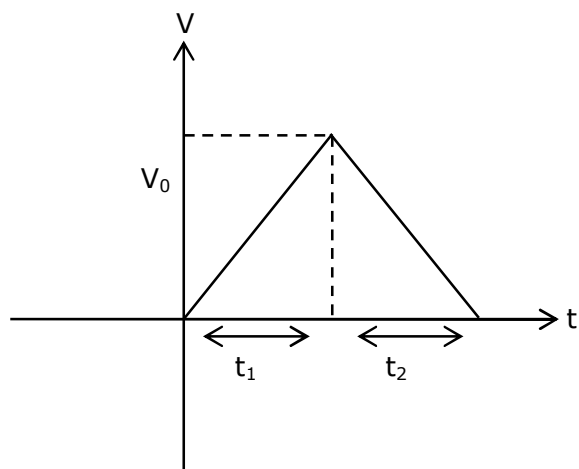
$$a_2 = \frac{v_0}{t_2}$$

$$v_0 = a_2 t_2 \quad \dots(2)$$

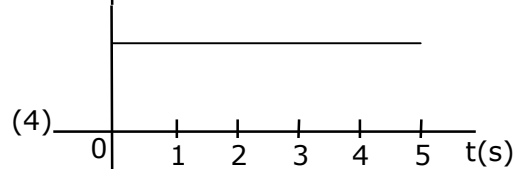
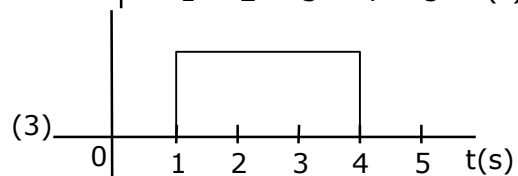
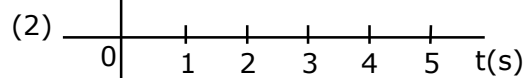
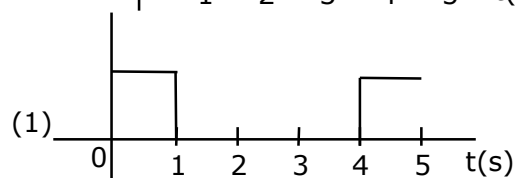
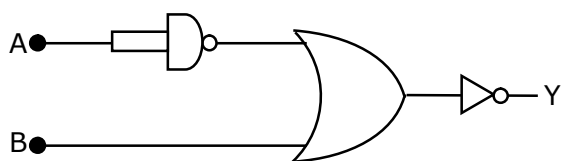
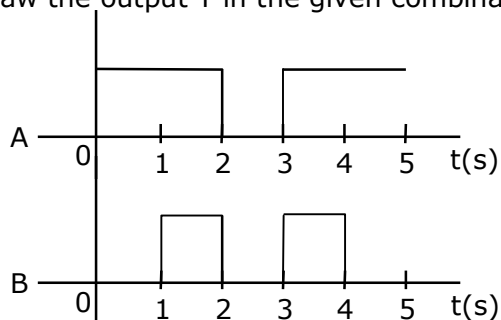
from (1) & (2)

$$a_1 t_1 = a_2 t_2$$

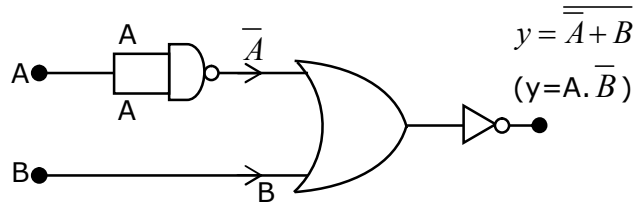
$$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$



9. Draw the output Y in the given combination of gates.



Sol. (1)



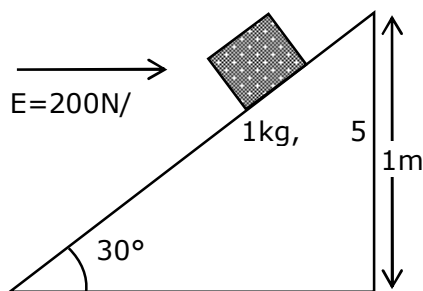
Find output expression $y = A \cdot \overline{B}$

Inputs

A	B	$y = A \cdot \overline{B}$
1	0	1
1	1	0
0	0	0
1	1	0
1	0	1

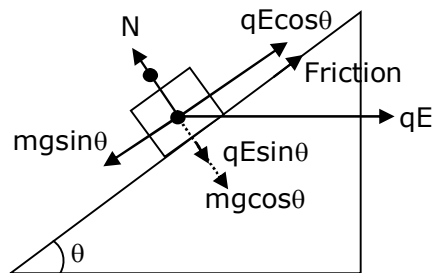
- 10.** An inclined plane making an angle of 30° with horizontal is placed in a uniform horizontal electric field $200 \frac{N}{C}$ as shown in the figure. A body of mass 1 kg and charge 5mC is allowed to slide down from rest at a height of 1m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom.

$$\left[g = 9.8 m/s^2, \sin 30^\circ = \frac{1}{2}; \cos 30^\circ = \frac{\sqrt{3}}{2} \right]$$



- (1) 2.3 s
(2) 0.46 s
(3) 1.3 s
(4) 0.92 s

Sol. (3)



$$F = mg \sin \theta - (\mu N + q E \cos \theta)$$

$$F = mg \sin \theta - \mu(mg \cos \theta + qE \sin \theta) - qE \cos \theta$$

$$F = 1 \times 10 \times \sin 30^\circ - 0.2 (1 \times 10 \times \cos 30^\circ + 200 \times 5 \times 10^{-3} \sin 30^\circ) - 200 \times 5 \times 10^{-3} \cos 30^\circ$$

$$F = 2.3 \text{ N}$$

$$a = \frac{F}{m} \Rightarrow \frac{2.3}{1} \Rightarrow 2.3 \text{ m / sec}^2$$

$$t = \sqrt{\frac{25}{9}} \Rightarrow \sqrt{\frac{2 \times 2}{2.3}} \Rightarrow 1.3 \text{ sec}$$

- 11.** If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ where $C/V = \lambda$?

(1) $[M^{-2}L^{-4}I^3T^7]$

(2) $[M^{-2}L^{-3}I^2T^6]$

(3) $[M^{-1}L^{-3}I^{-2}T^{-7}]$

(4) $[M^{-3}L^{-4}I^3T^7]$

Sol. (1)

$$\therefore v = \frac{w}{q} \text{ and } c = \frac{q}{v}$$

$$\text{dimension of } \frac{c}{v}$$

$$\Rightarrow \frac{q}{v^2}$$

$$\Rightarrow \frac{q}{w^2} \times q^2 \Rightarrow \frac{q^3}{w^2}$$

$$\Rightarrow \frac{I^3 T^3}{M^2 L^4 T^{-4}} \Rightarrow [M^{-2} L^{-4} T^7 I^3]$$

- 12.** Given below are two statements: One is labeled as Assertion A and the other is labeled as Reason R.

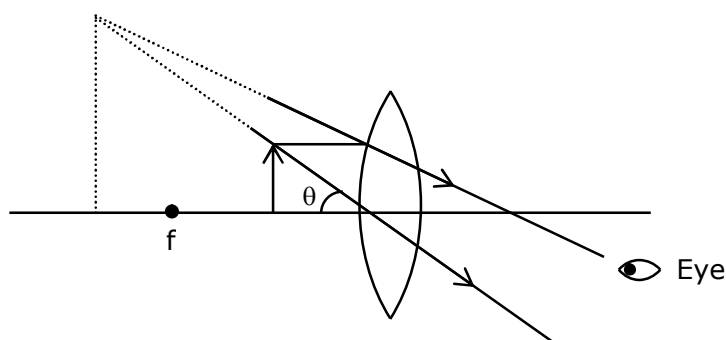
Assertion A : For a simple microscope, the angular size of the object equals the angular size of the image.

Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Both A and R are true but R is NOT the correct explanation of A
- (2) Both A and R are true and R is the correct explanation of A
- (3) A is true but R is false
- (4) A is false but R is true

Sol. (2)

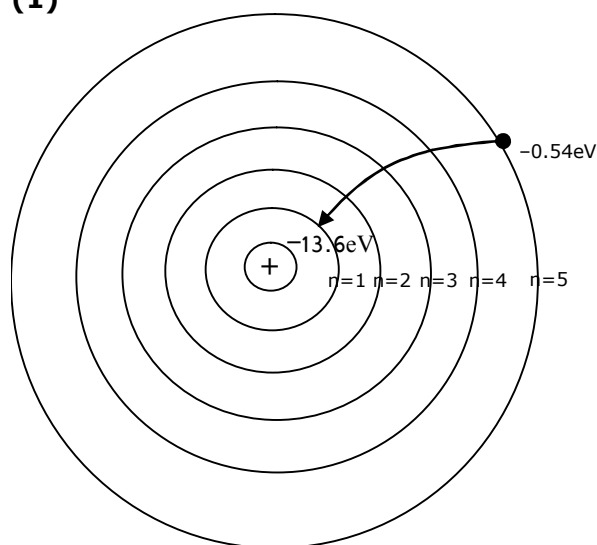


Both obtain same angle, since image can be at a distance greater than 25 cm, object can be moved closer to eye.

- 13.** The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state will be:

- (1) 4.17 m/s
- (2) 4.34 m/s
- (3) 219 m/s
- (4) 3.25 m/s

Sol. (1)

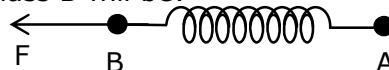


$$\text{momentum (P)} = \frac{\Delta E}{C} \Rightarrow \frac{(13.6 - 0.54)\text{eV}}{3 \times 10^8}$$

$$mv = \frac{(13.06) \times 1.6 \times 10^{-19}}{3 \times 10^8}$$

$$v = \frac{(13.06) \times 1.6 \times 10^{-19}}{3 \times 10^8 \times 1.67 \times 10^{-27}} \Rightarrow 4.17 \text{ m/sec}$$

- 14.** Two masses A and B, each of mass M are fixed together by a massless springs. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B will be:



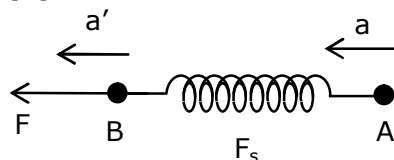
(1) $\frac{F + Ma}{M}$

(2) $\frac{F - Ma}{M}$

(3) $\frac{Ma - F}{M}$

(4) $\frac{MF}{F + Ma}$

Sol. (2)



$$F - F_s = Ma'$$

$$a' = \frac{F}{M} - a$$

$$\frac{F - Ma}{M}$$

- 15.** A wire of 1Ω has a length of 1 m. It is stretched till its length increases by 25%. The percentage change in a resistance to the nearest integer is:

(1) 25%

(2) 12.5%

(3) 76%

(4) 56%

Sol. (4)

For stretched or compressed wire

$$R \propto l^2$$

$$\frac{R_1}{R_2} = \frac{l_1^2}{l_2^2}$$

$$\Rightarrow \frac{R}{R_2} = \frac{l^2}{(1.25l)^2}$$

$$\Rightarrow R_2 = 1.5625 R$$

$$\% \text{ increase} \rightarrow 56.235\%$$

16. Given below are two statements :

Statement (1) :- A second's pendulum has a time period of 1 second.

Statement (2) :- It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options give below.

- (1) Both Statement I and Statement II are false
- (2) Statement I is true but Statement II is false
- (3) Statement I is false but Statement II is true
- (4) Both Statement I and Statement II is true

Sol. (3)

As we know time period of second's pendulum is 2 sec, so statement (1) is incorrect.

Time taken between two extreme points in second's pendulum is 1 sec.

Above statement is correct because time taken by particle performing SHM between two extreme position is $T/2$.

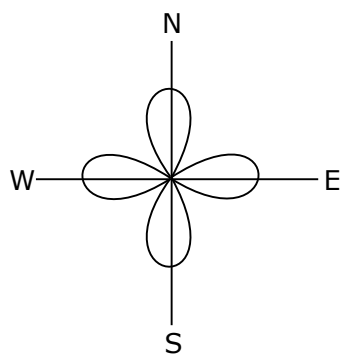
Here, $T = 2$ sec.

So, time = $2/2 = 1$ sec

17. An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is 2.5×10^{-4} Wb/m² and the angle of dip is 60°. The emf induced between the tips of the plane wings will be _____.

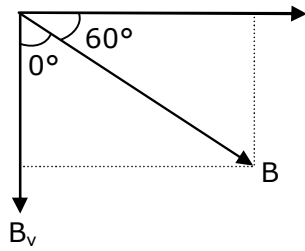
- (1) 88.37 mV
- (2) 62.50 mV
- (3) 54.125 mV
- (4) 108.25 mV

Sol. (4)



$$\Sigma = B \perp v \ell$$

$$\sin 60^\circ = \frac{B_v}{B}$$



$$\frac{\sqrt{3}}{2} = \frac{B_v}{B}$$

$$B_v = \frac{\sqrt{3}}{2} B$$

$$E = \frac{\sqrt{3}}{2} B \ell v$$

$$= \frac{\sqrt{3}}{2} \times 2.5 \times 10^{-4} \times 10 \times 180 \times \frac{5}{18}$$

$$= \frac{\sqrt{3}}{2} \times 2.5 \times 5 \times 10^{-2} = 10.825 \times 10^{-2} = 108.25 \text{ mV}$$

- 18.** The length of metallic wire is l_1 when tension in it is T_1 . It is l_2 when the tension is T_2 . The original length of the wire will be :

(1) $\frac{l_1 + l_2}{2}$

(2) $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$

(3) $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$

(4) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

Sol. (4)

From young's modulus relation $\left(y = \frac{\frac{F}{A}}{\left(\frac{\Delta l}{l} \right)} \right)$

we can write for 1st case

$$\frac{T_1}{A} = \frac{Y(l_1 - l)}{l}$$

we can write for 2nd case

$$\frac{T_2}{A} = \frac{Y(\ell_2 - \ell)}{\ell}$$

$$\frac{T_1}{T_2} = \frac{\ell_1 - \ell}{\ell_2 - \ell}$$

$$T_1\ell_2 - T_1\ell = T_2\ell_1 - T_2\ell$$

$$\frac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1} = \ell$$

- 19.** The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as $U = 3PV + 4$. The gas is :

- (1) polyatomic only
- (2) monoatomic only
- (3) either monoatomic or diatomic
- (4) diatomic only.

Sol. (1)

$$U = 3PV + 4$$

$$\frac{f}{2} PV = 3PV + 4$$

$$\therefore u = \frac{f}{2} nRT$$

$$f = 6 + \frac{8}{PV}$$

$$\therefore Pv = nRT$$

$f > 6$ $\therefore$ Polyatomic gas.

- 20.** Given below are two statements :

Statement – I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

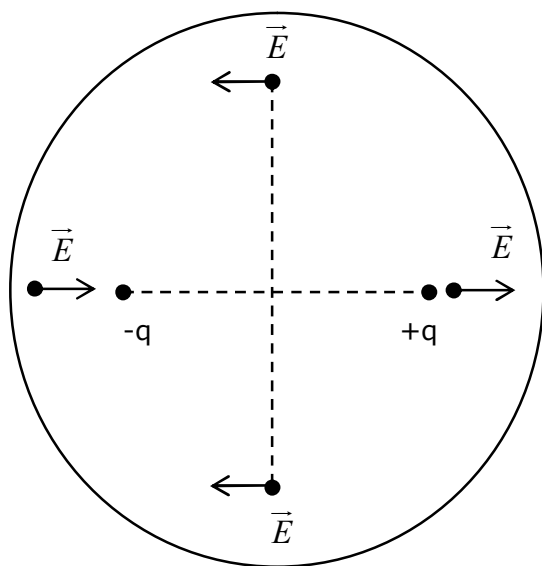
Statement – II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r (< R) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements. Choose the correct answer from the option given below :

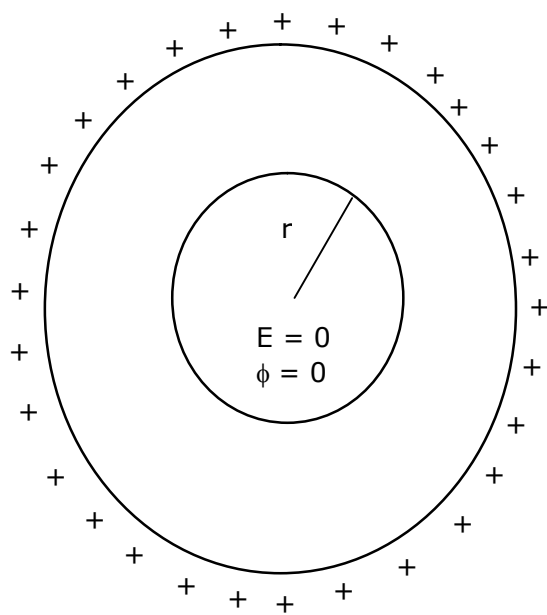
Option :

- (1) Statement I is true but Statement II is false
- (2) Statement I is false but Statement II is true
- (3) Both Statement I and Statement II are true
- (4) Both Statement I and Statement II are false

Sol. (1)



Statement - 1 $\rightarrow$ Correct



Statement - 2 $\rightarrow$ Incorrect

SECTION – B

1. If the highest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are _____.

Sol. (9)

$$\text{No. of station} = \frac{\text{Band width}}{2 \times \text{Highest Band width}}$$

$$\Rightarrow \frac{90}{2 \times 5}$$

$$\Rightarrow 9$$

2. 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$. The value of x is _____.

Sol. (25)

From thermodynamics law:

$$\Delta Q = \Delta U + \Delta W \quad \dots(1)$$

$$Q = nC_v \Delta T + \frac{Q}{5}$$

$$Q - \frac{Q}{5} = 1 \times \frac{5}{2} R \times \Delta T$$

$$Q = \frac{25}{8} R \Delta T \quad \dots(2) \quad \therefore Q = n c \Delta T$$

$$C = \frac{25}{8} R \quad \text{given } C = \frac{xR}{8}$$

$$x = 25$$

3. A particle executes S.H.M with amplitude 'a' and time period T. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}a}{2}$. The value of x is _____

Sol. (3)

For a particle executes S.H.M

$$V = \omega \sqrt{a^2 - x^2}$$

$$\text{Given } V = \frac{V_{\max}}{2} \Rightarrow \frac{A\omega}{2}$$

$$\frac{A^2 \omega^2}{4} = \omega^2 a^2 - \omega^2 x^2$$

$$x = \frac{\sqrt{3}}{2} a$$

4. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is $x : y$. The value of x is _____.

Sol. (1)

For photoelectric effect $k_{\max} = E - \phi$

$$E_1 = 2\phi, \quad k_1 = \phi$$

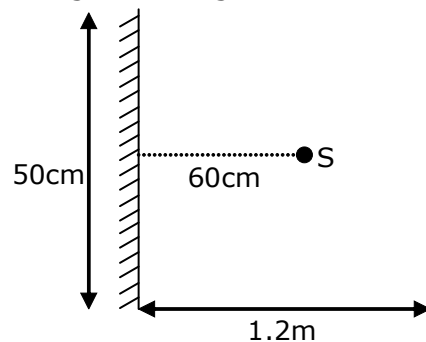
$$E_2 = 10\phi, \quad k_2 = 9\phi$$

$$\therefore V \propto \sqrt{k} \quad \left(k = \frac{1}{2}mv^2 \right)$$

$$\frac{v_1}{v_2} = \sqrt{\frac{1}{9}} \Rightarrow \frac{1}{3} = \frac{x}{y}$$

$$x = 1$$

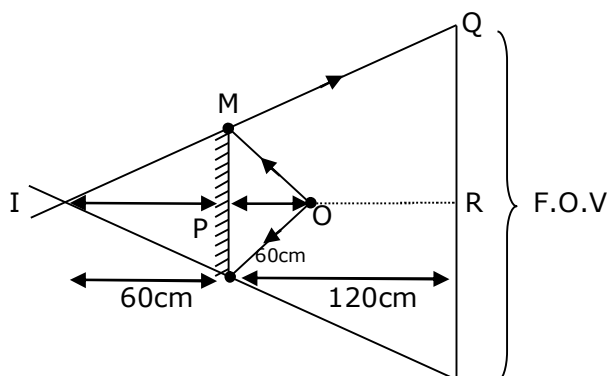
5. A point source of light S , placed at a distance 60 cm in front of the centre of plane mirror of width 50 cm, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is _____ cm



Sol. (150)

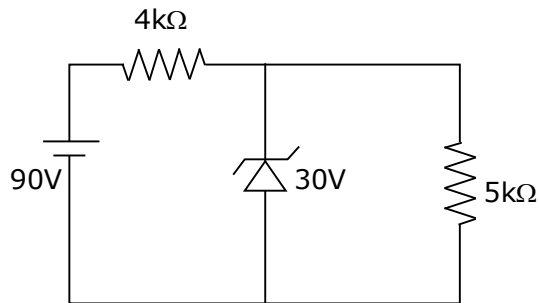
from similar triangle IMP and IQR

$$\frac{QR}{25} = \frac{180}{60} \Rightarrow QR = 75$$

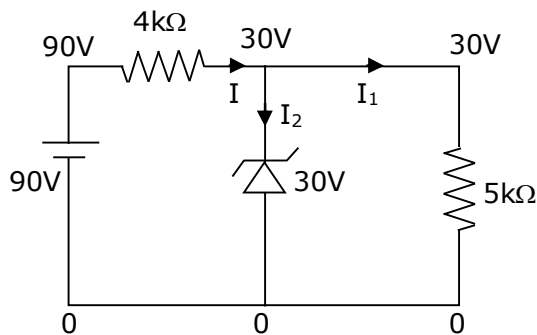


$$\text{F.O.V.} = 2 \times 75 \Rightarrow 150 \text{ cm}$$

6. The zener diode has a $V_Z = 30\text{ V}$. The current passing through the diode for the following circuit is _____ mA.



Sol. (9)

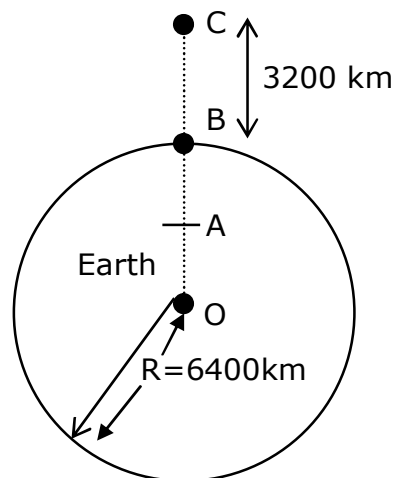


$$I = \frac{90 - 30}{4} = 15\text{mA}$$

$$I_1 = \frac{30}{5\text{k}\Omega} = 6\text{mA}$$

$$I_2 = 15\text{mA} - 6\text{mA} = 9\text{mA}$$

7. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is _____.



Sol. (4)

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM_r}{R^3}$$

$$OA = \frac{4R}{9} = r$$

$$AB = R - \frac{4R}{9} = \frac{5R}{9}$$

$$OA : AB$$

$$\frac{4R}{9} : \frac{5R}{9} \Rightarrow 4:5 = x:y$$

$$(x=4)$$

- 8.** 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is _____ times that of a smaller drop.

Sol. (243)

For self energy of sphere (conducting)

$$U = \frac{kq^2}{2r}$$

$$\text{For small drop} \rightarrow U_i = \frac{kq^2}{2r} \quad \dots\dots(1)$$

After combine small drops volume remains same as bigger drop

$$\therefore \frac{4}{3}\pi r^3 \times n = \frac{4}{3}\pi R^3$$

$$R = (n)^{\frac{1}{3}}r \quad \dots\dots(2)$$

$$\text{For large drop} \rightarrow U_f = \frac{k(nq)^2}{2 \times 3R} \quad \dots\dots(3)$$

From equation (1), (2), (3)

$$\frac{U_f}{U_i} = (n)^{5/3}$$

$$\Rightarrow (27)^{5/3}$$

$$\Rightarrow 243$$

9. The volume V of a given mass of monatomic gas changes with temperature T according to the relation $V = KT^{\frac{2}{3}}$. The work done when temperature changes by 90 K will be xR . The value of x is _____.
[R = universal gas constant]

Sol. (60)

Given: $V = k T^{2/3}$

$V^{3/2} = (k)^{3/2} T$

$TV^{-3/2} = \text{const.} \quad \dots\dots(1)$

and $TV^{\gamma-1} = \text{const.} \quad \dots\dots(2)$

From (1) & (2)

$$-\frac{3}{2} = \gamma - 1$$

$$\gamma = -\frac{1}{2}$$

$$\text{Work done (w)} = \frac{nR\Delta T}{\gamma - 1}$$

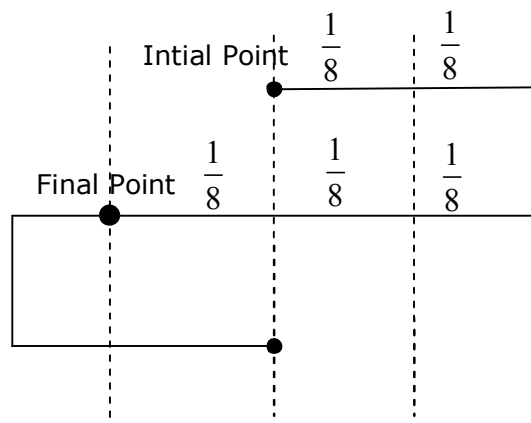
$$W = \frac{1 \times R \times 90}{-\frac{1}{2} - 1} \quad |W| = 60R \quad x = 60$$

10. Time period of a simple pendulum is T . The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta} T$. The value of α is _____.

Sol. (7)

For given $\left(\frac{5}{8}\right)$ oscillation, we can write it as $\rightarrow \left(\frac{1}{2} + \frac{1}{8}\right)$

And we know for half oscillations time $\rightarrow \frac{T}{2}$



For final point $\rightarrow \pi + \frac{\pi}{6} \Rightarrow \frac{7\pi}{6}$

Time $\rightarrow \frac{7T}{12} \rightarrow \text{given} \rightarrow \frac{\alpha}{\beta} T \alpha = 7p$

26th Feb. 2021 | Shift - 2
CHEMISTRY

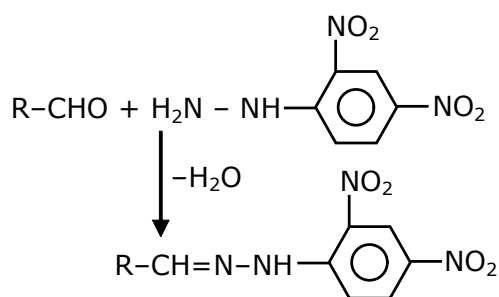
Section - A

1. 2,4-DNP test can be used to identify:

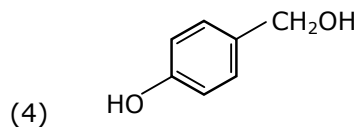
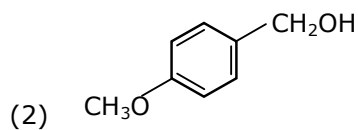
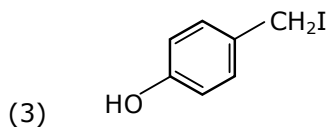
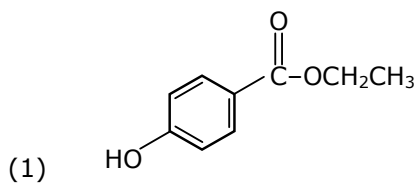
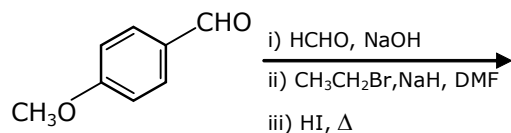
- (1) aldehyde
- (2) halogens
- (3) ether
- (4) amine

Ans. (1)

Sol.

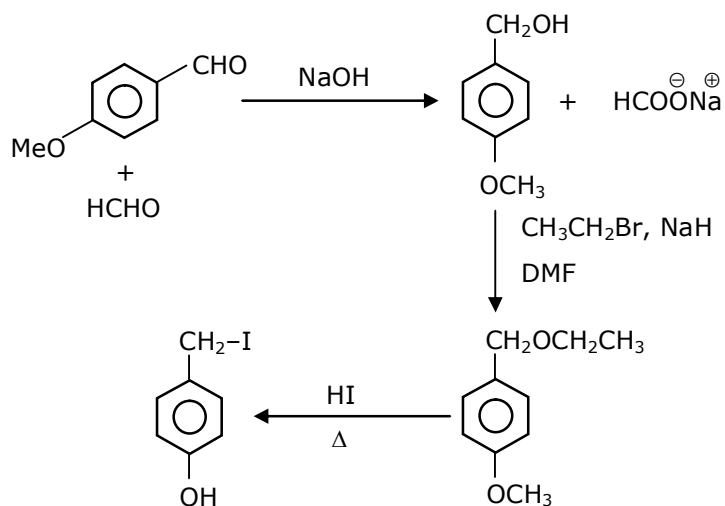


2. Identify A in the following chemical reaction.



Ans. (3)

Sol.



3. The nature of charge on resulting colloidal particles when FeCl₃ is added to excess of hot water is:

- (1) positive
- (2) neutral
- (3) sometimes positive and sometimes negative
- (4) negative

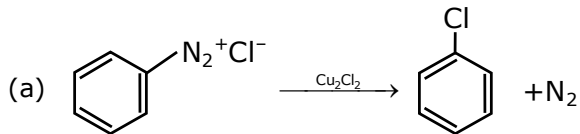
Ans. (1)

Sol. If FeCl₃ is added to excess of hot water, a positively charged sol of hydrated ferric oxide is formed due to adsorption of Fe³⁺ ions.

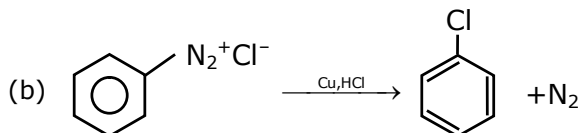
4. Match **List-I** with **List-II**

List-I

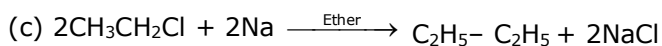
List-II



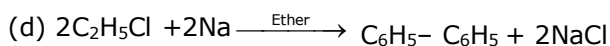
(i) Wurtz reaction



(ii) Sandmeyer reaction



(iii) Fitting reaction



(iv) Gatterman reaction

Choose the correct answer from the option given below:

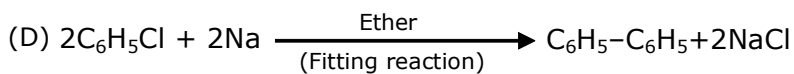
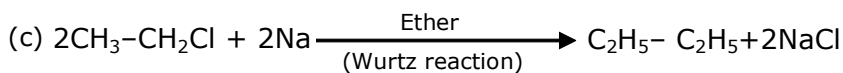
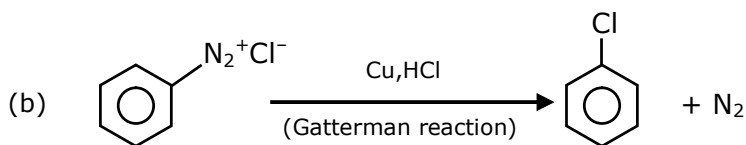
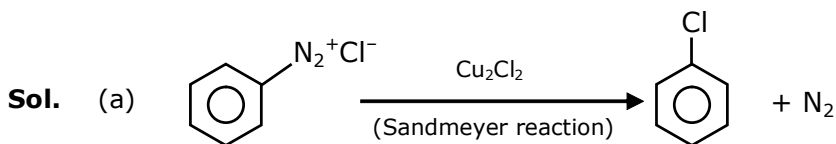
(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Ans. (3)



5. In $\overset{1}{\text{CH}_2} = \overset{2}{\text{C}} = \overset{3}{\text{CH}} - \overset{4}{\text{CH}_3}$ molecule, the hybridization of carbon 1, 2, 3 and 4 respectively are:

(1) sp^2 , sp , sp^2 , sp^3

(2) sp^2 , sp^2 , sp^2 , sp^3

(3) sp^2 , sp^3 , sp^2 , sp^3

(4) sp^3 , sp , sp^3 , sp^3

Ans. (1)

Sol. $\underset{\text{sp}^2}{\text{CH}_2} = \underset{\text{sp}}{\text{C}} = \underset{\text{sp}^2}{\text{CH}} - \underset{\text{sp}^3}{\text{CH}_3}$

6. Match List-I with List-II.

List-I

(a) Sucrose

(b) Lactose

(c) Maltose

List-II

(i) β -D-Galactose and β -D-Glucose

(ii) α -D-Glucose and β -D-Fructose

(iii) α -D-Glucose and α -D-Glucose

Choose the correct answer from the options given below:

(1) (a)-(iii), (b)-(ii), (c)-(i)

(2) (a)-(iii), (b)-(i), (c)-(ii)

(3) (a)-(i), (b)-(iii), (c)-(ii)

(4) (a)-(ii), (b)-(i), (c)-(iii)

Ans. (4)

Sol. Sucrose $\rightarrow$ α -D- Glucose and β -D- Fructose

Lactose $\rightarrow$ β -D- Galactose and β -D- Glucose

Maltose $\rightarrow$ α -D- Glucose and α -D- Glucose

7. Which pair of oxides is acidic in nature?

(1) N_2O , BaO

(2) CaO , SiO_2

(3) B_2O_3 , CaO

(4) B_2O_3 , SiO_2

Ans. (4)

Sol. B_2O_3 and SiO_2 both are oxides of non-metal and hence are acidic in nature.

8. Calgon is used for water treatment. Which of the following statement is NOT true about calgon?

- (1) Calgon contains the 2nd most abundant element by weight in the earth's crust.
- (2) It is also known as Graham's salt.
- (3) It is polymeric compound and is water soluble.
- (4) It doesnot remove Ca^{2+} ion by precipitation.

Ans. (1)

Sol. $\text{Na}_6(\text{PO}_3)_6$ or $\text{Na}_6\text{P}_6\text{O}_{18}$

Order of abundance of element in earth crust is

$\text{O} > \text{Si} > \text{Al} > \text{Fe} > \text{Ca} > \text{Na} > \text{Mg} > \text{K}$

So second most abundant element in earth crust is Si not Ca.

9. Ceric ammonium nitrate and $\text{CHCl}_3/\text{alc. KOH}$ are used for the identification of functional groups present in _____ and _____ respectively.

- (1) alcohol, amine
- (2) amine, alcohol
- (3) alcohol, phenol
- (4) amine, phenol

Ans. (1)

Sol. Alcohol give positive test with ceric ammonium nitrate and primary amines gives carbyl amine test with CHCl_3 , KOH.

10. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: In TlI_3 , isomorphous to CsI_3 , the metal is present in +1 oxidation state.

Reason R: Tl metals has fourteen *f* electrons in its electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Both A and R are correct and R is the correct explanation of A
- (2) A is not correct but R is correct
- (3) Both A and R are correct R is NOT the correct explanation of A
- (4) A is correct but R is not correct

Ans. (3)

Sol. TlI_3 is $\text{Tl}^+ \text{I}_3^-$

CsI_3 is $\text{Cs}^+ \text{I}_3^-$

Thallium shows Tl^+ state due to inert pair effect.

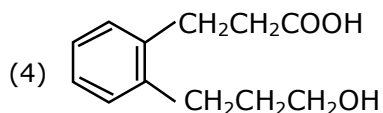
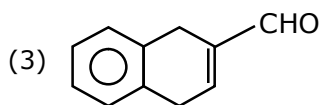
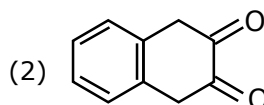
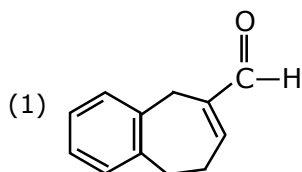
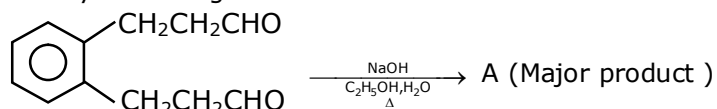
11. The correct order of electron gain enthalpy is:

- (1) $S > Se > Te > O$
- (2) $O > S > Se > Te$
- (3) $S > O > Se > Te$
- (4) $Te > Se > S > O$

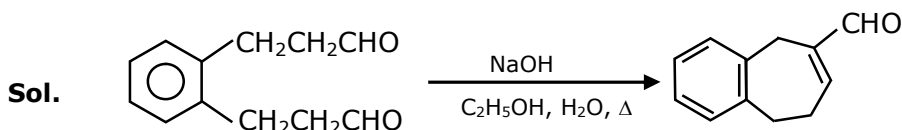
Ans. (1)

Sol. Electron gain enthalpy of O is very low due to small size.

12. Identify A in the given chemical reaction.



Ans. (1)



(Internal aldol condensation)

13. Match List-I with List-II

List-I

- (a) Siderite
- (b) Calamine
- (c) Malachite
- (d) Cryolite

List-II

- (i) Cu
- (ii) Ca
- (iii) Fe
- (iv) Al
- (v) Zn

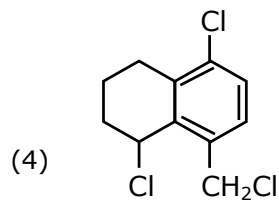
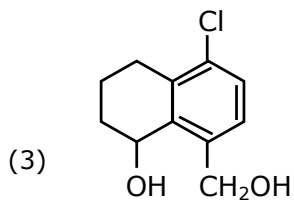
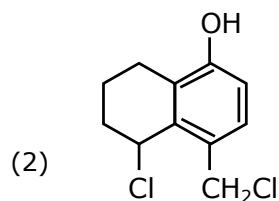
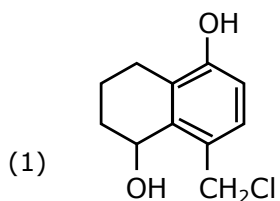
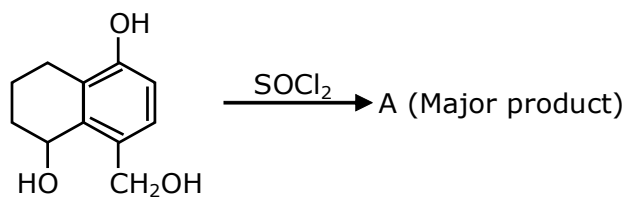
Choose the correct answer from the options given below:

- (1) (a)-(i), (b)-(ii), (c)-(v), (d)-(iii)
- (2) (a)-(iii), (b)-(v), (c)-(i), (d)-(iv)
- (3) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
- (4) (a)-(iii), (b)-(i), (c)-(v), (d)-(ii)

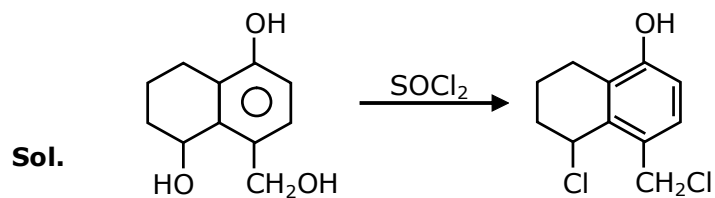
Ans. (2)

Sol. Siderite - FeCO_3
Calamine - ZnCO_3
Malachite - $\text{CuCO}_3 \cdot \text{Cu(OH)}_2$
Cryolite - Na_3AlF_6

14. Identify A in the given reaction



Ans. (2)



15. Match List-I with List-II.

List-I	List-II
(a) Sodium Carbonate	(i) Deacon
(b) Titanium	(ii) Caster-Kellner
(c) Chlorine	(iii) Van-Arkel
(d) Sodium hydroxide	(iv) Solvay

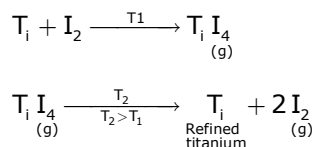
Choose the correct answer from the option given below:

- (1) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)
 (2) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)
 (3) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
 (4) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

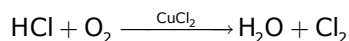
Ans. (2)

Sol. Sodium carbonate Na_2CO_3 & NaHCO_3

Titanium : Van arkel method



Chlorine : Deacon's process



Sodium hydroxide :- Caster-Kellner cell

16. Match List-I with List-II.

List-I (Molecule)	List-II (Bond order)
(a) Ne_2	(i) 1
(b) N_2	(ii) 2
(c) F_2	(iii) 0
(d) O_2	(iv) 3

Choose the correct answer from the options given below:

- (1) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii) (2) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
 (3) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii) (4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Ans. (1)

Sol. Ne_2O $\text{BO} = 0$
 N_2 $\text{BO} = 3$
 F_2 $\text{BO} = 1$
 O_2 $\text{BO} = 2$

As per molecular orbital theory

17. Which of the following forms of hydrogen emits low energy β^- particles?

- (1) Proton H^+
- (2) Deuterium ${}^2_1\text{H}$
- (3) Protium ${}^1_1\text{H}$
- (4) Tritium ${}^3_1\text{H}$

Ans. (4)

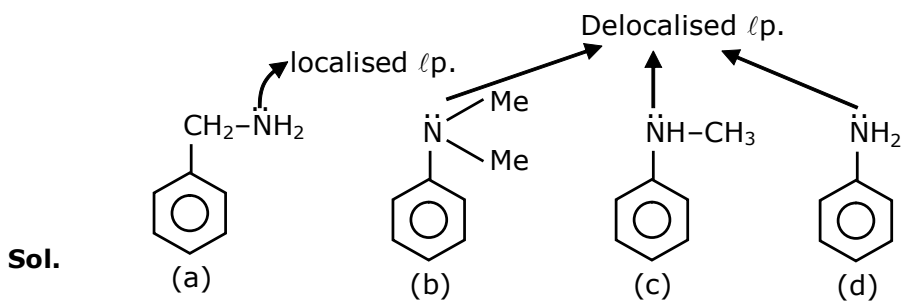
Sol. Tritium isotope of hydrogen is radioactive and emits low energy β^- particles. It is because of high n/p ratio of tritium which makes nucleus unstable.

- 18.** A. Phenyl methanamine
B. N, N-Dimethylaniline
C. N-Methyl aniline
D. Benzenamine

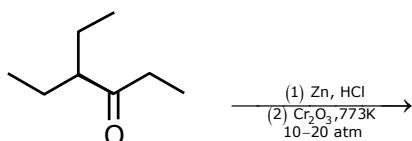
Choose the correct order of basic nature of the above amines.

- (1) $\text{D} > \text{C} > \text{B} > \text{A}$
- (2) $\text{D} > \text{B} > \text{C} > \text{A}$
- (3) $\text{A} > \text{C} > \text{B} > \text{D}$
- (4) $\text{A} > \text{B} > \text{C} > \text{D}$

Ans. (4)



19.

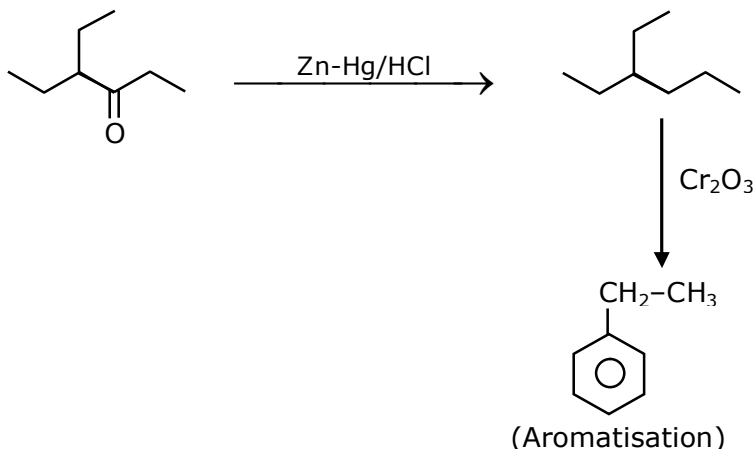


Considering the above reaction, the major product among the following is:

- (1)
- (2)
- (3)
- (4)

Ans. (3)

Sol.



20. Seliwanoff test and Xanthoproteic test are used for the identification of _____ and _____ respectively

(1) ketoses, proteins

(2) proteins, ketoses

(3) aldoses, ketoses

(4) ketoses, aldoses

Ans. (1)

Sol. Seliwanoff test and Xanthoproteic test are used for identification of 'Ketoses' and proteins respectively.

Section - B

1. The NaNO_3 weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^+ per mL is _____ g. (Rounded off to the nearest integer)
[Given: Atomic weight in g mol^{-1} . Na: 23; N: 14; O : 16]

Ans. 13

Sol. $\text{Na}^+ = 70 \text{ mg/mL}$

$$\begin{aligned} W_{\text{Na}^+} \text{ in 50mL solution} &= 70 \times 50\text{mg} \\ &= 3500 \text{ mg} \\ &= 3.5 \text{ gm} \end{aligned}$$

$$\text{Moles of } \text{Na}^+ \text{ in 50 ml solution} = \frac{3.5}{23}$$

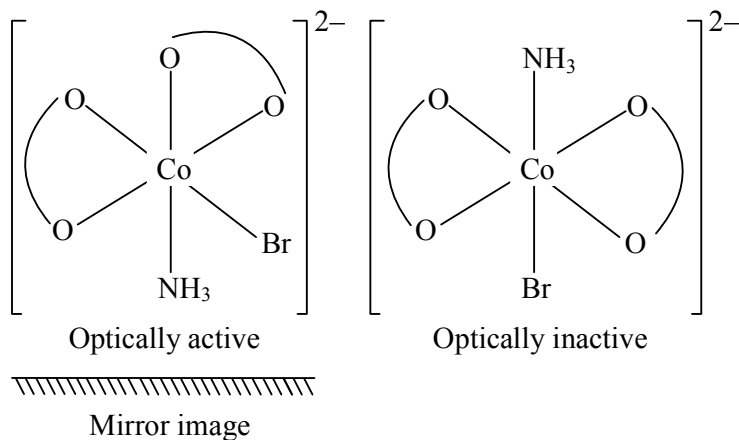
$$\begin{aligned} \text{Moles of } \text{NaNO}_3 &= \text{moles of } \text{Na}^+ \\ &= \frac{3.5}{23} \text{ mol} \end{aligned}$$

$$\begin{aligned} \text{Mass of } \text{NaNO}_3 &= \frac{3.5}{23} \times 85 = 12.934 \\ &\simeq 13\text{gm Ans.} \end{aligned}$$

2. The number of stereoisomers possible for $[\text{Co}(\text{ox})_2(\text{Br})(\text{NH}_3)]^{2-}$ is _____ [ox = oxalate]

Ans. 3

Sol. $[\text{Co}(\text{ox})_2\text{Br}(\text{NH}_3)]^{2-}$



Total stereoisomer = 2 (OI) + 1 POE (pair of enantiomers) = 3

3. The average S-F bond energy in kJ mol^{-1} of SF_6 is _____. (Rounded off to the nearest integer)

[Given : The values of standard enthalpy of formation of $\text{SF}_6(\text{g})$, $\text{S}(\text{g})$ and $\text{F}(\text{g})$ are - 1100, 275 and 80 kJ mol^{-1} respectively.]

Ans. 309

Sol. $\text{SF}_6(\text{g}) \longrightarrow \text{S}(\text{g}) + 6\text{F}(\text{g})$

$$\Delta H_{\text{reaction}}^{\circ} = 6 \times E_{\text{S-F}} = \Delta H_f^{\circ}[\text{S}(\text{g})] + 6 \times \Delta H_f^{\circ}[\text{F}(\text{g})] - \Delta H_f^{\circ}[\text{SF}_6(\text{g})]$$

$$6 \times E_{\text{S-F}} = 275 + 6 \times 80 - (-1100)$$

$$= 275 + 480 + 1100$$

$$6 \times E_{\text{S-F}} = 1855$$

$$E_{\text{S-F}} = \frac{1855}{6} = 309.1667$$

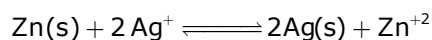
$\simeq 309 \text{ kJ/mol}$ Ans.

4. Emf of the following cell at 298 K in V is $x \times 10^{-2}$.
 $\text{Zn}|\text{Zn}^{2+} (0.1 \text{ M})||\text{Ag}^+(0.01 \text{ M})|\text{Ag}$
 The value of x is _____. (Rounded off to the nearest integer)

[Given: $E_{\text{Zn}^{2+}/\text{Zn}}^0 = -0.76\text{V}$; $E_{\text{Ag}^+/\text{Ag}}^0 = +0.80\text{V}$; $\frac{2.303RT}{F} = 0.059$]

Ans. 147

Sol. $\text{Zn(s)}|\text{Zn}^{2+}(0.1\text{M})||\text{Ag}^+(0.01\text{M})|\text{Ag(s)}$



$$E^0 = 0.80 + 0.76 = 1.56 ; Q = \left\{ \frac{\text{Zn}^{2+}}{(\text{Ag}^+)^2} \right\}$$

$$E = E^0 - \frac{0.059}{n} \log(Q)$$

$$E = 1.56 - \frac{0.059}{2} \log \left[\frac{0.1}{(0.01)^2} \right]$$

$$E = 1.56 - \frac{0.059}{2} \log \left[(10)^3 \right]$$

$$E = 1.4715 = 147.15 \times 10^{-2} \text{ volt}$$

$$= x \times 10^{-2}$$

$$X = 147.15 \simeq 147 \text{ Ans.}$$

5. A ball weighing 10g is moving with a velocity of 90ms^{-1} . If the uncertainty in its velocity is 5%, then the uncertainty in its position is _____ $\times 10^{-33}\text{m}$. (Rounded off to the nearest integer)
 [Given : $h = 6.63 \times 10^{-34} \text{ Js}$]

Ans. 1

Sol. $m = 10 \text{ g} = 10^{-2} \text{ Kg}$

$v = 90 \text{ m/sec.}$

$$\Delta v = v \times 5\% = 90 \times \frac{5}{100} = 4.5 \text{ m / sec}$$

$$m \cdot \Delta v \cdot \Delta x \geq \frac{h}{4\pi}$$

$$10^{-2} \times 4.5 \times \Delta x \geq \frac{6.63 \times 3 \times 10^{-34}}{4 \times \frac{22}{7}}$$

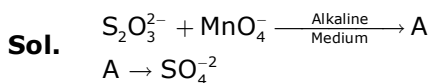
$$\Delta x \geq \frac{6.63 \times 7 \times 2 \times 10^{-34}}{9 \times 4 \times 22 \times 10^{-2}}$$

$$\Delta x \geq 1.17 \times 10^{-33} = x \times 10^{-33}$$

$$x = 1.17 \simeq 1$$

6. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^- to "A". The oxidation state of sulphur in "A" is_____.

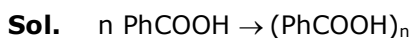
Ans. 6



$\therefore$ Oxidation no. of 'S' = +6 Ans.

7. When 12.2 g of benzoic acid is dissolved in 100g of water, the freezing point of solution was found to be -0.93°C ($K_f(\text{H}_2\text{O}) = 1.86 \text{ K kg mol}^{-1}$). The number (n) of benzoic acid molecules associated (assuming 100% association) is_____.

Ans. 2



$$N = \frac{1}{x} = i \{ \alpha = 1 \}$$

$$\Delta T_f = i \times k_f \times m$$

$$0.93 = \frac{1}{n} \times 1.86 \times \frac{12.2 \times 1000}{122 \times 100}$$

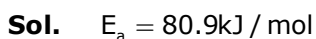
$$n = 2$$

8. If the activation energy of a reaction is 80.9 kJ mol^{-1} , the fraction of molecules at 700K, having enough energy to react to form products is e^{-x} . The value of x is _____.

(Rounded off to the nearest integer)

[Use $R = 8.31 \text{ JK}^{-1} \text{ mol}^{-1}$]

Ans. 14



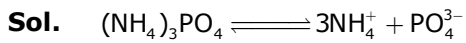
$$\text{Fraction of molecules able to cross energy barrier} = e^{-E_a/RT} = e^{-x}$$

$$x = \frac{E_a}{RT} = \frac{80.9 \times 1000}{8.31 \times 700} = 13.91$$

$$x \simeq 14 \text{ Ans}$$

9. The pH of ammonium phosphate solution, if pK_a of phosphoric acid and pK_b of ammonium hydroxide are 5.23 and 4.75 respectively, is_____.

Ans. 7



$$[H^+] = K_a \times \sqrt{\frac{K_w}{K_a \times K_b}}$$

$$pH = pK_a + \frac{1}{2} \{pK_w - pK_a - pK_b\}$$

$$pH = 5.23 + \frac{1}{2} \{14 - 5.23 - 4.75\}$$

$$pH = 5.23 + \frac{1}{2} (4.02) = 7.24 = 7 \text{ (Nearest integer)}$$

10. The number of octahedral voids per lattice site in a lattice is _____.
(Rounded off to the nearest integer)

Ans. 1

Sol. Assuming FCC

No of lattice sites = 6 face centre + 8 corner = 14

No. of octahedral voids = 13

$$\text{Ratio} = \frac{13}{14} = 0.92857 = 1 \text{ (Nearest integer)}$$

26th Feb. 2021 | Shift - 2

MATHEMATICS

1. Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from $(3, 2, 1)$ on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

(1) 142

(2) 68

(3) 136

(4) 102

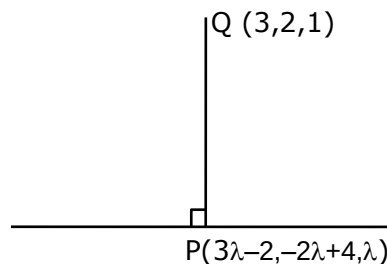
Ans. (4)

Sol. Dr's of line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$

Dr/s :- $(3, -2, 1)$

Points on the line $(-2, 4, 0)$

Equation of the line $\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z}{1} = \lambda$



Dr's of PQ ; $3\lambda - 5, -2\lambda + 2, \lambda - 1$

Dr's of y lines are $(3, -2, 1)$

Since $PQ \perp$ line

$3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$

$\lambda = \frac{10}{7}$

$P\left(\frac{16}{7}, \frac{8}{7}, \frac{10}{7}\right)$

$21(\alpha + \beta + \gamma) = 21\left(\frac{34}{7}\right) = 102$

2. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8} e + \frac{19}{8} e^{-1} - 10$

(2) $-\frac{41}{8} e + \frac{19}{8} e^{-1} - 10$

(3) $\frac{41}{8} e - \frac{19}{8} e^{-1} - 10$

(4) $\frac{41}{8} e + \frac{19}{8} e^{-1} + 10$

Ans. (3)

Sol. $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$

Put $2n + 1 = r$, where $r = 3, 5, 7, \dots$

$$\Rightarrow n = \frac{r-1}{2}$$

$$\frac{n^2 + 6n + 10}{(2n+1)!} = \frac{\left(\frac{r-1}{2}\right)^2 + 3r - 3 + 10}{r!} = \frac{r^2 + 10r + 29}{4r!}$$

$$\begin{aligned} \text{Now } \sum_{r=3,5,7} \frac{r(r-1) + 11r + 29}{4r!} &= \frac{1}{4} \sum_{r=3,5,7,\dots} \left(\frac{1}{(r-2)!} + \frac{11}{(r-1)!} + \frac{29}{r!} \right) \\ &= \frac{1}{4} \left\{ \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots \right) + 11 \left(\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots \right) + 29 \left(\frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \dots \right) \right\} \\ &= \frac{1}{4} \left\{ \frac{e - \frac{1}{e}}{2} + 11 \left(\frac{e + \frac{1}{e} - 2}{2} \right) + 29 \left(\frac{e - \frac{1}{e} - 2}{2} \right) \right\} \\ &= \frac{1}{8} \left\{ e - \frac{1}{e} + 11e + \frac{11}{e} - 22 + 29e - \frac{29}{e} - 58 \right\} \\ &= \frac{1}{8} \left\{ 41e - \frac{19}{e} - 80 \right\} \end{aligned}$$

3. Let $f(x)$ be a differentiable function at $x = a$ with $f'(a) = 2$ and $f(a) = 4$. Then $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a}$ equals :

(1) $2a + 4$

(2) $2a - 4$

(3) $4 - 2a$

(4) $a + 4$

Ans. (3)

Sol. By L-H rule

$$L = \lim_{x \rightarrow a} \frac{f(a) - af'(x)}{1}$$

$$\therefore L = 4 - 2a$$

4. Let A (1, 4) and B(1, -5) be two points. Let P be a point on the circle $(x - 1)^2 + (y - 1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points, P, A and B lie on :

(1) a parabola

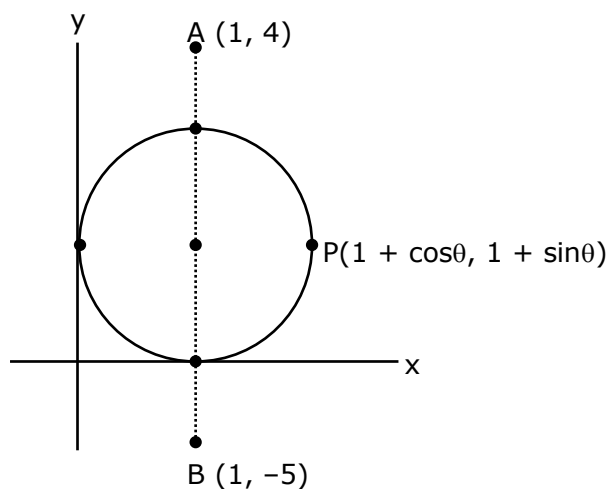
(2) a straight line

(3) a hyperbola

(4) an ellipse

Ans. (2)

Sol.



$$\therefore PA^2 = \cos^2\theta + (\sin\theta - 3)^2 = 10 - 6 \sin\theta$$

$$PB^2 = \cos^2\theta + (\sin\theta - 6)^2 = 37 - 12 \sin\theta$$

$$PA^2 + PB^2 \big|_{\max.} = 47 - 18 \sin\theta \big|_{\min.} \Rightarrow \theta = \frac{3\pi}{2}$$

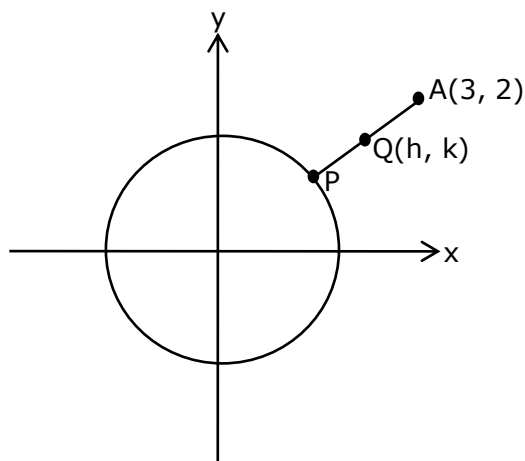
$\therefore$ P, A, B lie on a line $x = 1$

5. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of the radius r , then r is equal to :

- (1) $\frac{1}{4}$
 (2) $\frac{1}{2}$
 (3) 1
 (4) $\frac{1}{3}$

Ans. (2)

Sol.



$$\therefore P \equiv (2h - 3, 2k - 2) \rightarrow \text{on circle}$$

$$\therefore \left(h - \frac{3}{2}\right)^2 + (k - 1)^2 = \frac{1}{4}$$

$$\Rightarrow \text{radius} = \frac{1}{2}$$

6. Let slope of the tangent line to a curve at any point $P(x, y)$ be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y , for which the point $(3, y)$ lies on the curve, is :

- (1) $-\frac{18}{11}$
 (2) $-\frac{18}{19}$
 (3) $-\frac{4}{3}$

$$(4) \frac{18}{35}$$

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{xy^2 + y}{x}$
 $\Rightarrow \frac{xdy - ydx}{y^2} = xdx$

$$\Rightarrow -d\left(\frac{x}{y}\right) = d\left(\frac{x^2}{2}\right)$$

$$\Rightarrow \frac{-x}{y} = \frac{x^2}{2} + C$$

Curve intersect the line $x + 2y = 4$ at $x = -2$

$$\text{So, } -2 + 2y = 4 \Rightarrow y = 3$$

So the curve passes through $(-2, 3)$

$$\Rightarrow \frac{2}{3} = 2 + C$$

$$\Rightarrow C = \frac{-4}{3}$$

$$\therefore \text{curve is } \frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$$

It also passes through $(3, y)$

$$\frac{-3}{y} = \frac{9}{2} - \frac{4}{3}$$

$$\Rightarrow \frac{-3}{y} = \frac{19}{6}$$

$$\Rightarrow y = -\frac{18}{19}$$

7. Let A_1 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and y -axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, x -axis and $x = \frac{\pi}{2}$ in the first quadrant. Then,

$$(1) A_1 = A_2 \text{ and } A_1 + A_2 = \sqrt{2}$$

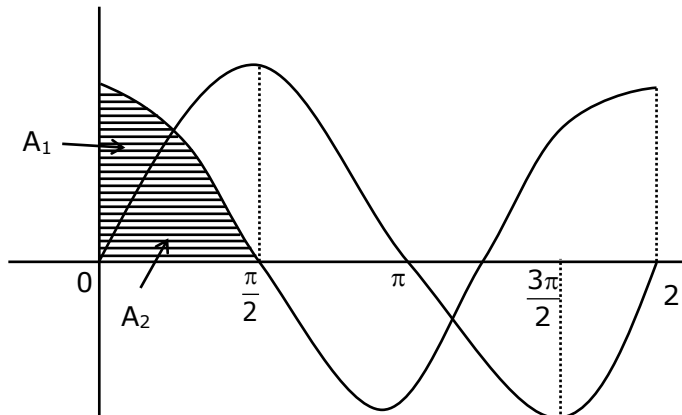
$$(2) A_1 : A_2 = 1 : 2 \text{ and } A_1 + A_2 = 1$$

$$(3) 2A_1 = A_2 \text{ and } A_1 + A_2 = 1 + \sqrt{2}$$

$$(4) A_1 : A_2 = 1 : \sqrt{2} \text{ and } A_1 + A_2 = 1$$

Ans. (4)

Sol. $A_1 + A_2 = \int_0^{\pi/2} \cos x \cdot dx = \sin x \Big|_0^{\pi/2} = 1$



$$A_1 = \int_0^{\pi/4} (\cos x - \sin x) dx = (\sin x + \cos x) \Big|_0^{\pi/4} = \sqrt{2} - 1$$

$$\therefore A_2 = 1 - (\sqrt{2} - 1) = 2 - \sqrt{2}$$

$$\therefore \frac{A_1}{A_2} = \frac{\sqrt{2} - 1}{\sqrt{2}(\sqrt{2} - 1)} = \frac{1}{\sqrt{2}}$$

8. If $0 < a, b < 1$, and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of

$$(a + b) - \left(\frac{a^2 + b^2}{2} \right) + \left(\frac{a^3 + b^3}{3} \right) - \left(\frac{a^4 + b^4}{4} \right) + \dots \text{ is :}$$

(1) $\log_e 2$

(2) $\log_e \left(\frac{e}{2} \right)$

(3) e

(4) $e^2 - 1$

Ans. (1)

Sol. $\tan^{-1} \left(\frac{a+b}{1-ab} \right) = \frac{\pi}{4} \Rightarrow a + b = 1 - ab \Rightarrow (1 + a)(1 + b) = 2$

$$\text{Now, } (a + b) - \left(\frac{a^2 + b^2}{2} \right) + \left(\frac{a^3 + b^3}{3} \right) - \dots \infty$$

$$= \left(a - \frac{a^2}{2} + \frac{a^3}{3} - \dots \right) + \left(b - \frac{b^2}{2} + \frac{b^3}{3} - \dots \right)$$

$$\log_e (1 + a) + \log_e (1 + b) = \log_e (1 + a) (1 + b) = \log_e 2$$

9. Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then :

- (1) F_1 is not a tautology but F_2 is a tautology
- (2) F_1 is a tautology but F_2 is not a tautology
- (3) F_1 and F_2 both are tautologies
- (4) Both F_1 and F_2 are not tautologies

Ans. (1)

Sol. Truth table for F_1

A	B	C	$\sim A$	$\sim B$	$\sim C$	$A \vee \sim B$	$A \vee B$	$\sim C \vee (A \vee B)$	$[\sim C \wedge (A \vee B)] \vee \sim A$	$(A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$
T	T	T	F	F	F	F	T	F	F	F
T	T	F	F	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	F	F	T
T	F	F	F	T	T	T	T	T	T	T
F	T	T	T	F	F	F	T	F	T	T
F	T	F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	F	F	F	T	T
F	F	F	T	T	T	F	F	F	T	T

Not a tautology

Truth table for F_2

A	B	$A \vee B$	$\sim A$	$B \rightarrow \sim A$	$(A \vee B) \vee (B \rightarrow \sim A)$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	T	T

F_1 not shows tautology and F_2 shows tautology

10. Consider the following system of equations :

$$x + 2y - 3z = a$$

$$2x + 6y - 11z = b$$

$$x - 2y + 7z = c,$$

Where a, b and c are real constants. Then the system of equations :

- (1) has a unique solution when $5a = 2b + c$
- (2) has infinite number of solutions when $5a = 2b + c$
- (3) has no solution for all a, b and c
- (4) has a unique solution for all a, b and c

Ans. (2)

Sol.
$$D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$$

$$= 20 - 2(25) - 3(-10)$$

$$= 20 - 50 + 30 = 0$$

$$D_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix}$$

$$= 20a - 2(7b + 11c) - 3(-2b - 6c)$$

$$= 20a - 14b - 22c + 6b + 18c$$

$$= 20a - 8b - 4c$$

$$= 4(5a - 2b - c)$$

$$D_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

$$= 7b + 11c - a(25) - 3(2c - b)$$

$$= 7b + 11c - 25a - 6c + 3b$$

$$= -25a + 10b + 5c$$

$$= -5(5a - 2b - c)$$

$$D_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

$$= 6c + 2b - 2(2c - b) - 10a$$

$$= -10a + 4b + 2c$$

$$= -2(5a - 2b - c)$$

for infinite solution

$$D = D_1 = D_2 = D_3 = 0$$

$$\Rightarrow 5a = 2b + c$$

- 11.** A seven digit number is formed using digit 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$

(2) $\frac{4}{7}$

(3) $\frac{3}{7}$

(4) $\frac{1}{7}$

Ans. (3)

Sol. $n(s) = \frac{7!}{2!3!2!}$

$$n(E) = \frac{6!}{2!2!2!}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6!}{7!} \times \frac{2!3!2!}{2!2!2!}$$

$$\frac{1}{7} \times 3 = \frac{3}{7}$$

- 12.** If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

(1) $\frac{1}{\sqrt{2}} (-\hat{j} + \hat{k})$

(2) $\frac{1}{\sqrt{2}} (\hat{i} - \hat{j})$

(3) $\frac{1}{\sqrt{3}} (\hat{i} - \hat{j} + \hat{k})$

(4) $\frac{1}{\sqrt{3}} (\hat{i} + \hat{j} - \hat{k})$

Ans. (3)

Sol. $\frac{x}{1} = -\frac{1}{y} = \frac{1}{z} = \lambda$ (let)

Unit vector parallel to $x\hat{i} + y\hat{j} + z\hat{k} = \pm \frac{\left(\lambda\hat{i} - \frac{1}{\lambda}\hat{j} + \frac{1}{\lambda}\hat{k}\right)}{\sqrt{\lambda^2 + \frac{2}{\lambda^2}}}$

For $\lambda = 1$, it is $\pm \frac{(\hat{i} - \hat{j} + \hat{k})}{\sqrt{3}}$

- 13.** For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to :

(1) $\frac{1}{2}$

(2) -1

(3) 1

(4) 0

Ans. (1)

Sol. $f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\ln t}{1+t} dt + \int_1^{1/e} \frac{\ln t}{1+t} dt = I_1 + I_2$

$I_2 = \int_1^{1/e} \frac{\ln t}{1+t} dt$ put $t = \frac{1}{z}, dt = -\frac{dz}{z^2}$

$= \int_1^e -\frac{\ln z}{1 + \frac{1}{z}} \times \left(-\frac{dz}{z^2}\right) = \int_1^e \frac{\ln z}{z(z+1)} dz$

$$\begin{aligned}
f(e) + f\left(\frac{1}{e}\right) &= \int_1^e \frac{\ln t}{1+t} dt + \int_1^e \frac{\ln t}{t(t+1)} dt = \int_1^e \frac{\ln t}{1+t} + \frac{\ln t}{t(t+1)} dt \\
&= \int_1^e \frac{\ln t}{t} dt \quad \left\{ \ln t = u, \frac{1}{t} dt \right\} \\
&= du = \int_0^1 u du = \frac{u^2}{2} \Big|_0^1 = \frac{1}{2}
\end{aligned}$$

14. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \leq x \leq 1 \\ \sin(\pi x) & \text{if } x > 1 \end{cases}$

If $f(x)$ is continuous on $\mathbb{R}$, then $a + b$ equals :

- (1) 3
- (2) -1
- (3) -3
- (4) 1

Ans. (2)

Sol. If f is continuous at $x = -1$, then
 $f(-1^-) = f(-1)$

$$\Rightarrow 2 = |a - 1 + b|$$

$$\Rightarrow |a + b - 1| = 2 \dots (i)$$

similarly

$$f(1^-) = f(1)$$

$$\Rightarrow |a + b + 1| = 0$$

$$\Rightarrow a + b = -1$$

15. Let $A = \{1, 2, 3, \dots, 10\}$ and $f: A \rightarrow A$ be defined as

$$f(k) = \begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases} \quad \text{Then the number of possible functions}$$

$g : A \rightarrow A$ such that $\text{gof} = f$ is :

- (1) 10^5
- (2) $^{10}C_5$
- (3) 5^5
- (4) $5!$

Ans. (1)

Sol. $g(f(x)) = f(x)$

$$\Rightarrow g(x) = x, \text{ when } x \text{ is even.}$$

5 elements in A can be mapped to any 10

$$\text{So, } 10^5 \times 1 = 10^5$$

- 16.** A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n , including 1, is :

(1) 11

(2) $6x$

(3) 12

(4) 6

Ans. (3)

Sol. $y + z = 5 \quad \dots(1)$

$$\frac{1}{y} + \frac{1}{z} = \frac{5}{6}$$

$$\Rightarrow \frac{y+z}{yz} = \frac{5}{6}$$

$$\Rightarrow \frac{5}{yz} = \frac{5}{6}$$

$$\Rightarrow yz = 6$$

$$\text{Also } (y - z)^2 = (y + z)^2 - 4yz$$

$$\Rightarrow (y - z)^2 = (y + z)^2 - 4yz$$

$$\Rightarrow (y - z)^2 = 25 - 4(6) = 1$$

$$\Rightarrow y - z = 1 \quad \dots(2)$$

from (1) and (2), $y = 3$ and $z = 2$

for calculating odd divisor of $p = 2^x \cdot 3^y \cdot 5^z$

x must be zero

$$P = 2^0 \cdot 3^3 \cdot 5^2$$

$$\therefore \text{ total odd divisors must be } (3 + 1)(2 + 1) = 12$$

- 17.** Let $f(x) = \sin^{-1} x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x \rightarrow 2} g(x)$, then the domain of the function fog is :

- (1) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$
 (2) $(-\infty, -1] \cup [2, \infty)$
 (3) $(-\infty, -2] \cup [-1, \infty)$
 (4) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

Ans. (1)

Sol. $g(2) = \lim_{x \rightarrow 2} \frac{(x-2)(x+1)}{(2x+3)(x-2)} = \frac{3}{7}$

For domain of fog (x)

$$\left| \frac{x^2 - x - 2}{2x^2 - x - 6} \right| \leq 1 \Rightarrow (x+1)^2 \leq (2x+3)^2 \Rightarrow 3x^2 + 10x + 8 \geq 0$$

$$\Rightarrow (3x+4)(x+2) \geq 0$$

$$x \in (-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right]$$

- 18.** If the mirror image of the point (1,3,5) with respect to the plane $4x-5y+2z=8$ is (α, β, γ) , then $5(\alpha + \beta + \gamma)$ equals:

- (1) 47
 (2) 39
 (3) 43
 (4) 41

Ans. (1)

Sol. Image of (1, 3, 5) in the plane $4x - 5y + 2z = 8$ is (α, β, γ)

$$\Rightarrow \frac{\alpha - 1}{4} = \frac{\beta - 3}{-5} = \frac{\gamma - 5}{2} = -2 \frac{(4(1) - 5(3) + 2(5) - 8)}{4^2 + 5^2 + 2^2} = \frac{2}{5}$$

$$\therefore \alpha = 1 + 4 \left(\frac{2}{5} \right) = \frac{13}{5}$$

$$\beta = 3 - 5 \left(\frac{2}{5} \right) = 1 = \frac{5}{5}$$

$$\gamma = 5 + 2 \left(\frac{2}{5} \right) = \frac{29}{5}$$

$$\text{Thus, } 5(\alpha + \beta + \gamma) = 5 \left(\frac{13}{5} + \frac{5}{5} + \frac{29}{5} \right) = 47$$

19. Let $f(x) = \int_0^x e^t f(t) dt + e^x$ be a differentiable function for all $x \in \mathbb{R}$. Then

$f(x)$ equals.

(1) $2e^{(e^x-1)} - 1$

(2) $e^{(e^x-1)}$

(3) $2e^{e^x} - 1$

(4) $e^{e^x} - 1$

Ans. (1)

Sol. Given, $f(x) = \int_0^x e^t f(t) dt + e^x \quad \dots(1)$

Differentiating both sides w.r.t x

$f'(x) = e^x \cdot f(x) + e^x \quad \text{(Using Newton Leibnitz Theorem)}$

$\Rightarrow \frac{f'(x)}{f(x) + 1} = e^x$

Integrating w.r.t x

$\int \frac{f'(x)}{f(x) + 1} dx = \int e^x dx$

$\Rightarrow \ln(f(x) + 1) = e^x + c$

Put $x = 0$

$\ln 2 = 1 + c \quad (\because f(0) = 1, \text{ from equation (1)})$

$\therefore \ln(f(x) + 1) = e^x + \ln 2 - 1$

$\Rightarrow f(x) + 1 = 2 \cdot e^{e^x-1}$

$\Rightarrow f(x) = 2e^{e^x-1} - 1$

20. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is:

(1) A right angle triangle having two of its sides of length $2r$ and r .

(2) An equilateral triangle of height $\frac{2r}{3}$.

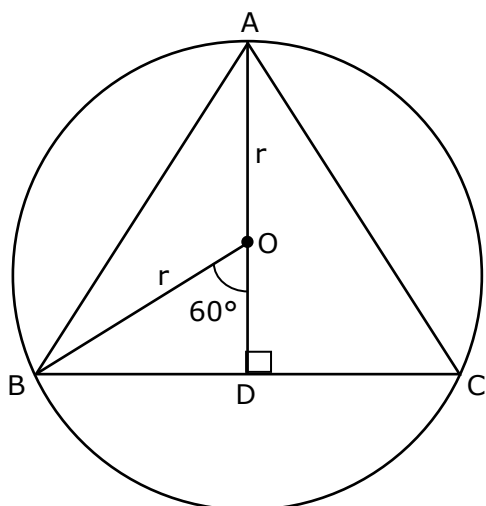
(3) An isosceles triangle with base equal to $2r$.

(4) An equilateral triangle having each of its side of length $\sqrt{3} r$.

Ans. (4)

Sol. Triangle of maximum area that can be inscribed in a circle is an equilateral triangle.

Let $\triangle ABC$ be inscribed in circle,



Now, in $\triangle OBD$

$$OD = r \cos 60^\circ = \frac{r}{2}$$

$$\text{Height} = AD = \frac{3r}{2}$$

Again in $\triangle ABD$

$$\text{Now } \sin 60^\circ = \frac{\frac{3r}{2}}{AB}$$

$$\Rightarrow AB = \sqrt{3} r$$

Section - B

1. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is

Ans. 1000

Sol. Since, required number has G.C.D with 18 as 3. It must be odd multiple of '3' but not a multiple of '9'.

(i) Now, 4-digit number which are odd multiple of '3' are,
1005, 1011, 1017, 9999 $\rightarrow$ 1499

(ii) 4-digit number which are odd multiple of 9 are,
1017, 1035, 9999 $\rightarrow$ 499

$$\therefore \text{Required numbers} = 1499 - 499 = 1000$$

2. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$.
Let $P_n = (\alpha)^n + (\beta)^n$, $P_{n-1} = 11$ and $P_{n+1} = 29$ for some integer $n \geq 1$.
Then, the value of P_n^2 is _____.

Ans. 324

Sol. Given, $\alpha + \beta = 1$, $\alpha\beta = -1$

$\therefore$ Quadratic equation with roots α, β is $x^2 - x - 1 = 0$

$$\Rightarrow \alpha^2 = \alpha + 1$$

Multiplying both sides by α^{n-1}

$$\alpha^{n+1} = \alpha^n + \alpha^{n-1} \quad \text{_____ (1)}$$

Similarly,

$$\beta^{n+1} = \beta^n + \beta^{n-1} \quad \text{_____ (2)}$$

Adding (1) & (2)

$$\alpha^{n+1} + \beta^{n+1} = (\alpha^n + \beta^n) + (\alpha^{n-1} + \beta^{n-1})$$

$$\Rightarrow P_{n+1} = P_n + P_{n-1}$$

$$\Rightarrow 29 = P_n + 11 \text{ (Given, } P_{n+1} = 29, P_{n-1} = 11)$$

$$\Rightarrow P_n = 18$$

$$\therefore P_n^2 = 18^2 = 324$$

- 3.** Let $X_1, X_2, \dots, X_{18}$ be eighteen observations such that $\sum_{i=1}^{18} (X_i - \alpha) = 36$ and $\sum_{i=1}^{18} (X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is _____.

Ans. 4

Sol. Given, $\sum_{i=1}^{18} (X_i - \alpha) = 36$

$$\Rightarrow \sum x_i - 18\alpha = 36$$

$$\Rightarrow \sum x_i - 18(\alpha + 2) \quad \dots(1)$$

Also, $\sum_{i=1}^{18} (X_i - \beta)^2 = 90$

$$\Rightarrow \sum x_i^2 + 18\beta^2 - 2\beta \sum x_i = 90$$

$$\Rightarrow \sum x_i^2 + 18\beta^2 + 2\beta \times 18(\alpha + 2) = 90 \quad \text{(using equation (1))}$$

$$\Rightarrow \sum x_i^2 = 90 - 18\beta^2 + 36\beta(\alpha + 2)$$

$$\sigma^2 = 1 \Rightarrow \frac{1}{18} \sum x_i^2 - \left(\frac{\sum x_i}{18} \right)^2 = 1 \quad (\because \sigma = 1, \text{ given})$$

$$\Rightarrow \frac{1}{18} (90 - 18\beta^2 + 36\alpha\beta + 72\beta) - \left(\frac{18(\alpha + 2)}{18} \right)^2 = 1$$

$$\Rightarrow 90 - 18\beta^2 + 36\alpha\beta + 72\beta - 18(\alpha + 2)^2 = 18$$

$$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - (\alpha + 2)^2 = 1$$

$$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4 - 4\alpha = 1$$

$$\Rightarrow \alpha^2 - \beta^2 + 2\alpha\beta + 4\beta - 4\alpha = 0$$

$$\Rightarrow (\alpha - \beta)(\alpha - \beta + 4) = 0$$

$$\Rightarrow \alpha - \beta = -4$$

$$\therefore |\alpha - \beta| = 4 \quad (\alpha \neq \beta)$$

4. In $I_{m,n} = \int_0^1 x^{m-1} (1-x)^{n-1} dx$, for $m, n \geq 1$ and $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \alpha I_{m,n}$, $\alpha \in \mathbb{R}$, then α equals_____.

Ans. 1

Sol. $I_{m,n} = \int_0^1 x^{m-1} \cdot (1-x)^{n-1} dx$

Put $x = \frac{1}{y+1} \Rightarrow dx = \frac{-1}{(y+1)^2} dy$

$1-x = \frac{y}{y+1}$

$\therefore I_{m,n} = \int_{\infty}^0 \frac{y^{n-1}}{(y+1)^{m+n}} (-1) dy = \int_0^{\infty} \frac{y^{n-1}}{(y+1)^{m+n}} dy \quad \dots(i)$

Similarly $I_{m,n} = \int_0^1 x^{n-1} \cdot (1-x)^{m-1} dx$

$\Rightarrow I_{m,n} = \int_0^{\infty} \frac{y^{m-1}}{(y+1)^{m+n}} dy \quad \dots(ii)$

From (i) & (ii)

$2I_{m,n} = \int_0^{\infty} \frac{y^{m-1} + y^{n-1}}{(y+1)^{m+n}} dy$

$\Rightarrow 2I_{m,n} = \int_0^1 \frac{y^{m-1} + y^{n-1}}{(y+1)^{m+n}} dy + \int_1^{\infty} \frac{y^{m-1} + y^{n-1}}{(y+1)^{m+n}} dy$
 $\quad \quad \quad I_1 \quad \quad \quad I_2$

Put $y = \frac{1}{z}$ in I_2

$dy = -\frac{1}{z^2} dz$

$\Rightarrow 2I_{m,n} = \int_0^1 \frac{y^{m-1} + y^{n-1}}{(y+1)^{m+n}} dy + \int_1^0 \frac{z^{m-1} + z^{n-1}}{(z+1)^{m+n}} (-dz)$

$\Rightarrow I_{m,n} = \int_0^1 \frac{y^{m-1} + y^{n-1}}{(y+1)^{m+n}} dy \Rightarrow \alpha = 1$

5. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is _____.

Ans. 3

Sol. E: $\frac{x^2}{9} + \frac{y^2}{4} = 1$ C: $x^2 + y^2 = \frac{31}{4}$

equation of tangent to ellipse is

$y = mx \pm \sqrt{9m^2 + 4} \quad \dots(i)$

equation of tangent to circle is

$$y = mx \pm \sqrt{\frac{31}{4}m^2 + \frac{31}{4}} \quad \dots(ii)$$

Comparing equation (i) & (ii)

$$9m^2 + 4 = \frac{31}{4}m^2 + \frac{31}{4}$$

$$\Rightarrow 36m^2 + 16 = 31m^2 + 31$$

$$\Rightarrow 5m^2 = 15$$

$$\Rightarrow m^2 = 3$$

6. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation

$$A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for some real numbers } \alpha \text{ and } \beta, \text{ then } \beta -$$

α is equal to _____.

Ans. 4

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

.
.
.
.
.

$$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}, A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{L.H.S} = A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 + \alpha + \beta & 0 & 0 \\ 0 & 2^{20} + \alpha 2^{19} + 2\beta & 0 \\ 3\alpha + 3\beta & 0 & 1 - \alpha - \beta \end{bmatrix}$$

$$\text{R.H.S} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \alpha + \beta = 0 \text{ and } 2^{20} + \alpha 2^{19} + 2\beta = 4$$

$$\Rightarrow 2^{20} + \alpha (2^{19} - 2) = 4$$

$$\Rightarrow \alpha = \frac{4 - 2^{20}}{2^{19} - 2} = -2$$

$$\Rightarrow \beta = 2$$

$$\therefore \beta - \alpha = 4$$

7. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence $-16, 8, -4, 2, \dots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then $p+q$ is equal to _____.

Ans. 10

Sol. Given, $4x^2 - 9x + 5 = 0$

$$\Rightarrow (x - 1)(4x - 5) = 0$$

$$\Rightarrow \text{A.M} = \frac{5}{4}, \text{G.M} = 1 \quad (\text{Q A.M} > \text{G.M})$$

Again, for the series

$-16, 8, -4, 2, \dots$

$$p^{\text{th}} \text{ term } t_p = -16 \left(\frac{-1}{2} \right)^{p-1}$$

$$q^{\text{th}} \text{ term } t_q = -16 \left(\frac{-1}{2} \right)^{q-1}$$

$$\text{Now, A.M} = \frac{t_p + t_q}{2} = \frac{5}{4} \text{ \& G.M} = \sqrt{t_p t_q} = 1$$

$$\Rightarrow 16^2 \left(-\frac{1}{2} \right)^{p+q-2} = 1$$

$$\Rightarrow (-2)^8 = (-2)^{(p+q-2)}$$

$$\Rightarrow p + q = 10$$

8. Let the normals at all the points on a given curve pass through a fixed point (a, b) . If the curve passes through $(3, -3)$ and $(4, -2\sqrt{2})$, and given that $a - 2\sqrt{2}b = 3$, then $(a^2 + b^2 + ab)$ is equal to _____.

Ans. 9

Sol. Let the equation of normal is $Y - y = -\frac{1}{m}(X - x)$, where, $m = \frac{dy}{dx}$

As it passes through (a, b)

$$b - y = -\frac{1}{m}(a - x) = -\frac{dx}{dy}(a - x)$$

$$\Rightarrow (b - y)dy = (x - a)dx$$

$$by - \frac{y^2}{2} = \frac{x^2}{2} - ax + c \quad \dots(i)$$

It passes through (3, -3) & (4, $-2\sqrt{2}$)

$$\therefore -3b - \frac{9}{2} = \frac{9}{2} - 3a + c$$

$$\Rightarrow -6b - 9 = 9 - 6a + 2c$$

$$\Rightarrow 6a - 6b - 2c = 18$$

$$\Rightarrow 3a - 3b - c = 9 \quad \dots(ii)$$

Also

$$-2\sqrt{2}b - 4 = 8 - 4a + c$$

$$4a - 2\sqrt{2}b - c = 12 \quad \dots(iii)$$

$$\text{Also } a - 2\sqrt{2}b = 3 \quad \dots(iv) \text{ (given)}$$

$$(ii) - (iii) \Rightarrow -a + (2\sqrt{2} - 3)b = -3 \quad \dots(v)$$

$$(iv) + (v) \Rightarrow b = 0, \quad a = 3$$

$$\therefore a^2 + b^2 + ab = 9$$

9. Let z be those complex number which satisfy

$$|z+5| \leq 4 \text{ and } z(1+i) + \bar{z}(1-i) \geq -10, i = \sqrt{-1}.$$

If the maximum value of $|z+1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$

is _____.

Ans. 48

Sol. Given, $|z + 5| \leq 4$

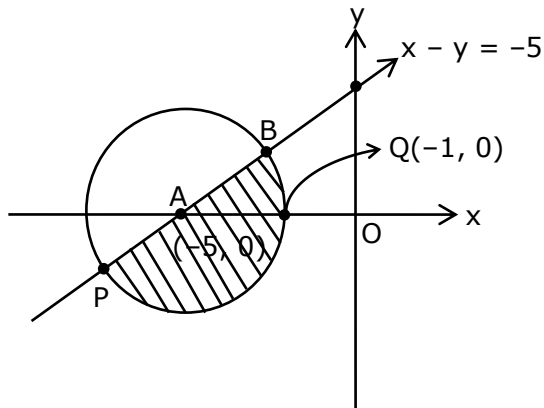
$$\Rightarrow (x + 5)^2 + y^2 \leq 16 \quad \dots(1)$$

$$\text{Also, } z(1+i) + \bar{z}(1-i) \geq -10.$$

$$\Rightarrow x - y \geq -5 \quad \dots(2)$$

From (1) and (2)

Locus of z is the shaded region in the diagram.



$|z + 1|$ represents distance of 'z' from $Q(-1, 0)$

Clearly 'p' is the required position of 'z' when $|z + 1|$ is maximum.

$$\therefore P \equiv (-5 - 2\sqrt{2}, -2\sqrt{2})$$

$$\therefore (PQ)^2 \Big|_{\max} = 32 + 16\sqrt{2}$$

$$\Rightarrow \alpha = 32$$

$$\Rightarrow \beta = 16$$

$$\text{Thus, } \alpha + \beta = 48$$

- 10.** Let a be an integer such that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval $(a, a + 1)$.
Then, $|a|$ is equal to _____.

Ans. 2

Sol. Let, $f(x) = 2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$

$$\Rightarrow f'(x) = 10(x^4 + 2x^3 + 3x^2 + 2x + 1)$$

$$= 10 \left(x^2 + \frac{1}{x^2} + 2 \left(x + \frac{1}{x} \right) + 3 \right)$$

$$= 10 \left(\left(x + \frac{1}{x} \right)^2 + 2 \left(x + \frac{1}{x} \right) + 1 \right)$$

$$= 10 \left(\left(x + \frac{1}{x} \right) + 1 \right)^2 > 0; \forall x \in \mathbb{R}$$

$\therefore f(x)$ is strictly increasing function. Since it is an odd degree polynomial it will have exactly one real root.

Now, by observation

$$f(-1) = 3 > 0$$

$$f(-2) = -64 + 80 - 80 + 40 - 20 + 10$$

$$= -34 < 0$$

$$\Rightarrow f(x) \text{ has at least one root in } (-2, -1) \equiv (a, a + 1)$$

$$\Rightarrow a = -2$$

$$\Rightarrow |a| = 2$$