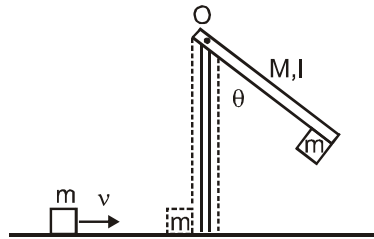


QUESTION PAPER WITH SOLUTION

PHYSICS _ 3 Sep. _ SHIFT - 1

1. A block of mass $m = 1 \text{ kg}$ slides with velocity $v = 6 \text{ m/s}$ on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass $M = 2 \text{ kg}$, and length $l = 1 \text{ m}$, the value of θ is approximately :
(take $g = 10 \text{ m/s}^2$)



- (1) 49° (2) 63° (3) 69° (4) 55°

Sol.

2

$$mv\ell = I\omega$$

$$\omega = \frac{mv\ell}{I}$$

$$\frac{1}{2}I\omega^2 = (m+M)g\ell_{\text{com}}(1 - \cos\theta)$$

$$\frac{1}{2} \frac{(mv\ell)^2}{I} = (m+M)g\ell_{\text{com}}(1 - \cos\theta)$$

$$I = \left(\frac{M\ell^2}{3} + m\ell^2 \right)$$

$$I = \left(\frac{2}{3} + 1 \right) = \frac{5}{3}$$

$$\frac{36 \times 3}{2 \times 5} = \frac{3 \times 10 \times 2}{3} (1 - \cos\theta)$$

$$\frac{27}{50} = (1 - \cos\theta)$$

$$\cos\theta = 1 - \frac{27}{50}$$

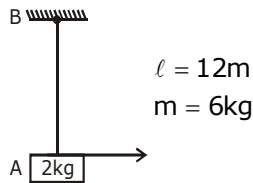
$$\cos\theta = \frac{23}{50}$$

$$\theta = 63^\circ$$

2. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ?

(1) 12 (2) 3 (3) 9 (4) 6

Sol. 1



$$\mu = \frac{6}{12} = \frac{1}{2} \text{ kg / m}$$

$$A \rightarrow V = \sqrt{\frac{T}{\mu}} = f\lambda$$

$$B \rightarrow \sqrt{\frac{T'}{\mu}} = f\lambda'$$

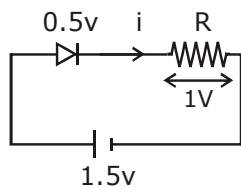
$$\sqrt{\frac{T}{T'}} = \frac{\lambda}{\lambda'}$$

$$\sqrt{\frac{20}{80}} = \frac{6}{\lambda'} = \lambda' = 12$$

3. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is :

(1) 300 Ω (2) 200 Ω (3) 50 Ω (4) 100 Ω

Sol. 4



$$V = iR$$

$$R = \frac{V}{i} = \frac{1}{10\text{mA}}$$

$$= \frac{1000}{10} = 100\Omega$$

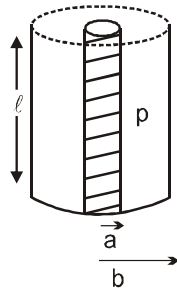
4. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as :
 (1) 2.125 cm (2) 2.124 cm (3) 2.123 cm (4) 2.121 cm

Sol. 2

$$\text{Least count} = \frac{0.1}{50} = \frac{1}{500} = 0.2 \times 10^{-2} = 0.002$$

when we multiply by division no.'s
 it must be even because L.C. is 0.002.

5. Model a torch battery of length ℓ to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R, the maximum joule heating in R will take place for :



(1) $R = \frac{\rho}{2\pi\ell} \ln\left(\frac{b}{a}\right)$ (2) $R = \frac{2\rho}{\pi\ell} \ln\left(\frac{b}{a}\right)$ (3) $R = \frac{\rho}{\pi\ell} \ln\left(\frac{b}{a}\right)$ (4) $\frac{\rho}{2\pi\ell} \ln\left(\frac{b}{a}\right)$

Sol. 4

$$dR = \rho \frac{dr}{2\pi r \ell}$$

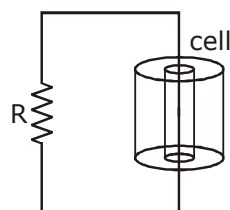
$$\int dR = \frac{\rho}{2\pi\ell} \int_a^b \frac{1}{r} dr$$

$$R = \frac{\rho}{2\pi\ell} [\ln(r)]_a^b$$

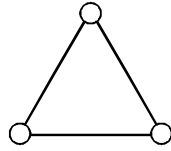
$$R = \frac{\rho}{2\pi\ell} \ln\left(\frac{b}{a}\right)$$

$$r = R$$

For Max heat transfer



6. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :



- (1) $\frac{3}{2} RT$ (2) $3RT$ (3) $\frac{5}{2} RT$ (4) $\frac{9}{2} RT$

Sol. 2

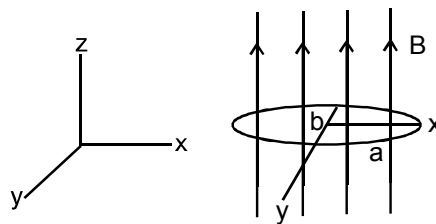
$$U = \frac{f}{2} nRT$$

$$U = \frac{6}{2} nRT$$

$$U = 3nRT \quad n = 1$$

$$U = 3RT$$

7. An elliptical loop having resistance R , of semi major axis a , and semi minor axis b is placed in magnetic field as shown in the figure. If the loop is rotated about the x -axis with angular frequency ω , the average power loss in the loop due to Joule heating is :



- (1) $\frac{\pi ab B \omega}{R}$ (2) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$ (3) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$ (4) Zero

Sol. 3

$$\phi = BA \cos \omega t$$

$$\epsilon = \frac{d\phi}{dt} = BA \omega \sin \omega t$$

$$\epsilon = B \pi a b \omega \sin \omega t$$

$$\epsilon_0 = B \pi a b \omega$$

$$P = \frac{\epsilon^2}{R} = \frac{B^2 \pi^2 a^2 b^2 \omega^2}{R} (\sin^2 \omega t)$$

$$P_{av} = \frac{B^2 \pi^2 a^2 b^2 \omega^2}{2R}$$

8. A balloon filled with helium (32° C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as :

(1) reversible isothermal (2) irreversible isothermal
(3) reversible adiabatic (4) irreversible adiabatic

Sol. 4

irreversible adiabatic → Because Energy can not be restored.

9. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to :

(1) 1.02 eV (2) 0.61 eV (3) 0.52 eV (4) 0.81 eV

Sol. 2

$$KE_{\max} = \frac{hc}{\lambda_1} - \phi$$

$$3KE_{\max} = \frac{hc}{\lambda_2} - \phi$$

$$3\left(\frac{hc}{\lambda_1} - \phi\right) = \frac{hc}{\lambda_2} - \phi$$

$$\frac{3hc}{\lambda_1} - \frac{hc}{\lambda_2} = 2\phi$$

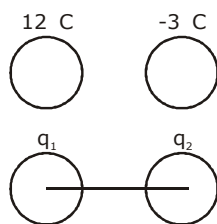
$$\frac{3 \times 1240}{500} - \frac{1240}{200} = 2\phi$$

$$\phi = 0.6$$

10. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and $-3\mu\text{C}$ charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively :

(1) 6 μC and 3 μC (2) 4.5 μC on both
(3) + 4.5 μC and $-4.5 \mu\text{C}$ (4) 3 μC and 6 μC

Sol. 1



$$q_1 + q_2 = 9\mu\text{C} \quad \dots(1)$$

$$\frac{Kq_1}{\frac{2}{3}R} = \frac{Kq_2}{\frac{1}{3}R}$$

$$q_1 = 2q_2$$

$$3q_2 = 9\mu\text{C}$$

$$q_2 = 3\mu\text{C}$$

$$q_1 = 6\mu\text{C}$$

- 11.** In a radioactive material, fraction of active material remaining after time t is $9/16$. The fraction that was remaining after $t/2$ is :

(1) $\frac{3}{4}$ (2) $\frac{7}{8}$ (3) $\frac{4}{5}$ (4) $\frac{3}{5}$

Sol. 1

$$N = N_0 e^{-\lambda t}$$

$$\left(\frac{N}{N_0}\right) = e^{-\lambda t}$$

$$\frac{9}{16} = e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t/2}$$

$$\frac{N}{N_0} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$$

- 12.** Moment of inertia of a cylinder of mass M , length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M \left(\frac{R^2}{4} + \frac{L^2}{12} \right)$. If such a cylinder is to be made for a given mass of a material, the ratio L/R for it to have minimum possible I is :

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\sqrt{\frac{3}{2}}$

Sol. 4

$$M = d.V$$

$$M = d\pi R^2 L$$

$$I = M \left(\frac{R^2}{4} + \frac{L^2}{12} \right)$$

$$I = M \left(\frac{M}{4d\pi L} + \frac{L^2}{12} \right)$$

$$I = \left(\frac{M^2}{4d\pi L} + \frac{ML^2}{12} \right)$$

$$\frac{dI}{dL} = \frac{M^2}{4d\pi} \left(\frac{-1}{L^2} \right) + \frac{2LM}{12} = 0$$

$$\frac{M^2}{4d\pi L^2} = \frac{2LM}{12}$$

$$\frac{d\pi R^2 L}{4d\pi L^2} = \frac{L}{6}$$

$$\frac{R^2}{L^2} = \frac{2}{3}$$

$$\frac{R}{L} = \sqrt{\frac{2}{3}} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

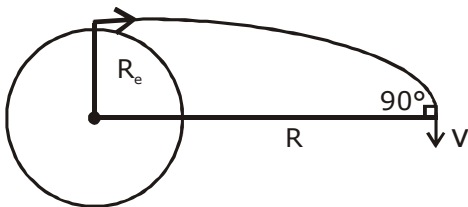
- 13.** A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e . By firing rockets attached to it, its speed is instantaneously increased

in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R . Value of R is :

- (1) $2R_e$ (2) $3R_e$ (3) $4R_e$ (4) $2.5 R_e$

Sol. **2**

$$V_0 = \sqrt{\frac{GM}{R_e}} \times \sqrt{\frac{3}{2}}$$



$$mV_0 R_e = mVR$$

$$\sqrt{\frac{3}{2}} \sqrt{\frac{GM}{R_e}} R_e = VR$$

$$-\frac{GMm}{R_e} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$-\frac{GMm}{R_e} + \frac{1}{2}m \frac{3}{2} \frac{GM}{R_e} = -\frac{GMm}{R} + \frac{1}{2}m \frac{3}{2} \frac{GM R_e^2}{R^2}$$

$$-\frac{1}{R_e} + \frac{3}{4R_e} = -\frac{1}{R} + \frac{3}{4} \frac{R_e}{R^2}$$

$$-\frac{1}{4R_e} = -\frac{1}{R} + \frac{3}{4} \frac{R_e}{R^2}$$

By further calculating

$$R = 3R_e$$

- 14.** Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :

(1) 4 : 1 (2) 2 : 1 (3) 0.8 : 1 (4) 8 : 1

Sol. 4

$$P_{in} = P_0 + \frac{4T}{R_1}$$

$$1.01 = 1 + \frac{4T}{R_1}$$

$$0.01 = \frac{4T}{R_1}$$

$$0.02 = \frac{4T}{R_2}$$

$$\frac{1}{2} = \frac{R_2}{R_1}$$

$$\frac{V_1}{V_2} = \frac{8}{1}$$

- 15.** In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :

(1) 0.17° (2) 0.07° (3) 0.57° (4) 1.7°

Sol. 3

$$B = \frac{\lambda D}{d}$$

$$\theta = \frac{\beta}{D} = \left(\frac{\lambda}{d} \right)$$

$$\theta = \frac{500 \times 10^{-9}}{(0.05 \times 10^{-3})}$$

$$\theta = \frac{5 \times 10^{-2}}{5} = \frac{5}{100 \times 5}$$

$$\theta^\circ = \frac{5}{100} \times \frac{180}{5 \times \pi}$$

$$\theta^\circ = 0.57^\circ$$

- 16.** A 750 Hz, 20 V (rms) source is connected to a resistance of $100\ \Omega$, an inductance of $0.1803\ \text{H}$ and a capacitance of $10\ \mu\text{F}$ all in series. The time in which the resistance (heat capacity $2\ \text{J}/^\circ\text{C}$) will get heated by 10°C . (assume no loss of heat to the surroundings) is close to :
- (1) 245 s (2) 365 s (3) 418 s (4) 348 s

Sol. 4

$$f = 750\text{Hz}, V_{\text{rms}} = 20\text{V}, R = 100\Omega, L = 0.1803\text{H}$$

$$C = 10\mu\text{f}$$

$$S = 2\text{J} / ^\circ\text{C}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$|Z| = \sqrt{(100)^2 + \left(2 \times 3.14 \times 750 \times 0.1803 - \frac{1}{2 \times 3.14 \times 750 \times 10^{-5}}\right)^2}$$

$$|Z| = 834\Omega$$

In AC

$$P = V_{\text{rms}} i_{\text{rms}} \cos \phi$$

$$P = \left(V_{\text{rms}} \cdot \frac{V_{\text{rms}}}{|Z|} \cdot \frac{R}{|Z|} \right)$$

$$P = \left(\frac{V_{\text{rms}}}{|Z|} \right)^2 R$$

$$P = \left(\frac{20}{834} \right)^2 \times 100 = 0.0575\text{J} / \text{S}$$

$$P \times t = S \Delta \theta$$

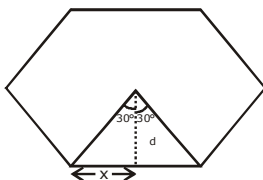
$$t = \frac{2(10)}{0.0575}$$

$$t = 348\text{ sec}$$

- 17.** Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is :

- (1) $250\sqrt{3}$ (2) $50\sqrt{3}$ (3) $500\sqrt{3}$ (4) $5\sqrt{3}$

Sol. 3



$$\tan 30 = \frac{x}{d}$$

$$d = \frac{x}{\tan 30}$$

$$d = \frac{5 \times 10^{-2}}{\frac{1}{\sqrt{3}}}$$

$$d = 5\sqrt{3} \times 10^{-2}$$

$$B = \frac{6 \times \mu_0 I N}{4\pi d} (\sin \theta_1 + \sin \theta_2)$$

$$B = \frac{\mu_0 I}{4\pi} \times \frac{50 \times 6}{5\sqrt{3} \times 10^{-2}} (\sin 30 + \sin 30)$$

$$B = \frac{10 \times 6 \times 100}{\sqrt{3} \times 4} \left(\frac{\mu_0 I}{\pi} \right)$$

$$B = 500\sqrt{3}$$

- 18.** The magnetic field of a plane electromagnetic wave is

$$\vec{B} = 3 \times 10^{-8} \sin[200\pi(y + ct)] \hat{i} \text{ T}$$

where $c = 3 \times 10^8 \text{ ms}^{-1}$ is the speed of light.

The corresponding electric field is :

(1) $\vec{E} = -9 \sin [200\pi(y + ct)] \hat{k} \text{ V/m}$

(2) $\vec{E} = 9 \sin [200\pi(y + ct)] \hat{k} \text{ V/m}$

(3) $\vec{E} = -10^{-6} \sin [200\pi(y + ct)] \hat{k} \text{ V/m}$

(4) $\vec{E} = 3 \times 10^{-8} \sin [200\pi(y + ct)] \hat{k} \text{ V/m}$

Sol.

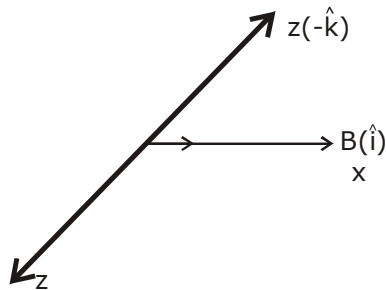
1

$$E = BC$$

$$E = 3 \times 10^{-8} \times 3 \times 10^8$$

$$E = 9$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{r}$$



$$E = 9 \sin [200\pi(y + ct)(-\hat{k})]$$

- 19.** A charged particle carrying charge $1 \mu\text{C}$ is moving with velocity $(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ ms}^{-1}$. If an external magnetic field of $(5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3} \text{ T}$ exists in the region where the particle is moving then the force on the particle is $\vec{F} \times 10^{-9} \text{ N}$. The vector $\vec{F}$ is :

(1) $-0.30\hat{i} + 0.32\hat{j} - 0.09\hat{k}$

(2) $-3.0\hat{i} + 3.2\hat{j} - 0.9\hat{k}$

(3) $-30\hat{i} + 32\hat{j} - 9\hat{k}$

(4) $-300\hat{i} + 320\hat{j} - 90\hat{k}$

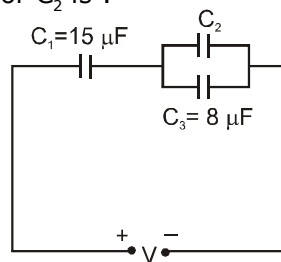
Sol. 3

$$\vec{F} = q(\vec{V} \times \vec{B})$$

$$\vec{F} = 10^{-6}(2\hat{i} + 3\hat{j} + 4\hat{k}) \times (5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$$

$$\vec{F} = (-30\hat{i} + 32\hat{j} - 9\hat{k}) \times 10^{-9}$$

- 20.** In the circuit shown in the figure, the total charge is $750 \mu\text{C}$ and the voltage across capacitor C_2 is 20 V . Then the charge on capacitor C_2 is :



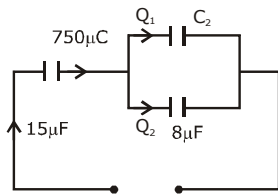
(1) $650 \mu\text{C}$

(2) $450 \mu\text{C}$

(3) $590 \mu\text{C}$

(4) $160 \mu\text{C}$

Sol. 3



$$Q_2 = CV = 8\mu\text{f} \times 20 = 160\mu\text{C}$$

$$Q_1 = 750 - 160 = 590\mu\text{C}$$

- 21.** A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at $5 \text{ revolutions per minute (rpm)}$. The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _____.

Sol. 9

$$I_1\omega_1 = I_2\omega_2$$

$$\left(\frac{MR^2}{2} + mR^2\right)\omega_1 = \omega_2 \frac{MR^2}{2}$$

$$\left(1 + \frac{2mR^2}{MR^2}\right)\omega_1 = \omega_2$$

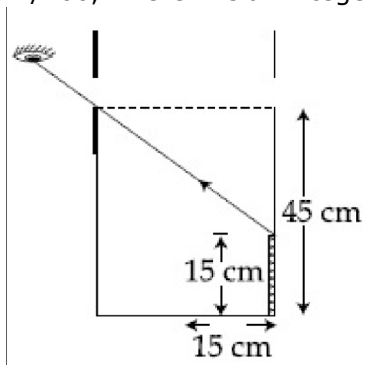
$$\left(1 + \frac{2 \times 80}{200}\right)\omega_1 = \omega_2$$

$$\omega_2 = \omega_1 \times 1.8$$

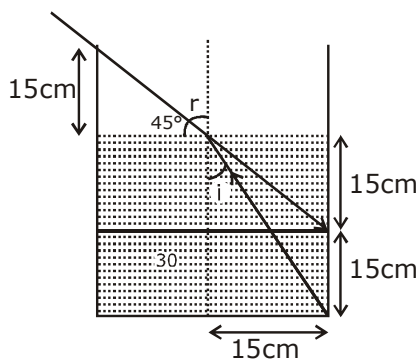
$$2\pi f_2 = 2\pi f_1 \times 1.8$$

$$f_2 = 5 \times 1.8 = 9$$

22. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is $N/100$, where N is an integer, the value of N is _____.



Sol. 1.58



$$\mu \sin i = 1 \sin 45$$

$$\mu \frac{15}{\sqrt{1125}} = \frac{1}{\sqrt{2}}$$

$$\mu = \frac{\sqrt{1125}}{15\sqrt{2}}$$

$$\mu = \sqrt{\frac{1125}{450}}$$

$$\mu = 1.58$$

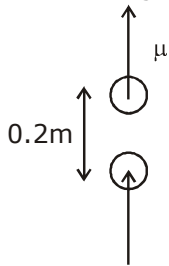
$$\mu = \frac{N}{100} = 1.58$$

$$\Rightarrow N = 158$$

- 23.** A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is ($g = 10 \text{ ms}^{-2}$)_____.

Sol. 150

$$m = 0.15 \text{ kg}$$



$$\mu = \sqrt{2gh}$$

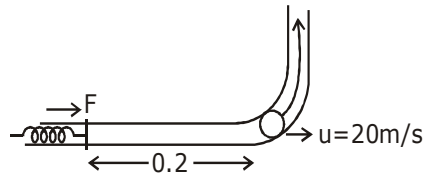
$$\mu = \sqrt{2 \times 10 \times 20}$$

$$\mu = 20 \text{ m / sec}$$

$$Fd = \frac{1}{2}mv^2$$

$$F \times 0.2 = \frac{1}{2} \times \frac{0.15}{100} \times 400$$

$$F = \frac{15 \times 200 \times 10}{100 \times 0.2} = 150$$

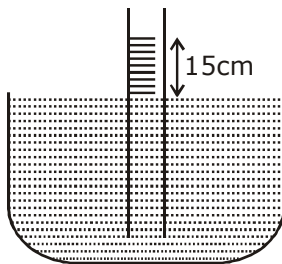


- 24.** When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass is close to 0° , the surface tension of the liquid, in milliNewton m^{-1} , is

[$\rho_{\text{(liquid)}} = 900 \text{ kgm}^{-3}$, $g = 10 \text{ ms}^{-2}$] (Give answer in closest integer)_____.

Sol. 101

$$r = 0.015$$



$$h = \frac{2T}{\rho g r}$$

$$T = \frac{h \rho g r}{2}$$

$$T = \frac{15 \times 10^{-2} \times 900 \times 10 \times 0.015 \times 10^{-2}}{2 \times 10^3}$$

$$T = 101 \text{ milli N/m}$$

- 25.** A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{\text{(beaker)}} = 6 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$ and $\gamma_{\text{(mercury)}} = 1.5 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to _____.

Sol. 20

$$\Delta V = v \gamma \Delta T$$

$$V_1 \gamma_1 = V_2 \gamma_2$$

$$500\text{cc} \times 6 \times 10^{-6} = V_m \times 1.5 \times 10^{-4}$$

$$V_m = \frac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}} = \frac{30}{1.5}$$

$$V_m = 20\text{cc}$$

QUESTION PAPER WITH SOLUTION

CHEMISTRY _ 3 Sep. _ SHIFT - 1

1. It is true that :
- (1) A second order reaction is always a multistep reaction
 - (2) A first order reaction is always a single step reaction
 - (3) A zero order reaction is a multistep reaction
 - (4) A zero order reaction is a single step reaction

Sol. 3
Factual

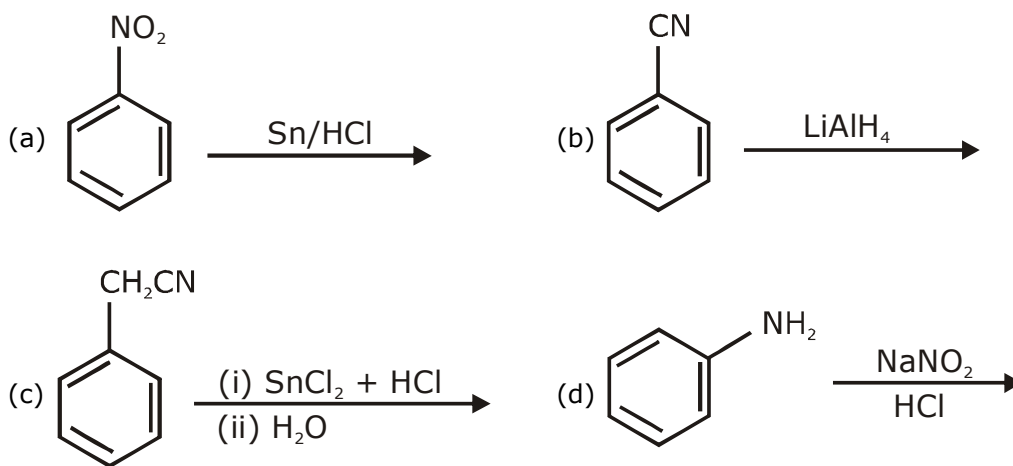
2. An acidic buffer is obtained on mixing :
- (1) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH_3COONa
 - (2) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl
 - (3) 100 mL of 0.1 M CH_3COOH and 100 mL of 0.1 M NaOH
 - (4) 100 mL of 0.1 M CH_3COOH and 200 mL of 0.1 M NaOH

Sol. 1

$$\begin{array}{rcccl} 2\text{HCl} & + & \text{CH}_3\text{COO}^- & \longrightarrow & \text{CH}_3\text{COOH} & + & \text{OH}^- \\ 10 & & 20 & & & & \\ \text{X} & & 10 & & 10 & & \end{array}$$

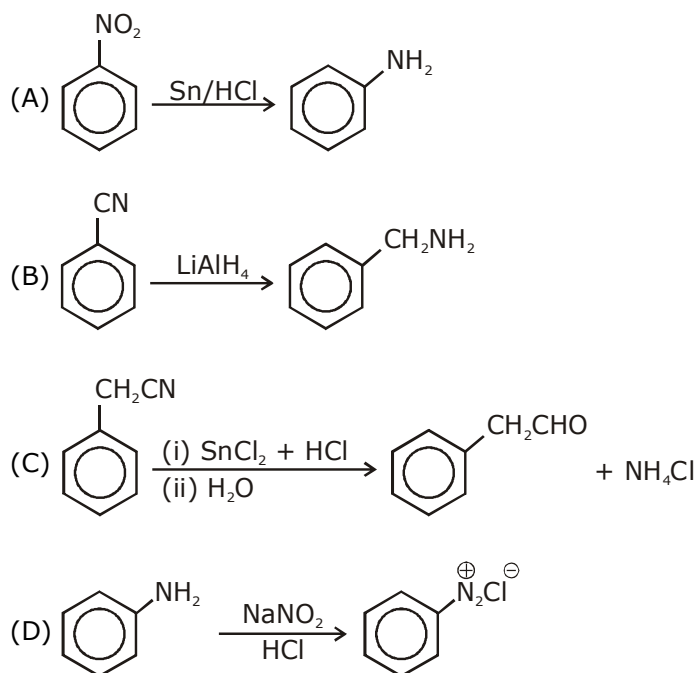
└──────────┘
Acidic buffer

3. The Kjeldahl method of Nitrogen estimation fails for which of the following reaction products?



- (1) (a), (c) and (d)
- (2) (b) and (c)
- (3) (c) and (d)
- (4) (a) and (d)

Sol. 3



Diazo compound and inorganic nitrogen can't be estimated by kjeldal method.

- 4.** If the boiling point of H_2O is 373 K, the boiling point of H_2S will be :
- (1) greater than 300 K but less than 373 K
 - (2) equal to 373 K
 - (3) more than 373 K
 - (4) less than 300 K

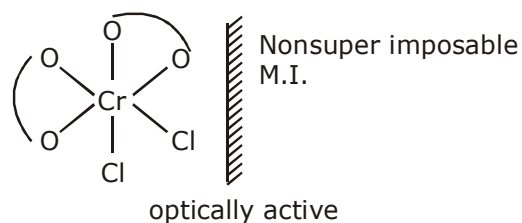
Sol. 4

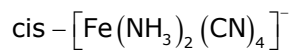
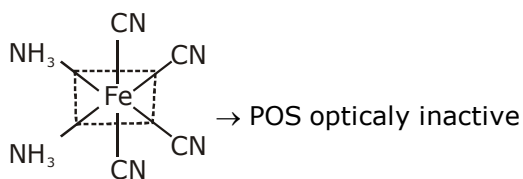
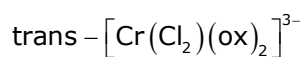
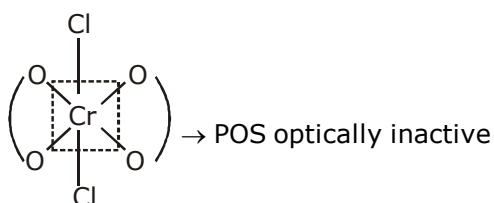
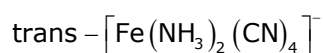
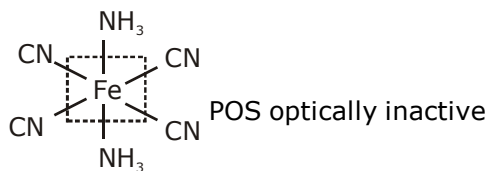
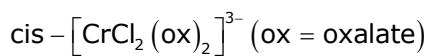
Less than 300 K (factual)

- 5.** The complex that can show optical activity is :

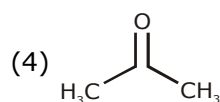
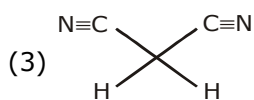
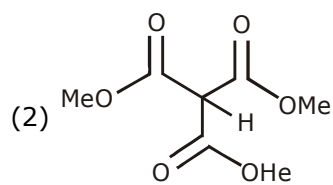
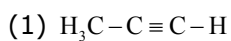
- (1) $\text{cis} - [\text{CrCl}_2(\text{ox})_2]^{3-}$ (ox = oxalate)
- (2) $\text{trans} - [\text{Fe}(\text{NH}_3)_2(\text{CN})_4]^-$
- (3) $\text{trans} - [\text{Cr}(\text{Cl}_2)(\text{ox})_2]^{3-}$
- (4) $\text{cis} - [\text{Fe}(\text{NH}_3)_2(\text{CN})_4]^-$

Sol. 1

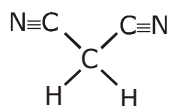




6. Which one of the following compounds possesses the most acidic hydrogen?



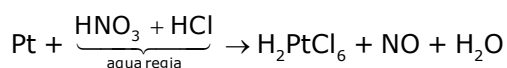
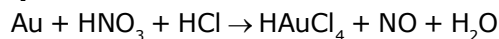
Sol. 3



has most acidic hydrogen among given compound, this is due to strong -M effect of -CN group which stabilize -ve charge significantly.

7. Aqua regia is used for dissolving noble metals (Au, Pt, etc.). The gas evolved in this process is :
 (1) N_2O_3 (2) N_2 (3) N_2O_5 (4) NO

Sol. 4

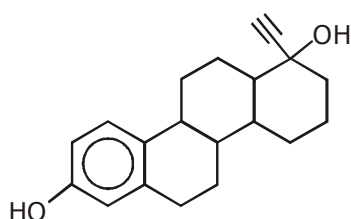


8. The antifertility drug "Novestrol" can react with :

- (1) Br_2 /water; $ZnCl_2$ /HCl; $FeCl_3$ (2) Br_2 /water; $ZnCl_2$ /HCl; NaOCl
 (3) Alcoholic HCN; NaOCl; $ZnCl_2$ /HCl (4) $ZnCl_2$ /HCl; $FeCl_3$; Alcoholic HCN

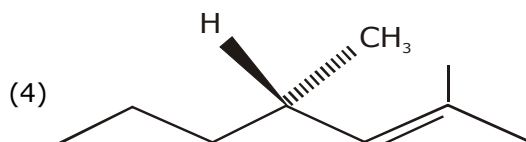
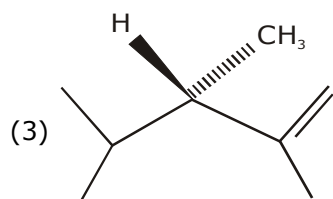
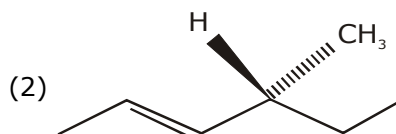
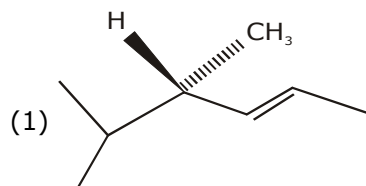
Sol. 1

Novestrol

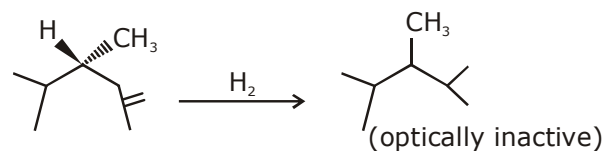


It can react with Br_2 /water due to presence of unsaturation, with $ZnCl_2$ /HCl due to -OH group and with $FeCl_3$ due to phenol.

9. Which of the following compounds produces an optically inactive compound on hydrogenation?



Sol. 3



10. Of the species, NO, NO^+ , NO^{2+} and NO^- , the one with minimum bond strength is :
 (1) NO^- (2) NO^+ (3) NO^{2+} (4) NO

Sol. 1

$$\begin{aligned} \text{B.O. NO}^- &= 2 \\ \text{BO NO}^+ &= 3 \\ \text{BO NO}^{2+} &= 2.5 \\ \text{BO NO} &= 2.5 \end{aligned}$$

$$\text{B.O} \propto \frac{1}{\text{B.L}}$$

11. Glycerol is separated in soap industries by :

- | | |
|-----------------------------|---|
| (1) Fractional distillation | (2) Distillation under reduced pressure |
| (3) Differential extraction | (4) Steam distillation |

Sol. 2

conceptual

Glycerol is separated in soap industries by distillation under reduced pressure

12. Thermal power plants can lead to :

- | | |
|---------------------------|------------------------|
| (1) Ozone layer depletion | (2) Blue baby syndrome |
| (3) Eutrophication | (4) Acid rain |

Sol. 4

Refer environmental chemistry

It emits CO_2 that combine with moisture of atmosphere and forms H_2CO_3 (carbonic acid)

13. Henry's constant (in kbar) for four gases α , β , γ and δ in water at 298 K is given below :

	α	β	γ	δ
K_H	50	2	2×10^{-5}	0.5

(density of water = 10^3 kg m^{-3} at 298 K)

This table implies that :

- (1) solubility of γ at 308 K is lower than at 298 K
- (2) The pressure of a 55.5 molal solution of δ is 250 bar
- (3) α has the highest solubility in water at a given pressure
- (4) The pressure of a 55.5 molal solution of γ is 1 bar

Sol. 1

$p = K_H X$ mol fraction of gas in liquid.

On increasing temp, ' K'_H ' increases

Hence solubility $\downarrow$

therefore, option 1

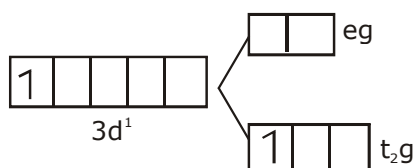
- 14.** The electronic spectrum of $[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$ shows a single broad peak with a maximum at $20,300 \text{ cm}^{-1}$. The crystal field stabilization energy (CFSE) of the complex ion, in kJ mol^{-1} , is :

(1 $\text{kJ mol}^{-1} = 83.7 \text{ cm}^{-1}$)

- (1) 83.7 (2) 242.5 (3) 145.5 (4) 97

Sol. 4

$[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$ Ti^{3+} $3d^1$ in octahedral field of ligand



$$\text{CFSE} = -0.4 \Delta_0$$

$$\text{CFSE} = \frac{-0.4 \times 20300}{83.7}$$

$$= 97 \text{ kJ mol}^{-1}$$

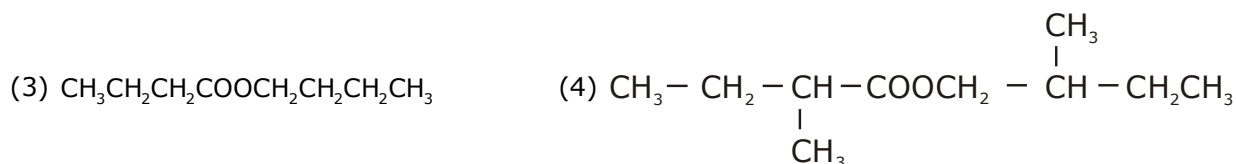
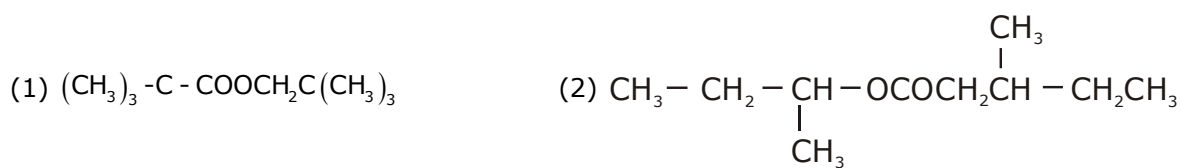
- 15.** The atomic number of the element unnilennium is :

- (1) 109 (2) 102 (3) 119 (4) 108

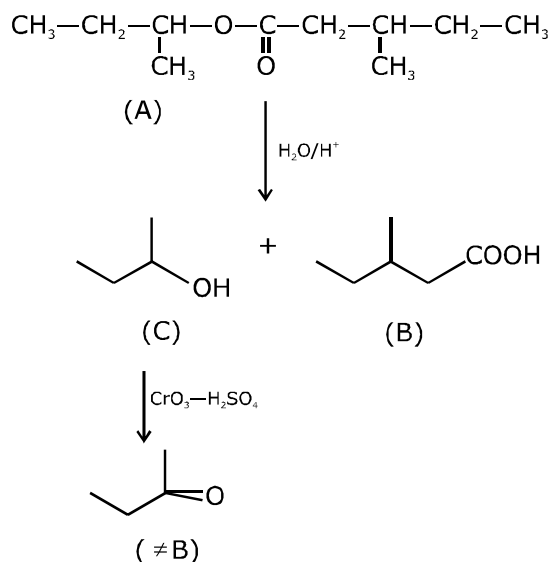
Sol. 1

Unnilennium 109

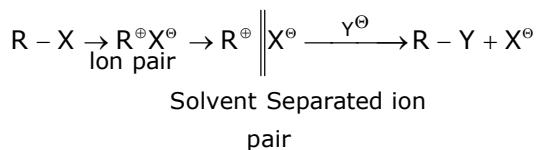
- 16.** An organic compound [A], molecular formula $\text{C}_{10}\text{H}_{20}\text{O}_2$ was hydrolyzed with dilute sulphuric acid to give a carboxylic acid [B] and an alcohol [C]. Oxidation of [C] with $\text{CrO}_3 - \text{H}_2\text{SO}_4$ produced [B]. Which of the following structures are not possible for [A]?



Sol. 2



17. The mechanism of $\text{S}_{\text{N}}1$ reaction is given as :



A student writes general characteristics based on the given mechanism as :

(a) The reaction is favoured by weak nucleophiles.

(b) R^+ would be easily formed if the substituents are bulky.

(c) The reaction is accompanied by racemization.

(d) The reaction is favoured by non-polar solvents.

Which observations are correct?

(1) (a) and (b)

(2) (a), (b) and (c)

(3) (a) and (c)

(4) (b) and (d)

Sol. 2

Statement (a), (b) & (c) are correct for $\text{S}_{\text{N}}1$ reaction mechanism.

18. Tyndall effect is observed when:

(1) The diameter of dispersed particles is much smaller than the wavelength of light used.

(2) The diameter of dispersed particles is much larger than the wavelength of light used.

(3) The refractive index of dispersed phase is greater than that of the dispersion medium.

(4) The diameter of dispersed particles is similar to the wavelength of light used.

Sol. 4

Diameter of dispersed particles should not be much smaller than wavelength of light used.
Refer topic surface chemistry

- 19.** Let C_{NaCl} and C_{BaSO_4} be the conductances (in S) measured for saturated aqueous solutions of NaCl and BaSO_4 , respectively, at a temperature T. Which of the following is false?

- (1) $C_{\text{NaCl}}(T_2) > C_{\text{NaCl}}(T_1)$ for $T_2 > T_1$
 (2) $C_{\text{BaSO}_4}(T_2) > C_{\text{BaSO}_4}(T_1)$ for $T_2 > T_1$
 (3) Ionic mobilities of ions from both salts increase with T.
 (4) $C_{\text{NaCl}} \gg C_{\text{BaSO}_4}$ at a given T

Sol. 4

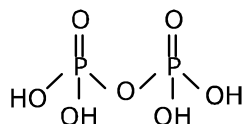
Ionic

$C_{\text{NaCl}} \gg C_{\text{BaSO}_4}$ at temp 'T'

- 20.** In a molecule of pyrophosphoric acid, the number of P-OH, P = O and P - O - P bonds/moiety(ies) respectively are :

- (1) 3, 3 and 3 (2) 4, 2 and 1 (3) 2, 4 and 1 (4) 4, 2 and 0

Sol. 2



P - OH bonds = 4

P = O bonds = 2

P - O - P linkage = 1

Ans. 4, 2, 1

- 21.** The mole fraction of glucose ($\text{C}_6\text{H}_{12}\text{O}_6$) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is _____.

Sol. 47 %

$$x_{\text{Glucose}} = 0.1$$

$$\text{mass\% of glucose} = \frac{0.1 \times 180}{0.1 \times 180 + 0.9 \times 18} \times 100$$

$$= \frac{1800}{18 + 16.2}$$

$$= \frac{1800}{34.2} \%$$

$$= 52.63\%$$

$$= 53\%$$

$$\therefore \text{mass \% of H}_2\text{O} = 47\%$$

- 22.** The volume strength of 8.9 M H_2O_2 solution calculated at 273 K and 1 atm is _____. ($R = 0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1}$) (rounded off to the nearest integer)

Sol. 100

$$\begin{aligned}\text{Vol. strength} &= \frac{8.9}{2} \times \frac{0.0821 \times 273}{1} \\ &= 99.73 \\ &= 100\end{aligned}$$

- 23.** An element with molar mass $2.7 \times 10^{-2} \text{ kg mol}^{-1}$ forms a cubic unit cell with edge length 405 pm. If its density is $2.7 \times 10^3 \text{ kg m}^{-3}$, the radius of the element is approximately _____ $\times 10^{-12} \text{ m}$ (to the nearest integer).

Sol. 143

$$\text{Density} = \frac{Z \times \text{GMM}}{N_A \times a^3}$$

$$2.7 \times 10^3 = \frac{Z \times 2.7 \times 10^{-2}}{6.023 \times 10^{23} \times (405 \times 10^{-12})^3}$$

$$Z = 6.023 \times 405 \times 405 \times 405 \times 10^{23-36+3+2}$$

$$Z = 6.023 \times 405 \times 405 \times 405 \times 10^{-8}$$

$$Z = 4$$

FCC

$$4R = \sqrt{2} \times a$$

$$R = \frac{405}{2\sqrt{2}} \times 10^{-12} = 143.21 \times 10^{-12} \text{ m}$$

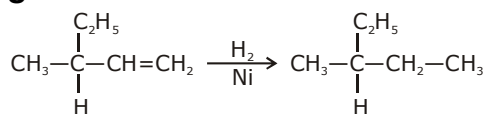
$$= 143 \text{ ans}$$

- 24.** The total number of monohalogenated organic products in the following (including stereoisomers) reaction is _____.

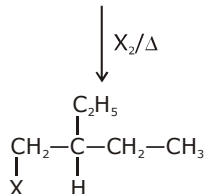


(Simplest
optically
active
alkene)

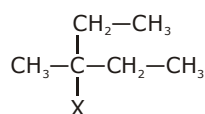
Sol. 8



(Simplest optically active alkene)

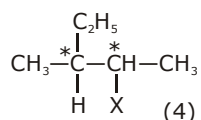


+

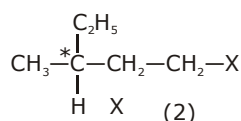


+

Total 8 organic products are possible



+



25. The photoelectric current from Na (Work function, $w_0 = 2.3 \text{ eV}$) is stopped by the output voltage of the cell $\text{Pt(s)} \mid \text{H}_2(\text{g}, 1 \text{ Bar}) \mid \text{HCl (aq. pH} = 1) \mid \text{AgCl(s)} \mid \text{Ag(s)}$.

The pH of aq. HCl required to stop the photoelectric current from K ($w_0 = 2.25 \text{ eV}$), all other conditions remaining the same, is _____ $\times 10^{-2}$ (to the nearest integer).

Given,

$$2.303 \frac{RT}{F} = 0.06 \text{ V}; E_{\text{AgCl}|\text{Ag}|\text{Cl}^-}^0 = 0.22 \text{ V}$$

Sol. 58

Energy of photon = $2.3 - E_{\text{cell}} \{\text{for Na}\}$

Energy of photon = $2.25 - E_{\text{cell}} \{\text{for K}\}$

$E_{\text{cell}} \{\text{for 'Na'}\} + 0.05 = E_{\text{cell}} \{\text{for 'K'}\}$

$0.22 + 0.06 \log [\text{H}^+][\text{Cl}^-] + 0.05 = 0.22 + 0.06 \log [\text{H}^+][\text{Cl}^-]$

$6 \log (10^{-2}) + 5 = 6 \log [\text{H}^+][\text{Cl}^-]$

$\log (10^{-12}) + \log (10^5) = \log \{[\text{H}^+][\text{Cl}^-]\}^6$

$\{[\text{H}^+][\text{Cl}^-]\}^6 = 10^{-7}$

$[\text{H}^+]^{12} = 10^{-7}$

$$\text{pH} = \frac{7}{12} = 0.58$$

$$= 58 \times 10^{-2} = 58 \text{ Ans}$$

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 3 Sep. _ SHIFT - 1

Q.1 The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots$ up to 51^{th} term) $+(1! - 2! + 3! - \dots$ up to 51^{th} term) is equal to:

- (1) $1 - 51(51)!$ (2) $1 + (52)!$ (3) 1 (4) $1 + (51)!$

Sol. 2

$$2. {}^1P_0 = \underline{2}$$

$$3. {}^2P_1 = \underline{3}$$

$$4. {}^3P_2 = \underline{4}$$

$$(\underline{2} - \underline{3} + \underline{4} - \underline{5} + \dots \dots \dots \underline{52}) + (1 - 2 + 3 - 4 \dots \dots + 51)$$

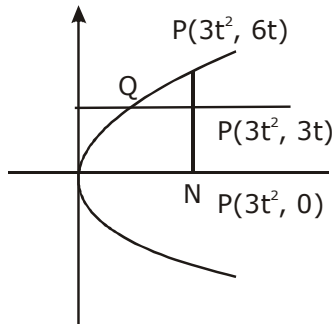
$$= \underline{52} + 1$$

Q.2 Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis

which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then:

- (1) $PN=4$ (2) $MQ=\frac{1}{3}$ (3) $PN=3$ (4) $MQ=\frac{1}{4}$

Sol. 4



Q (h, 3t) lie on

Parabola

$$9t^2 = 12h$$

$$h = \frac{3t^2}{4}$$

$$Q = \left(\frac{3t^2}{4}, 3t \right)$$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3t^2}{4} - 3t^2\right)} \quad (x - 3t^2)$$

$$y = \frac{-4t}{3t^2}(x - 3t^2)$$

$$\text{put } x = 0$$

$$y = \frac{-4}{3t}(-3t^2) = 4t$$

$$4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$$

$$PN = 6t = 6 \cdot \frac{1}{3} = 2$$

$$M = \left[\frac{1}{3}, 1\right], Q\left[\frac{1}{12}, 1\right]$$

$$MQ = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$$

Q.3 If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$, then $B+C$ is equal to:

(1) 1

(2) -1

(3) -3

(4) 9

Sol. **3**

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 0 & -1 & 3x-8 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$$\Rightarrow -1[(3-2x)(x-2) - (3x-4)] + (3x-8)[(x-2)(-x+2) - (2x-3)] = Ax^3+Bx^2+Cx+D$$

$$\Rightarrow 3x - 2x^2 - 6 + 4x - 3x + 4 + (3x-8)[-x^2 + 4x - 4 - 2x + 3] = Ax^3+Bx^2+Cx+D$$

$$A = -3, B = 12, C = -15$$

$$B + C = -3$$

Q.4 The foot of the perpendicular drawn from the point (4,2,3) to the line joining the points (1,-2,3) and (1,1,0) lies on the plane:

(1) $x-y-2z=1$

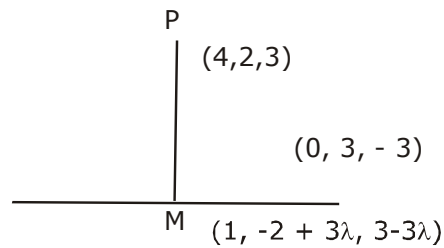
(2) $x-2y+z=1$

(3) $2x+y-z=1$

(4) $x+2y-z=1$

Sol. 3

$$\vec{r} = (1, -2, 3) + \lambda(0, 3, -3)$$



$$\overrightarrow{PM} \perp \vec{b}$$

$$\overrightarrow{PM} \cdot \vec{b} = 0$$

$$(-3, 3\lambda - 4, -3\lambda) \cdot (0, 3, -3) = 0$$

$$\Rightarrow 0 + 9\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{12}{18} = \frac{2}{3}$$

$$m = (1, 0, 1) \text{ are on } 2x + y - z = 1$$

Q.5 If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then

(1) $|y'(0)| + |y''(0)| = 1$

(2) $y''(0) = 0$

(3) $|y'(0)| + |y''(0)| = 3$

(4) $|y''(0)| = 2$

Sol. 4

$$2yy' + 2(-\tan x) = y'$$

diff. w.r.t.x

$$\dots(1)$$

$$2yy'' + 2(y')^2 - 2\sec^2 x = y''$$

$$\dots(2)$$

Put $x = 0$ in given equation we get $y = 0, 1$

from (1) $x = 0, y = 0 \Rightarrow y'(0) = 0$

$x = 0, y = 1, \Rightarrow y'(0) = 0$

from (2) $x = 0, y = 0, y'(0) = 0 \Rightarrow y''(0) = -2$
 $x = 0, y = 1, y'(0) = 0 \Rightarrow y''(0) = 2$
 $|y''(0)| = 2$

Q.6 $2\pi - \left(\sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{25} \right)$ is equal to:

- (1) $\frac{5\pi}{4}$ (2) $\frac{3\pi}{2}$ (3) $\frac{7\pi}{4}$ (4) $\frac{\pi}{2}$

Sol. 2

$$2\pi - \left[\tan^{-1} \left(\frac{4}{3} \right) + \tan^{-1} \left(\frac{5}{12} \right) + \tan^{-1} \frac{16}{63} \right]$$

$$2\pi - \tan^{-1} \left(\frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} \right) - \tan^{-1} \left(\frac{16}{63} \right)$$

$$\Rightarrow 2\pi - \tan^{-1} \left(\frac{48 + 15}{36 - 20} \right) - \tan^{-1} \left(\frac{16}{63} \right)$$

$$\Rightarrow 2\pi - \left[\tan^{-1} \left(\frac{63}{16} \right) + \cot^{-1} \left(\frac{63}{16} \right) \right]$$

$$\Rightarrow 2\pi - \frac{\pi}{2}$$

$$= \frac{3\pi}{2}$$

Q.7 A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?

- (1) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}} \right)$ (2) $\left(1, -\frac{1}{\sqrt{2}} \right)$ (3) $\left(\frac{1}{\sqrt{2}}, 0 \right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1 \right)$

Sol. 1

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$b_1^2 = a_1^2 (1 - e_1^2)$$

$$3 = 4(1 - e_1^2)$$

$$e_1 = \frac{1}{2}$$

$$\text{focus} = (\pm a_1 e_1, 0) \\ = (\pm 1, 0)$$

$$\text{Length of transverse axis } 2a_2 = \sqrt{2} \rightarrow a_2 = \frac{1}{\sqrt{2}}$$

$$a_2 e_2 = 1$$

$$= e_2 = \sqrt{2}$$

$$b_2^2 = a_2^2 (e_2^2 - 1)$$

$$b_2^2 = \frac{1}{2} (2 - 1) = \frac{1}{2}$$

equation of Hyperbola

$$x^2 - y^2 = \frac{1}{2}$$

Q.8 For the frequency distribution:

Variate(x):	x_1	x_2	$x_3 \dots x_{15}$
Frequency(f):	f_1	f_2	$f_3 \dots f_{15}$

where $0 < x_1 < x_2 < x_3 < \dots < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be:

(1) 1

(2) 4

(3) 6

(4) 2

Sol. 3

$$\sigma^2 \leq \frac{1}{4} (M - m)^2$$

(M = upper bound of value of any random variable,
m = Lower bound of value of any random variable)

$$\sigma^2 \leq \frac{1}{4} (10 - 0)^2$$

$$\sigma^2 < 25$$

$$-5 < \sigma < 5$$

$$\sigma \neq 6$$

Q.9 A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is:

(1) $\frac{1}{3}$

(2) $\frac{1}{4}$

(3) $\frac{1}{8}$

(4) $\frac{1}{9}$

Sol. 4

Total Possibilities = (1, 3), (3, 1), (2, 2),
(2, 6), (6, 2) (4, 4)
(3, 5), (5, 3) (6, 6)
fav. = 1 = (4, 4)

$$\text{prob.} = \frac{1}{9}$$

Q.10 If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value

of n is:

(1) 128

(2) 248

(3) 256

(4) 264

Sol. 3

$$T_{r+1} = {}^nC_r \left(3^{\frac{1}{2}}\right)^{n-r} \left(5^{\frac{1}{8}}\right)^r$$

$$\left. \begin{array}{l} \frac{n-r}{2} \rightarrow n-r = 0, 2, 4, 6, 8, \dots \end{array} \right]$$

$$\left. \begin{array}{l} \frac{r}{8} \rightarrow r = 0, 8, 16, 24, \dots \end{array} \right]$$

common r = 0, 8, 16, 24,

no. of integral term = 33.

$$L = 0 + (33 - 1) \times 8 \rightarrow L = 32 \times 8 \\ = 256$$

Q.11 $\int_{-\pi}^{\pi} |\pi - |x|| dx$ is equal to:

(1) π^2

(2) $\frac{\pi^2}{2}$

(3) $\sqrt{2}\pi^2$

(4) $2\pi^2$

Sol. 1

$$\int_{-\pi}^{\pi} |\pi - |x|| dx$$

even function

$$2 \int_0^{\pi} |\pi - x| dx$$

$$= 2 \int_0^{\pi} (\pi - x) dx \Rightarrow 2 \left[\pi x - \frac{x^2}{2} \right]_0^{\pi}$$

$$= 2 \left[\frac{\pi^2}{2} \right] = \pi^2$$

Q.12 Consider the two sets:

$A = \{m \in \mathbb{R} : \text{both the roots of } x^2 - (m+1)x + m+4 = 0 \text{ are real}\}$ and $B = [-3, 5]$.

Which of the following is not true ?

(1) $A - B = (-\infty, -3) \cup (5, \infty)$

(2) $A \cap B = \{-3\}$

(3) $B - A = (-3, 5)$

(4) $A \cup B = \mathbb{R}$

Sol. 1

$$D \geq 0$$

$$(m+1)^2 - 4(m+4) \geq 0$$

$$\Rightarrow m^2 - 2m - 15 \geq 0$$

$$(m-5)(m+3) \geq 0$$

$$m \in (-\infty, -3] \cup [5, \infty)$$

$$A = (-\infty, -3] \cup [5, \infty)$$

$$B = [-3, 5]$$

$$A - B = (-\infty, -3) \cup (5, \infty)$$

$$A \cup B = \mathbb{R}$$

Q.13 The proposition $p \rightarrow \neg(p \wedge \sim q)$ is equivalent to :

(1) $(\sim p) \vee (\sim q)$

(2) $(\sim p) \wedge q$

(3) q

(4) $(\sim p) \vee q$

Sol. 4

$$\sim(p \wedge \sim q) \rightarrow \sim p \vee q$$

$$p \rightarrow (\sim p \vee q)$$

$$\Rightarrow \sim p \vee (\sim p \vee q)$$

$$\Rightarrow \sim p \vee q$$

Q.14 The function, $f(x) = (3x-7)x^{2/3}$, $x \in \mathbb{R}$ is increasing for all x lying in:

(1) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

(2) $\left(-\infty, \frac{14}{15}\right)$

(3) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(4) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

Sol. 3

$$f(x) = (3x - 7) \cdot \frac{2}{3x^{\frac{1}{3}}} + x^{\frac{2}{3}} \cdot 3$$

$$= \frac{6x - 14 + 9x}{3x^{\frac{1}{3}}}$$

$$= \frac{15x - 14}{3x^{\frac{1}{3}}}$$

$$\begin{array}{c} \boxed{+} \quad \boxed{-} \quad \boxed{+} \\ \hline 0 \qquad \qquad 14/15 \end{array}$$

$$f(x) > 0 \uparrow \Rightarrow x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$$

Q.15 If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:

(1) $\frac{1}{6}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{1}{7}$

Sol. 1
a = 3

$$\frac{25}{2} [2a + 24d] = \frac{15}{2} [2 \times (a + 25d) + 14d]$$

$$\Rightarrow 50a + 600d = 15 [2a + 50d + 14d]$$

$$\Rightarrow 20a + 600d = 960d$$

$$\Rightarrow 60 = 360d$$

$$d = \frac{1}{6}$$

Q.16 The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$, which passes through the point (0,1), is:

(1) $y^2 = 1 + y \log_e \left(\frac{1 + e^{-x}}{2} \right)$

(2) $y^2 + 1 = y \left(\log_e \left(\frac{1 + e^{-x}}{2} \right) + 2 \right)$

(3) $y^2 + 1 = y \left(\log_e \left(\frac{1 + e^x}{2} \right) + 2 \right)$

(4) $y^2 = 1 + y \left(\log_e \left(\frac{1 + e^x}{2} \right) \right)$

Sol. 4

$$\int \left(\frac{1 + y^2}{y^2} \right) dy = \int \left(\frac{1}{1 + e^{-x}} \right) dx$$

$$\int \frac{1}{y^2} dy + \int dy = \int \left(\frac{e^x}{e^x + 1} \right) dx$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x + 1| + c$$

$$x = 0, y = 1$$

$$\Rightarrow -1 + 1 = \ln 2 + c \Rightarrow c = -\ln 2$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x + 1| - \ln 2$$

$$\Rightarrow y^2 = 1 + y \left[\ln \left(\frac{e^x + 1}{2} \right) \right]$$

Q.17 The area (in sq. units) of the region $\left\{ (x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2 \right\}$ is

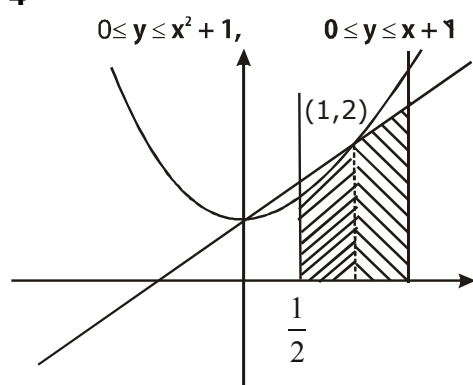
(1) $\frac{23}{16}$

(2) $\frac{79}{16}$

(3) $\frac{23}{6}$

(4) $\frac{79}{24}$

Sol. 4



$$A = \int_{\frac{1}{2}}^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx$$

$$\left(\frac{x^3}{3} + x \right)_{\frac{1}{2}}^1 + \left(\frac{x^2}{2} + x \right)_1^2$$

$$= \left(\frac{1}{3} + 1 \right) - \left(\frac{1}{24} + \frac{1}{2} \right) + \left((2 + 2) - \left(\frac{3}{2} \right) \right)$$

$$= \left(\frac{4}{3} - \frac{13}{24} \right) + \left(\frac{5}{2} \right)$$

$$= \left(\frac{32-13}{24} \right) + \left(\frac{5}{2} \right) = \frac{19+60}{24} = \frac{79}{24}$$

Q.18 If α and β are the roots of the equation $x^2+px+2=0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the

equation $2x^2+2qx+1=0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to :

(1) $\frac{9}{4}(9+p^2)$ (2) $\frac{9}{4}(9+q^2)$ (3) $\frac{9}{4}(9-p^2)$ (4) $\frac{9}{4}(9-q^2)$

Sol. 3

$$\alpha + \beta = -p, \alpha\beta = 2$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = -q, \frac{1}{\alpha\beta} = \frac{1}{2}$$

$$\frac{\alpha + \beta}{\alpha\beta} = -q \Rightarrow \frac{-p}{2} = -q$$

$$\Rightarrow p = 2q$$

$$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2$$

$$= 2 + \frac{1}{2} + 2 = \frac{9}{2}$$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right) = \alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}$$

$$= 2 + \frac{1}{2} - \left[\frac{\alpha^2 + \beta^2}{\alpha\beta} \right]$$

$$= \frac{5}{2} - \left[\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \right]$$

$$= \frac{5}{2} - \left[\frac{p^2 - 4}{2} \right]$$

$$= \frac{9 - p^2}{2}$$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \left(\frac{9 - p^2}{2}\right)\left(\frac{9}{2}\right)$$

$$= \frac{9}{4}(9 - p^2)$$

Q.19 The lines $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

(1) do not intersect for any values of l and m

(2) intersect when $l=1$ and $m=2$

(3) intersect when $l=2$ and $m=\frac{1}{2}$

(4) intersect for all values of l and m

Sol. 1

$$\frac{2}{1} \neq \frac{0}{1} \neq \frac{1}{-1} \rightarrow \text{lines are intersecting}$$

$$\vec{r} = (1 + 2l)\hat{i} - \hat{j} + l\hat{k} \quad \dots(1)$$

$$\vec{r} = (2 + m)\hat{i} + (m - 1)\hat{j} - m\hat{k} \quad \dots(2)$$

compare coeff. of $\hat{i}, \hat{j}, \hat{k}$

$$1 + 2l = 2 + m \quad \left| \begin{array}{l} -1 = m - l \\ m = 0 \end{array} \right| \quad l = 0$$

Lines do not intersect

Q.20 Let $[t]$ denote the greatest integer $\leq t$. if for some $\lambda \in \mathbb{R} - \{0, 1\}$

$$\lim_{x \rightarrow 0} \left| \frac{1 - x + |x|}{\lambda - x + [x]} \right| = L, \text{ then } L \text{ is equal to:}$$

- (1) 0 (2) 2 (3) $\frac{1}{2}$ (4) 1

Sol. 2

$$\lim_{x \rightarrow 0} \left| \frac{1-x+|x|}{\lambda-x+[x]} \right| = L$$

$$\lim_{h \rightarrow 0} \left| \frac{1-h+h}{\lambda-h+[h]} \right|$$

$$\lim_{h \rightarrow 0} \left| \frac{1}{\lambda-h+0} \right| = \left| \frac{1}{\lambda} \right| \quad [x] = 0$$

$$\lim_{h \rightarrow 0} \left| \frac{1+h+h}{\lambda+h+[-h]} \right|$$

$$= \left| \frac{1}{\lambda-1} \right| \quad [-h] = -1$$

$$\therefore |\lambda| = |\lambda-1|$$

$$\lambda^2 = \lambda^2 - 2\lambda + 1 \Rightarrow \lambda = \frac{1}{2}$$

$$L = 2$$

Q.21 If $\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$, then the value of k is

Sol. 8

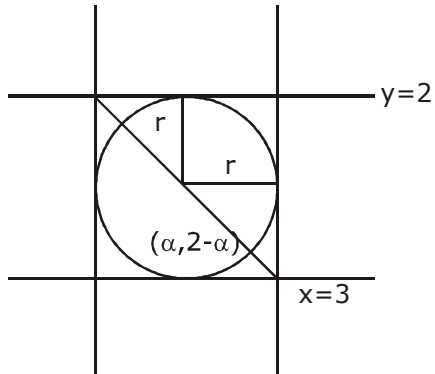
$$\lim_{x \rightarrow 0} \frac{\left(1 - \cos \frac{x^2}{2} \right) \left(1 - \cos \frac{x^2}{4} \right) \left(\frac{x^2}{2} \right) \cdot \left(\frac{x^2}{4} \right)^2}{\left(\frac{x^2}{2} \right)^2 \left(\frac{x^2}{4} \right)^2} \cdot \frac{1}{x^8}$$

$$\lim_{x \rightarrow 0} \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{16} \Rightarrow \frac{1}{256} = 2^{-k}$$

$$2^{-8} = 2^{-k} \Rightarrow k = 8$$

Q.22 The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x=3$ and $y=2$, is

Sol. 2



$$p = r \\ \text{for } y = 2$$

$$r = \left| \frac{2 - \alpha - 2}{1} \right| = |\alpha|$$

$$\text{for } x = 3$$

$$r = \left| \frac{\alpha - 3}{1} \right| = |\alpha - 3|$$

$$|\alpha| = |\alpha - 3|$$

$$\Rightarrow \alpha^2 + \alpha^2 - 6\alpha + 9 \Rightarrow \alpha = \frac{3}{2}$$

$$2\alpha = 3 = 2r$$

Q.23 The value of $(0.16)^{\log_{2.5} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty \right)}$ is equal to.....

Sol. 4

$$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$$

$$\log_{2.5} \left(\frac{1}{2} \right) \Rightarrow \log_{\frac{5}{2}} \frac{1}{2}$$

$$.16 = \frac{16}{100} = \frac{4}{25} = \left(\frac{2}{5} \right)^2$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2\log_{\frac{5}{2}} \frac{1}{2}} = \left(\frac{5}{2}\right)^{-2\log_{\frac{5}{2}} \frac{1}{2}}$$

$$\Rightarrow \left(\frac{5}{2}\right)^{\log_{\frac{5}{2}} \left(\frac{1}{2}\right)^{-2}}$$

$$= 4$$

Q.24 Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbb{R}$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to

Sol. 10

$$A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x^3 + x + x & x^2 + 1 \\ x^2 + 1 & x \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^3 + 2x & x^2 + 1 \\ x^2 + 1 & x \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x^4 + 2x^2 + x^2 + 1 & x^3 + 2x \\ x^3 + x + x & x^2 + 1 \end{bmatrix}$$

$$a_{11} \Rightarrow x^4 + 3x^2 + 1 = 109$$

$$x^4 + 3x^2 - 108 = 0$$

$$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$$

$$x = \pm 3$$

$$a_{11} = x^2 + 1 = 10$$

Q.25 If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$, ($m, n \in \mathbb{N}$) then the greatest common divisor of the least values of m and n is

Sol. 4

$$\left[\frac{(1+i)(1+i)}{(1+i)(1-i)}\right]^{\frac{m}{2}} = \left[\left(\frac{1+i}{-1+i}\right)\left(\frac{-1-i}{-1-i}\right)\right]^{\frac{n}{3}} = 1$$

$$= \left(\frac{2i}{2}\right)^{\frac{m}{2}} = 1 \quad \left| \quad \left(\frac{-1-i-i+1}{1+1}\right)^{\frac{n}{3}} = 1\right.$$

$$m = 8$$

$$\begin{aligned} (-i)^{n/3} &= 1 \\ n &= 12 \end{aligned}$$

greatest common divisor of m & n is 4