

JEE Main 2020 Paper

Date: 7th January 2020 (Shift 2)

Time: 2:30 P.M. to 5:30 P.M.

Subject: Mathematics

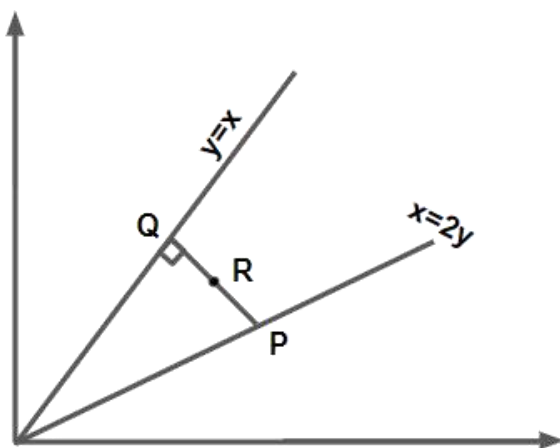
1. From any point P on the line $x = 2y$, a perpendicular is drawn on $y = x$. Let the foot of perpendicular be Q . Find the locus of mid point of PQ .

- a. $5x = 7y$
c. $7x = 5y$

- b. $2x = 3y$
d. $3x = 2y$

Answer: (a)

Solution:



Let R be the midpoint of PQ

PQ is perpendicular on line $y = x$

$\therefore$ Equation of the line PQ can be written as $y = -x + c$

$y = -x + c$ intersects $y = x$ at $Q: \left(\frac{c}{2}, \frac{c}{2}\right)$

$y = -x + c$ intersects $x = 2y$ at $P: \left(\frac{2c}{3}, \frac{c}{3}\right)$

$\therefore$ Midpoint $R: \left(\frac{7c}{12}, \frac{5c}{12}\right)$

Locus of $R: x = \frac{7c}{12}$

$$y = \frac{5c}{12}$$

$$\Rightarrow 5x = 7y$$

2. Let θ_1 and θ_2 (where $\theta_1 < \theta_2$) are two solutions of $2 \cot^2 \theta - \frac{5}{\sin \theta} + 4 = 0, \theta \in [0, 2\pi)$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to

a. $\frac{\pi}{9}$

b. $\frac{2\pi}{3}$

c. $\frac{\pi}{3} + \frac{1}{6}$

d. $\frac{\pi}{3}$

Answer: (d)

Solution:

$$2 \cot^2 \theta - \frac{5}{\sin \theta} + 4 = 0, \theta \in [0, 2\pi)$$

$$\Rightarrow 2 \operatorname{cosec}^2 \theta - 2 - 5 \operatorname{cosec} \theta + 4 = 0$$

$$\Rightarrow 2 \operatorname{cosec}^2 \theta - 4 \operatorname{cosec} \theta - \operatorname{cosec} \theta + 2 = 0$$

$$\Rightarrow \operatorname{cosec} \theta = 2 \text{ or } \frac{1}{2} \text{ (Not possible)}$$

As $\theta \in [0, 2\pi)$,

$$\theta_1 = \frac{\pi}{6}, \theta_2 = \frac{5\pi}{6}$$

$$\Rightarrow \int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{(1 + \cos 6\theta)}{2} \, d\theta$$

$$= \frac{1}{2} \left(\frac{5\pi}{6} - \frac{\pi}{6} \right) + \frac{\sin 6\theta}{12} \Bigg|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{\pi}{3}$$

3. Coefficient of x^7 in $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \dots + x^{10}$ is

- a. 260
b. 210
c. 420
d. 330

Answer: (d)

Solution:

Coefficient of x^7 in $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \dots + x^{10}$

$$\text{Applying sum of terms of G.P.} = \frac{(1+x)^{10} \left(1 - \left(\frac{x}{1+x} \right)^{11} \right)}{\left(1 - \frac{x}{1+x} \right)} = (1+x)^{11} - x^{11}$$

$$\text{Coefficient of } x^7 \Rightarrow {}^{11}C_7 = 330$$

4. Let α and β are the roots of $x^2 - x - 1 = 0$ such that $P_k = \alpha^k + \beta^k, k \geq 1$ then which one is incorrect?

- a. $P_5 = P_2 \times P_3$
b. $P_1 + P_2 + P_3 + P_4 + P_5 = 26$
c. $P_5 = 11$
d. $P_3 = P_5 - P_4$

Answer: (a)

Solution:

Given α, β are the roots of $x^2 - x - 1 = 0$

$$\Rightarrow \alpha + \beta = 1 \text{ \& } \alpha\beta = -1$$

$$\Rightarrow \alpha^2 = \alpha + 1 \text{ \& } \beta^2 = \beta + 1$$

$$P_k = \alpha^{k-2}\alpha^2 + \beta^{k-2}\beta^2$$

$$P_k = \alpha^{k-2}(\alpha + 1) + \beta^{k-2}(\beta + 1)$$

$$P_k = \alpha^{k-1} + \beta^{k-1} + \alpha^{k-2} + \beta^{k-2}$$

$$\Rightarrow P_k = P_{k-1} + P_{k-2}$$

$$\Rightarrow P_3 = P_2 + P_1 = 4$$

$$P_4 = P_3 + P_2 = 7$$

$$P_5 = P_4 + P_3 = 11$$

$$\therefore P_5 \neq P_2 P_3 \text{ \& } P_1 + P_2 + P_3 + P_4 + P_5 = 26$$

$$\text{\& } P_3 = P_5 - P_4$$

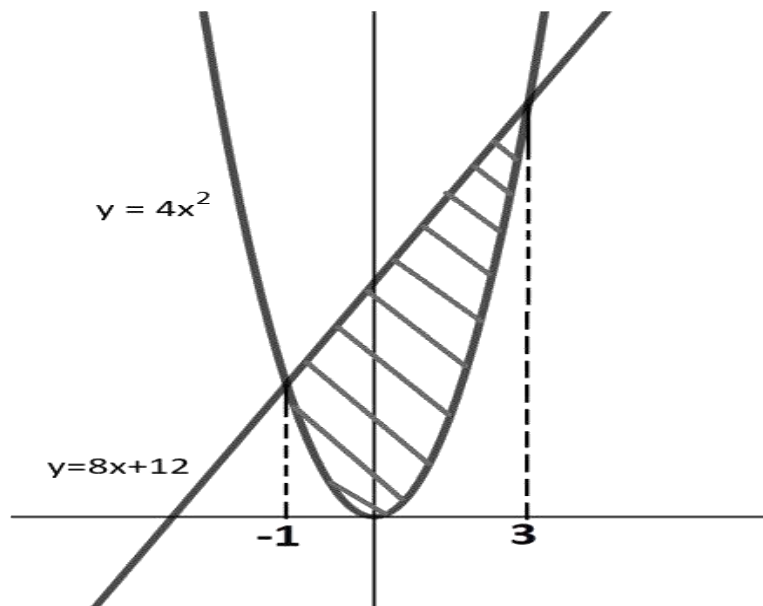
5. The area bounded by $4x^2 \leq y \leq 8x + 12$ is

a. $\frac{127}{3}$
c. $\frac{128}{3}$

b. $\frac{125}{3}$
d. $\frac{124}{3}$

Answer: (c)

Solution:



For point of intersection,

$$4x^2 = 8x + 12$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow x = 3, -1$$

Area bounded is given by

$$A = \int_{-1}^3 (8x + 12 - 4x^2) dx$$

$$A = \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3} \right]_{-1}^3$$

$$A = (36 + 36 - 36) - \left(4 - 12 + \frac{4}{3} \right)$$

$$A = 44 - \frac{4}{3} = \frac{128}{3}$$

6. Contrapositive of $A \subset B$ and $B \subset C$ then $C \subset D$

- a. $C \not\subset D$ or $A \not\subset B$ or $B \not\subset C$
 c. $C \subset D$ and $A \not\subset B$ or $B \not\subset C$

- b. $C \subset D$ or $A \not\subset B$ and $B \not\subset C$
 d. $C \subset D$ or $A \not\subset B$ or $B \not\subset C$

Answer: (d)

Solution:

Given statements: $A \subset B$ and $B \subset C$

Let $A \subset B$ be p

$B \subset C$ be q

$C \subset D$ be r

Modified statement: $(p \wedge q) \Rightarrow r$

Contrapositive: $\sim r \Rightarrow \sim (p \wedge q)$

$\therefore r \vee (\sim p \vee \sim q)$

$\Rightarrow C \subset D$ or $A \not\subset B$ or $B \not\subset C$

7. Let $3 + 4 + 8 + 9 + 13 + 14 + 18 + \dots \dots \dots 40$ terms $= S$. If $S = (102)m$ then $m =$

- a. 5
 c. 25
 b. 10
 d. 20

Answer: (d)

Solution:

$S = \underline{3+4} + \underline{8+9} + 13 + 14 + \dots \dots 40$ terms

$S = 7 + 17 + 27 + 37 + \dots \dots \dots 20$ terms

$$S = \frac{20}{2}[14 + (19)10] = 20 \times 102$$

$$\therefore m = 20$$

8. $({}^{36}C_{r+1}) \times (k^2 - 3) = {}^{35}C_r \times 6$, then the number of ordered pairs (r, k) , where $k \in \mathbf{I}$, are

- | | |
|------|------|
| a. 2 | b. 6 |
| c. 3 | d. 4 |

Answer: (d)

Solution:

$$\text{using } {}^{36}C_{r+1} = \frac{36}{r+1} \times {}^{35}C_r$$

$$\frac{36}{r+1} \times {}^{35}C_r \times (k^2 - 3) = {}^{35}C_r \times 6$$

$$k^2 - 3 = \frac{r+1}{6}$$

$$k^2 = \frac{r+1}{6} + 3$$

$$k \in \mathbf{I}$$

$$r \rightarrow \text{Non-negative integer } 0 \leq r \leq 35$$

$$r = 5 \Rightarrow k = \pm 2$$

$$r = 35 \Rightarrow k = \pm 3$$

$$\text{No. of ordered pairs } (r, k) = 4$$

9. Let $f(x)$ be a five-degree polynomial which has critical points $x = \pm 1$ and $\lim_{x \rightarrow 0} \left(2 + \frac{f(x)}{x^3} \right) = 4$ then which one is incorrect.

- $f(x)$ has minima at $x = 1$ and maxima at $x = -1$
- $f(1) - 4f(-1) = 4$
- $f(x)$ has maxima at $x = 1$ and minima at $x = -1$
- $f(x)$ is odd

Answer: (a)

Solution:

Given $\lim_{x \rightarrow 0} \left(2 + \frac{f(x)}{x^3} \right) = 4$

$$\lim_{x \rightarrow 0} \frac{f(x)}{x^3} = 2$$

$\lim_{x \rightarrow 0} \frac{f(x)}{x^3}$ Limit exists and it is finite

$$\therefore f(x) = ax^5 + bx^4 + cx^3$$

$$\Rightarrow \lim_{x \rightarrow 0} (ax^2 + bx + c) = 2$$

$$c = 2$$

Also $f'(x) = 5ax^4 + 4bx^3 + 6x^2$

$$f'(1) = 5a + 4b + 6 = 0$$

$$f'(-1) = 5a - 4b + 6 = 0$$

$$b = 0, \quad a = -\frac{6}{5}$$

$$f(x) = -\frac{6}{5}x^5 + 2x^3 \Rightarrow f(x) \text{ is odd}$$

$$f'(x) = -6x^4 + 6x^2$$

$$f''(x) = -24x^3 + 12x \quad (f''(1) < 0, \quad f''(-1) > 0)$$

At $x = -1$ local minima at $x = 1$ local maxima

$$\text{And } f(1) - 4f(-1) = 4$$

10. If LMVT is applicable on $f(x) = x^3 - 4x^2 + 8x + 11$ in $[0,1]$, the value of c is

a. $\frac{4+\sqrt{5}}{3}$

b. $\frac{4+\sqrt{7}}{3}$

c. $\frac{4-\sqrt{7}}{3}$

d. $\frac{4-\sqrt{5}}{3}$

Answer: (c)

Solution:

LMVT is applicable on $f(x)$ in $[0,1]$, therefore it is continuous and differentiable in $[0,1]$

Now, $f(0) = 11, f(1) = 16$

$$f'(x) = 3x^2 - 8x + 8$$

$$\therefore f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1}$$

$$\Rightarrow 3c^2 - 8c + 8 = 5$$

$$\Rightarrow 3c^2 - 8c + 3 = 0$$

$$\Rightarrow c = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

As $c \in (0,1)$

$$\text{We get, } c = \frac{4-\sqrt{7}}{3}$$

11. Consider there are 5 machines. Probability of a machine being faulty is $\frac{1}{4}$. Probability of at most two machines being faulty is $\left(\frac{3}{4}\right)^3 k$, then the value of k is

a. $\frac{17}{4}$

b. $\frac{17}{8}$

c. $\frac{17}{2}$

d. 4

Answer: (b)

Solution:

$$P(\text{machine being faulty}) = p = \frac{1}{4}$$

$$\therefore q = \frac{3}{4}$$

$$P(\text{at most two machines being faulty}) = P(\text{zero machine being faulty})$$

$$+ P(\text{one machine being faulty}) + P(\text{two machines being faulty})$$

$$= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4 + {}^5C_2 p^2 q^3$$

$$= q^5 + 5pq^4 + 10p^2q^3$$

$$= \left(\frac{3}{4}\right)^5 + 5 \times \frac{1}{4} \left(\frac{3}{4}\right)^4 + 10 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3$$

$$\therefore k = \frac{17}{8}$$

- $$\lambda = -171$$

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$$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$$

Differentiating w.r.t. x on both the sides, we get:

$$y'\sqrt{1-x^2} + y \times \frac{1}{2\sqrt{1-x^2}} \times (-2x) = -\sqrt{1-y^2} - x \times \frac{1}{2\sqrt{1-y^2}} \times (-2y)y'$$

$$\Rightarrow y'\sqrt{1-x^2} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - \frac{xy}{\sqrt{1-y^2}}y' = 0$$

$$\Rightarrow y' \left[\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}} \right] = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$$

$$\text{Putting } x = \frac{1}{2}, y = -\frac{1}{4}$$

$$\Rightarrow y' \left[\frac{\sqrt{3}}{2} + \frac{\frac{1}{8}}{\frac{\sqrt{15}}{4}} \right] = -\frac{\frac{1}{8}}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{15}}{4}$$

$$\Rightarrow y' \left[\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{15}} \right] = -\frac{1}{4\sqrt{3}} - \frac{\sqrt{15}}{4}$$

$$\Rightarrow y' \left[\frac{\sqrt{45}+1}{2\sqrt{15}} \right] = -\frac{1+\sqrt{45}}{4\sqrt{3}}$$

$$\Rightarrow y' = -\frac{\sqrt{5}}{2}$$

14. Let $A = [a_{ij}]$, $B = [b_{ij}]$ are two 3×3 matrices such that $b_{ij} = \lambda^{i+j-2}a_{ij}$ and $|B| = 81$. Find $|A|$ if $\lambda = 3$

a. $\frac{1}{81}$

b. $\frac{1}{27}$

c. $\frac{1}{9}$

d. 3

Answer: (c)

Solution:

$$b_{ij} = \lambda^{i+j-2}a_{ij}, \lambda = 3$$

$$B = \begin{bmatrix} 3^0 a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3^0 a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}$$

Taking 3^2 Common each from C_3 & R_3

$$|B| = 81 \begin{vmatrix} a_{11} & 3a_{12} & a_{13} \\ 3a_{21} & 3^2 a_{22} & 3a_{23} \\ a_{31} & 3a_{32} & a_{33} \end{vmatrix}$$

Taking 3 common each from C_2 & R_2

$$|B| = 81(9) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\text{Given } |B| = 81$$

$$\Rightarrow 81 = 81(9)|A|$$

$$\Rightarrow |A| = \frac{1}{9}$$

15. Pair of tangents are drawn from the origin to the circle $x^2 + y^2 - 8x - 4y + 16 = 0$, then the square of length of chord of contact is

a. $\frac{8}{5}$

b. $\frac{8}{13}$

c. $\frac{24}{5}$

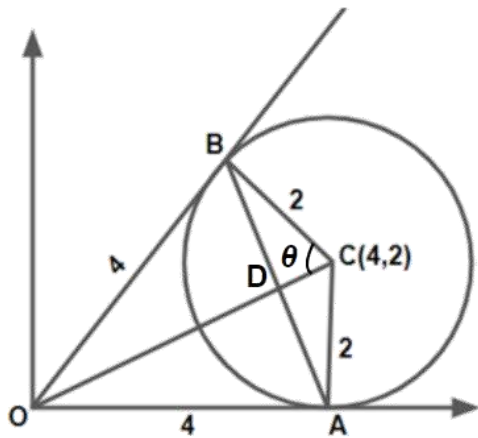
d. $\frac{64}{5}$

Answer: (d)

Solution:

$$x^2 + y^2 - 8x - 4y + 16 = 0$$

$$(x - 4)^2 + (y - 2)^2 = 4 \Rightarrow \text{Centre } (4, 2), \text{ radius } (2)$$



$$OA = 4 = OB$$

In $\triangle OBC$

$$\tan \theta = \frac{4}{2} = 2 \Rightarrow \sin \theta = \frac{2}{\sqrt{5}}$$

In $\triangle BDC$

$$\sin \theta = \frac{BD}{2} \Rightarrow BD = \frac{4}{\sqrt{5}}$$

$$\text{Length of chord of contact } (AB) = \frac{8}{\sqrt{5}}$$

Alternative

(l) length of tangent = 4

(r) radius = 2

$$\Rightarrow \text{Length of chord of contact} = \frac{2lr}{\sqrt{l^2 + r^2}}$$

$$\text{Square of length of chord of contact} = \frac{64}{5}$$

16. Let $y(x)$ is the solution of differential equation $(y^2 - x) \frac{dy}{dx} = 1$ and $y(0) = 1$, then find the value of x where the curve cuts the x -axis.

- | | |
|------------|--------|
| a. $2 - e$ | b. 2 |
| c. $2 + e$ | d. e |

Answer: (a)

Solution:

$$(y^2 - x) \frac{dy}{dx} = 1$$

$$\frac{dx}{dy} + x = y^2$$

$$xe^y = \int y^2 e^y dy$$

$$x = y^2 - 2y + 2 + ce^{-y}$$

$$\text{Given } y(0) = 1$$

$$\Rightarrow c = -e$$

$$\therefore \text{Solution is } x = y^2 - 2y + 2 - e^{-y+1}$$

$$\therefore \text{The value of } x \text{ where the curve cuts the } x\text{-axis will be at } x = 2 - e$$

17. Let $4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$ then $\alpha =$

a. $\ln \sqrt{2}$

b. $\ln \frac{3}{4}$

c. $\ln 2$

d. $\ln \frac{4}{3}$

Answer: (c)

Solution:

$$4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$$

$$4\alpha \left[\int_{-1}^0 e^{-\alpha|x|} dx + \int_0^2 e^{-\alpha|x|} dx \right] = 5$$

$$= 4\alpha \left[\int_{-1}^0 e^{\alpha x} dx + \int_0^2 e^{-\alpha x} dx \right] = 5$$

$$= 4\alpha \left[\left(\frac{1 - e^{-\alpha}}{\alpha} \right) + \left(\frac{e^{-2\alpha} - 1}{-\alpha} \right) \right] = 5$$

$$= 4[1 - e^{-2\alpha} - e^{-\alpha} + 1] = 5$$

$$\text{Let } e^{-\alpha} = t$$

$$\Rightarrow -4t^2 - 4t + 3 = 0$$

$$\Rightarrow t = \frac{1}{2} = e^{-\alpha} \Rightarrow \alpha = \ln 2$$

18. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ then $(\lambda, d) =$

a. $\left(\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

b. $\left(-\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

c. $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

d. $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

Answer: (c)

Solution:

$$\text{Given } \vec{a} + \vec{b} + \vec{c} = \vec{0} \quad \lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{0}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\lambda = -\frac{3}{2}$$

$$\text{Also } \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$$

$$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a}$$

$$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$$

$$\Rightarrow \vec{d} = 3(\vec{a} \times \vec{b}) = 3\vec{a} \times \vec{b}$$

- a. $2\sqrt{5}$
c. 4

Solution:

Equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ is $y = mx + \sqrt{a^2m^2 + 9}$

$$\Rightarrow m = -\frac{3}{4} \text{ and } \sqrt{a^2 m^2 + 9} = 3\sqrt{2}$$

$$\Rightarrow a^2 \left(-\frac{3}{4}\right)^2 + 9 = 18$$

$$\Rightarrow a^2 \times \frac{9}{16} = 9$$

$$\Rightarrow a^2 = 16 \Rightarrow a = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

Distance between foci is $2ae = 2 \times 4 \times \frac{\sqrt{7}}{4} = 2\sqrt{7}$

- Answer: (124)**

$$\text{Mean} = 10 \Rightarrow \frac{61+x+y}{8} = 10$$

$$\Rightarrow x + y = 19$$

$$\text{Variance} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$\Rightarrow 25 = \frac{2^2 + 3^2 + 16^2 + 20^2 + 13^2 + 7^2 + x^2 + y^2}{8} - 100$$

$$\Rightarrow 1000 = 887 + x^2 + y^2$$

$$\Rightarrow x^2 + y^2 = 113$$

$$\Rightarrow (x + y)^2 - 2xy = 113$$

$$\Rightarrow 361 - 2xy = 113$$

$$\text{So, } xy = 124$$

21. If $Q \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ is foot of perpendicular drawn from $P(1, 0, 3)$ onto a line L and line L is passing through $(\alpha, 7, 1)$, then value of α is _____.

Answer: (4)

Solution:

$$\text{Direction ratios of line } L: \left(\alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3}\right)$$

$$= \left(\frac{3\alpha - 5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$$

$$\text{Direction ratios of } PQ: \left(-\frac{2}{3}, -\frac{7}{3}, -\frac{8}{3}\right)$$

As line L is perpendicular to PQ

$$\text{So, } \left(\frac{3\alpha - 5}{3}\right)\left(-\frac{2}{3}\right) + \left(\frac{14}{3}\right)\left(-\frac{7}{3}\right) + \left(-\frac{14}{3}\right)\left(-\frac{8}{3}\right) = 0$$

$$\Rightarrow -6\alpha + 10 - 98 + 112 = 0$$

$$\Rightarrow 6\alpha = 24 \Rightarrow \alpha = 4$$

22. If system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $3x + 2y + \lambda z = \mu$ has more than 2 solutions, then $(\mu - \lambda^2)$ is _____.

Answer: (13)

Solution:

The system of equations has more than 2 solutions

$$\therefore D = D_3 = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow 2\lambda - 6 - \lambda + 9 + 2 - 6 = 0$$

$$\Rightarrow \lambda = 1$$

$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow 2\mu - 20 - \mu + 30 - 24 = 0$$

$$\Rightarrow \mu = 14$$

$$\text{So, } \mu - \lambda^2 = 13$$

23. If $f(x)$ is defined in $x \in \left(-\frac{1}{3}, \frac{1}{3}\right)$ &

$$f(x) = \begin{cases} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) & x \neq 0 \\ k & x = 0 \end{cases}$$

The value of k such that $f(x)$ is continuous is _____.

Answer: (5)

Solution:

As $f(x)$ is continuous

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = f(0) = k$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) = k$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{3 \log(1+3x)}{3x} - \lim_{x \rightarrow 0} \frac{(-2) \log(1-2x)}{(-2x)} = k$$

$$\Rightarrow 3 + 2 = k \Rightarrow k = 5$$

24. Let $X = \{x: 1 \leq x \leq 50, x \in \mathbf{N}\}$, $A = \{x: x \text{ is a multiple of } 2\}$, $B = \{x: x \text{ is a multiple of } 7\}$. Then the number of elements in the smallest subset of X which contain elements of both A and B is _____.

Answer: (29)

Solution:

$$A = \{x: x \text{ is multiple of } 2\} = \{2, 4, 6, 8, \dots\}$$

$$B = \{x: x \text{ is multiple of } 7\} = \{7, 14, 21, \dots\}$$

$$X = \{x: 1 \leq x \leq 50, x \in \mathbf{N}\}$$

Smallest subset of X which contains elements of both A and B is a set with multiples of 2 or 7 less than 50.

$$P = \{x: x \text{ is a multiple of } 2 \text{ less than or equal to } 50\}$$

$$Q = \{x: x \text{ is a multiple of } 7 \text{ less than or equal to } 50\}$$

$$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$$

$$= 25 + 7 - 3$$

$$= 29$$