

JEE Main 2020 Paper

Date: 8th January 2020 (Shift 1)

Time: 9:30 A.M. to 12:30 P.M.

Subject: Mathematics

1. The maximum values of ${}^{19}C_p$, ${}^{20}C_q$, ${}^{21}C_r$ are a, b, c respectively. Then, the relation between a, b, c is

a. $\frac{a}{22} = \frac{b}{42} = \frac{c}{11}$
c. $\frac{a}{22} = \frac{b}{11} = \frac{c}{42}$

b. $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$
d. $\frac{a}{21} = \frac{b}{11} = \frac{c}{22}$

Answer: (b)

Solution:

We know that, nC_r is maximum when $r = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{n+1}{2} \text{ or } \frac{n-1}{2}, & n \text{ is odd} \end{cases}$

$$\text{Therefore, } \max({}^{19}C_p) = {}^{19}C_9 = a$$

$$\max({}^{20}C_q) = {}^{20}C_{10} = b$$

$$\max({}^{21}C_r) = {}^{21}C_{11} = c$$

$$\therefore \frac{a}{{}^{19}C_9} = \frac{b}{{}^{20}C_{10}} = \frac{c}{{}^{21}C_{11}}$$

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{11}$$

$$\Rightarrow \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

2. Let $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{6}$ where A and B are independent events, then

a. $P\left(\frac{A}{B}\right) = \frac{2}{3}$

b. $P\left(\frac{A}{B'}\right) = \frac{5}{6}$

c. $P\left(\frac{A}{B'}\right) = \frac{1}{3}$

d. $P\left(\frac{A}{B}\right) = \frac{1}{6}$

Answer: (c)

Solution:

If X and Y are independent events, then

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X)P(Y)}{P(Y)} = P(X)$$

Therefore, $P\left(\frac{A}{B}\right) = P(A) = \frac{1}{3} \Rightarrow P\left(\frac{A}{B'}\right) = P(A) = \frac{1}{3}$.

3. If $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, then inverse of $f(x)$ is

a. $\frac{1}{2} \log_8 \left(\frac{1+x}{1-x} \right)$

b. $\frac{1}{2} \log_8 \left(\frac{1-x}{1+x} \right)$

c. $\frac{1}{4} \log_8 \left(\frac{1-x}{1+x} \right)$

d. $\frac{1}{4} \log_8 \left(\frac{1+x}{1-x} \right)$

Answer: (d)

Solution:

$$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}} = \frac{8^{4x} - 1}{8^{4x} + 1}$$

Put $y = \frac{8^{4x} - 1}{8^{4x} + 1}$

Applying componendo-dividendo on both sides

$$\frac{y+1}{y-1} = \frac{2 \times 8^{4x}}{-2}$$

$$\frac{y+1}{y-1} = -8^{4x} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$$

$$\Rightarrow x = \frac{1}{4} \log_8 \left(\frac{1+y}{1-y} \right)$$

$$f^{-1}(x) = \frac{1}{4} \log_8 \left(\frac{1+x}{1-x} \right).$$

4. Roots of the equation $x^2 + bx + 45 = 0$, $b \in \mathbf{R}$ lies on the curve $|z + 1| = 2\sqrt{10}$, where z is a complex number, then

a. $b^2 + b = 12$

b. $b^2 - b = 36$

c. $b^2 - b = 30$

d. $b^2 + b = 30$

Answer: (c)

Solution:

Given $x^2 + bx + 45 = 0$, $b \in \mathbf{R}$, let roots of the equation be $p \pm iq$

Then, sum of roots $= 2p = -b$

Product of roots $= p^2 + q^2 = 45$

As $p \pm iq$ lies on $|z + 1| = 2\sqrt{10}$, we get

$$(p+1)^2 + q^2 = 40$$

$$\Rightarrow p^2 + q^2 + 2p + 1 = 40$$

$$\Rightarrow 45 - b + 1 = 40$$

$$\Rightarrow b = 6$$

$$\Rightarrow b^2 - b = 30.$$

5. Rolle's theorem is applicable on $f(x) = \ln\left(\frac{x^2 + \alpha}{7x}\right)$ in $[3, 4]$. The value of $f''(c)$ is equal to

a. $\frac{1}{12}$
c. $\frac{-1}{6}$

b. $\frac{-1}{12}$
d. $\frac{1}{6}$

Answer: (a)

Solution:

Rolle's theorem is applicable on $f(x)$ in $[3, 4]$

$$\Rightarrow f(3) = f(4)$$

$$\Rightarrow \ln\left(\frac{9 + \alpha}{21}\right) = \ln\left(\frac{16 + \alpha}{28}\right)$$

$$\Rightarrow \frac{9 + \alpha}{21} = \frac{16 + \alpha}{28}$$

$$\Rightarrow 36 + 4\alpha = 48 + 3\alpha$$

$$\Rightarrow \alpha = 12$$

Now,

$$f(x) = \ln\left(\frac{x^2 + 12}{7x}\right) \Rightarrow f'(x) = \frac{7x}{x^2 + 12} \times \frac{7x \times 2x - (x^2 + 12) \times 7}{(7x)^2}$$

$$f'(x) = \frac{x^2 - 12}{x(x^2 + 12)}$$

$$f'(c) = 0 \Rightarrow c = 2\sqrt{3}$$

$$f''(x) = \frac{-x^4 + 48x^2 + 144}{x^2(x^2 + 12)^2}$$

$$f''(c) = \frac{1}{12}.$$

6. Let $f(x) = x \cos^{-1}(\sin(-|x|))$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then

a. $f'(0) = -\frac{\pi}{2}$

b. $f'(x)$ is not defined at $x = 0$

c. $f'(x)$ is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and $f'(x)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$

d. $f'(x)$ is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and $f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$

Answer: (c)

Solution:

$$f(x) = x \cos^{-1}(\sin(-|x|))$$

$$\Rightarrow f(x) = x \cos^{-1}(-\sin |x|)$$

$$\Rightarrow f(x) = x[\pi - \cos^{-1}(\sin |x|)]$$

$$\Rightarrow f(x) = x \left[\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin |x|) \right) \right]$$

$$\Rightarrow f(x) = x \left(\frac{\pi}{2} + |x| \right)$$

$$\Rightarrow f(x) = \begin{cases} x \left(\frac{\pi}{2} + x \right), & x \geq 0 \\ x \left(\frac{\pi}{2} - x \right), & x < 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \left(\frac{\pi}{2} + 2x \right), & x \geq 0 \\ \left(\frac{\pi}{2} - 2x \right), & x < 0 \end{cases}$$

Therefore, $f'(x)$ is decreasing $\left(-\frac{\pi}{2}, 0 \right)$ and increasing in $\left(0, \frac{\pi}{2} \right)$.

7. Ellipse $2x^2 + y^2 = 1$ and $y = mx$ meet at a point P in the first quadrant. Normal to the ellipse at P meets x -axis at $\left(-\frac{1}{3\sqrt{2}}, 0 \right)$ and y -axis at $(0, \beta)$, then $|\beta|$ is

a. $\frac{2}{3}$

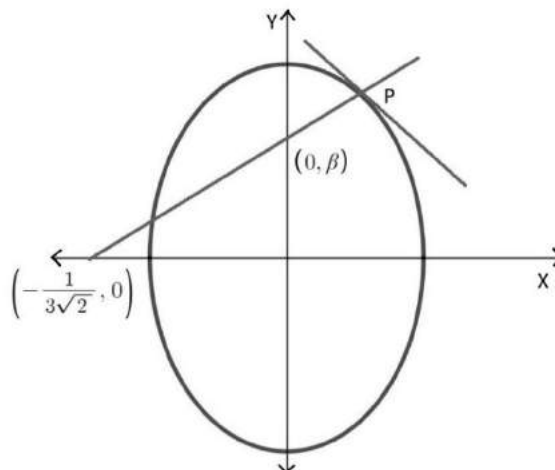
b. $\frac{2\sqrt{2}}{3}$

c. $\frac{\sqrt{2}}{3}$

d. $\frac{2}{\sqrt{3}}$

Answer: (c)

Solution:



Let $P \equiv (x_1, y_1)$

$2x^2 + y^2 = 1$ is given equation of ellipse.

$$\Rightarrow 4x + 2yy' = 0$$

$$\Rightarrow y'|_{(x_1, y_1)} = -\frac{2x_1}{y_1}$$

Therefore, slope of normal at $P(x_1, y_1)$ is $\frac{y_1}{2x_1}$

Equation of normal at $P(x_1, y_1)$ is

$$(y - y_1) = \frac{y_1}{2x_1}(x - x_1)$$

It passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right)$

$$\Rightarrow -y_1 = \frac{y_1}{2x_1}\left(-\frac{1}{3\sqrt{2}} - x_1\right)$$

$$\Rightarrow x_1 = \frac{1}{3\sqrt{2}}$$

$$\Rightarrow y_1 = \frac{2\sqrt{2}}{3} \text{ as } P \text{ lies in first quadrant}$$

Since $(0, \beta)$ lies on the normal of the ellipse at point P , hence we get

$$\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$$

8. If ABC is a triangle whose vertices are $A(1, -1)$, $B(0, 2)$, $C(x', y')$ and area of ΔABC is 5, and $C(x', y')$ lies on $3x + y - 4\lambda = 0$, then

a. $\lambda = 3$

b. $\lambda = 4$

c. $\lambda = -3$

d. $\lambda = 2$

Answer: (a)

Solution:

Area of triangle is

$$A = \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix} = \pm 5$$

$$-2(1 - x') + (y' + x') = \pm 10$$

$$-2 + 2x' + y' + x' = \pm 10$$

$$3x' + y' = 12 \text{ or } 3x' + y' = -8$$

$$\Rightarrow \lambda = 3 \text{ or } -2$$

- d. $4\sqrt{30}$

Solution:

$$\text{Shortest distance} = \frac{|\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} = \frac{|36+225+9|}{\sqrt{36+225+9}} = 3\sqrt{30}.$$

- d. 4

Solution:

$$\Rightarrow \lambda f\left(\frac{\pi}{3}\right) = -2.$$

11. If $y(x)$ is a solution of the differential equation $\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0$, such that

$$y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}, \text{ then}$$

a. $y\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$

b. $y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$

c. $y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

d. $y\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2}$

Answer: (c)

Solution:

$$\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow \sin^{-1} y + \sin^{-1} x = c$$

If $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$ then,

$$\sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{2} = c$$

$$\therefore \frac{\pi}{3} + \frac{\pi}{6} = c \Rightarrow c = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} y = \frac{\pi}{2} - \sin^{-1} x = \cos^{-1} x$$

$$\therefore \sin^{-1} y = \cos^{-1} \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin^{-1} y = \frac{\pi}{4}$$

$$\Rightarrow y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$$

12. $\lim_{x \rightarrow 0} \left(\frac{3x^2+2}{7x^2+2} \right)^{\frac{1}{x^2}}$ is equal to

a. e^{-2}

b. e^2

c. $e^{\frac{3}{7}}$

d. $e^{\frac{2}{7}}$

Answer: (a)

Solution:

$$\text{Let } L = \lim_{x \rightarrow 0} \left(\frac{3x^2+2}{7x^2+2} \right)^{\frac{1}{x^2}} \quad [\text{Intermediate form } 1^\infty]$$

$$\begin{aligned}\therefore L &= e^{\lim_{x \rightarrow 0} \frac{1}{x^2} \left(\frac{3x^2+2}{7x^2+2} - 1 \right)} \\ &= e^{\lim_{x \rightarrow 0} \frac{1}{x^2} \left(-\frac{4x^2}{7x^2+2} \right)} \\ &= e^{-2}.\end{aligned}$$

13. In a bag there are 5 red balls, 3 white balls and 4 black balls. Four balls are drawn from the bag. Find the number of ways in which at most 3 red balls are selected.
- a. 360 b. 450
c. 490 d. 510

Answer: (c)

Solution:

Number of ways to select at most 3 red balls = $P(0 \text{ red balls}) + P(1 \text{ red ball}) + P(2 \text{ red balls}) + P(3 \text{ red balls})$

$$= {}^7C_4 + {}^5C_1 \times {}^7C_3 + {}^5C_2 \times {}^7C_2 + {}^5C_3 \times {}^7C_1$$
$$= 35 + 175 + 210 + 70 = 490.$$

14. Let $f(x) = [\sin(\tan^{-1} x) + \sin(\cot^{-1} x)]^2 - 1$ where $|x| > 1$ and
 $\frac{dy}{dx} = \frac{1}{2} \frac{d}{dx} (\sin^{-1} f(x))$. If $y(\sqrt{3}) = \frac{\pi}{6}$ then $y(-\sqrt{3})$ is equal to
- a. $\frac{5\pi}{6}$ b. $-\frac{\pi}{6}$
- c. $\frac{2\pi}{3}$ d. $\frac{\pi}{3}$

Answer: (b)

Solution:

$$\begin{aligned} f(x) &= [\sin(\tan^{-1} x) + \sin(\cot^{-1} x)]^2 - 1 \\ \Rightarrow f(x) &= [\sin(\tan^{-1} x) + \cos(\tan^{-1} x)]^2 - 1 \\ \Rightarrow f(x) &= \sin(2 \tan^{-1} x) \end{aligned}$$

Now, $\frac{dy}{dx} = \frac{1}{2} \frac{d}{dx} (\sin^{-1}(f(x)))$

$$\Rightarrow 2y = \sin^{-1}(f(x)) + c$$

If $x = \sqrt{3}, y = \frac{\pi}{6}$

$$\therefore \frac{\pi}{3} = \sin^{-1}(\sin(2 \tan^{-1} \sqrt{3})) + c$$

$$\Rightarrow \frac{\pi}{3} = \sin^{-1} \left(\sin \left(\frac{2\pi}{3} \right) \right) + c$$

$$\Rightarrow \frac{\pi}{3} = \frac{\pi}{3} + c \Rightarrow c = 0$$

$$\Rightarrow 2y = \sin^{-1} \sin(2 \tan^{-1} x)$$

$$\text{When } x = -\sqrt{3}$$

$$2y = \sin^{-1} \left(\sin \left(2 \tan^{-1} (-\sqrt{3}) \right) \right) = \sin^{-1} \left(\sin \left(-\frac{2\pi}{3} \right) \right) = -\frac{\pi}{3}$$

$$\Rightarrow y = -\frac{\pi}{6}.$$

15. The system of equation $3x + 4y + 5z = \mu$

$$x + 2y + 3z = 1$$

$$4x + 4y + 4z = \delta$$

is inconsistent, then (μ, δ) can be

a. (4, 6)

b. (1, 0)

c. (4, 3)

d. (3, 4)

Answer: (c)

Solution:

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$$

$$R_3 \rightarrow R_3 - 2R_1 + 2R_2$$

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

For inconsistent system, one of D_x, D_y, D_z should not be equal to 0

$$D_x = \begin{vmatrix} \mu & 4 & 5 \\ 1 & 2 & 3 \\ \delta & 4 & 4 \end{vmatrix} \quad D_y = \begin{vmatrix} 3 & \mu & 5 \\ 1 & 1 & 3 \\ 4 & \delta & 4 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 3 & 4 & \mu \\ 1 & 2 & 1 \\ 4 & 4 & \delta \end{vmatrix}$$

For inconsistent system, $2\mu \neq \delta + 2$

$\therefore$ The system will be inconsistent for $\mu = 4, \delta = 3$.

16. If volume of parallelepiped whose conterminous edges are $\vec{u} = \hat{i} + \hat{j} + \lambda \hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{w} = \hat{i} + \hat{j} + 3\hat{k}$ is 1 cubic units. Then, the cosine of angle between $\vec{u}$ and $\vec{v}$ is

a. $\frac{5}{3\sqrt{10}}$
c. $\frac{7}{6\sqrt{3}}$

b. $\frac{5}{7}$
d. $\frac{7}{3\sqrt{3}}$

Answer: (c)

Solution:

Volume of parallelepiped = $[\vec{u} \ \vec{v} \ \vec{w}]$

$$\Rightarrow \begin{vmatrix} 1 & 1 & \lambda \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \pm 1$$

$$\Rightarrow \lambda = 2 \text{ or } 4$$

For $\lambda = 4$,

$$\cos \theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$$

17. If $2^{1-x} + 2^{1+x}$, $f(x)$, $3^x + 3^{-x}$ are in A.P. then the minimum value of $f(x)$ is

a. 4
c. 3

b. 2
d. 1

Answer: (c)

Solution:

$2^{1-x} + 2^{1+x}$, $f(x)$, $3^x + 3^{-x}$ are in A.P.

$$\therefore f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{(3^x + 3^{-x})}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Applying A.M. $\geq$ G.M. inequality, we get

$$\frac{(3^x + 3^{-x})}{2} \geq \sqrt{3^x \cdot 3^{-x}}$$

$$\Rightarrow \frac{(3^x + 3^{-x})}{2} \geq 1 \quad \dots (1)$$

Also, Applying A.M. $\geq$ G.M. inequality, we get

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq \sqrt{2^{1+x} \cdot 2^{1-x}}$$

$$\Rightarrow \frac{2^{1+x} + 2^{1-x}}{2} \geq 2 \quad \dots (2)$$

Adding (1) and (2), we get

$$f(x) \geq 1 + 2 = 3$$

Thus, minimum value of $f(x)$ is 3.

18. Which of the following is tautology?

a. $(p \wedge (p \rightarrow q)) \rightarrow q$

b. $q \rightarrow p \wedge (p \rightarrow q)$

c. $(p \wedge (p \vee q))$

d. $(p \vee (p \wedge q))$

Answer: (a)

Solution:

$$\begin{aligned} & (p \wedge (p \rightarrow q)) \rightarrow q \\ &= (p \wedge (\sim p \vee q)) \rightarrow q \\ &= [(p \wedge \sim p) \vee (p \wedge q)] \rightarrow q \\ &= (p \wedge q) \rightarrow q \\ &= \sim (p \wedge q) \vee q \\ &= \sim p \vee \sim q \vee q \\ &= T \end{aligned}$$

19. A is a 3×3 matrix whose elements are from the set $\{-1, 0, 1\}$. Find the number of matrices A such that $tr(AA^T) = 3$ where $tr(A)$ is sum of diagonal elements of matrix A

a. 612

b. 572

c. 672

d. 682

Answer: (c)

Solution:

$$tr(AA^T) = 3$$

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$tr(AA^T) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2 = 3$$

So out of 9 elements (a_{ij}) 's, 3 elements must be equal to 1 or -1 and rest elements must be 0.

So, the total possible cases will be

$$h = \frac{2t}{3}, k = \frac{-2 + t^2}{3}$$

Now, eliminating t from the above equations we get:

$$3k + 2 = \left(\frac{3h}{2}\right)^2$$

Replacing h and k by x and y , we get the locus of the curve as $9x^2 = 12y + 8$.

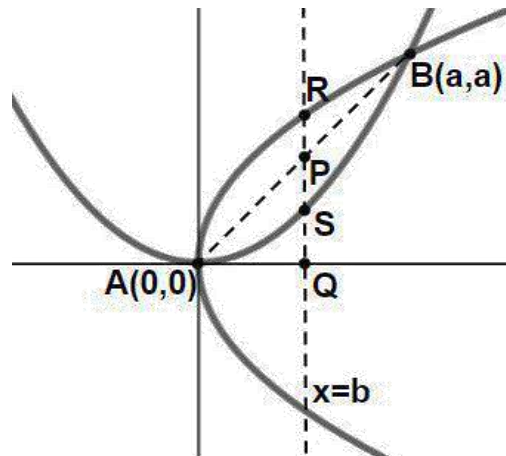
22. If the curves $y^2 = ax$ and $x^2 = ay$ intersect each other at A and B such that the area bounded by the curves is bisected by the line $x = b$ (given $a > b > 0$) and the area of triangle formed by the lines AB , $x = b$ and the x -axis is $\frac{1}{2}$. Then

- a. $a^6 + 12a^3 + 4 = 0$
c. $a^6 - 12a^3 + 4 = 0$

- b. $a^6 + 12a^3 - 4 = 0$
d. $a^6 - 12a^3 - 4 = 0$

Answer: (c)

Solution:



$$\text{Given, } ar(\Delta APQ) = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} \times b \times b = \frac{1}{2}$$

$$\Rightarrow b = 1$$

As per the question

$$\Rightarrow \int_0^1 \left(\sqrt{ax} - \frac{x^2}{a} \right) dx = \frac{1}{2} \int_0^a \left(\sqrt{ax} - \frac{x^2}{a} \right) dx$$

$$\Rightarrow \frac{2}{3} \sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$$

$$\Rightarrow 2a\sqrt{a} - 1 = \frac{a^3}{2}$$

$$\Rightarrow 4a\sqrt{a} = 2 + a^3$$

$$\Rightarrow 16a^3 = 4 + a^6 + 4a^3$$

$$\Rightarrow a^6 - 12a^3 + 4 = 0.$$

23. The sum $\sum_{k=1}^{20}(1 + 2 + 3 + \dots + k)$ is _____.

Answer: (1540)

Solution:

$$\begin{aligned} &= \sum_{k=1}^{20} \frac{k(k+1)}{2} \\ &= \frac{1}{2} \sum_{k=1}^{20} k^2 + k \\ &= \frac{1}{2} \left[\frac{20(21)(41)}{6} + \frac{20(21)}{2} \right] \\ &= \frac{1}{2} [2870 + 210] \\ &= 1540. \end{aligned}$$

24. If $2x^2 + (a - 10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+$ has real roots, then minimum value of ' a ' is equal to _____.

Answer: (8)

Solution:

$$\because 2x^2 + (a - 10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+ \text{ has real roots}$$

$$\Rightarrow D \geq 0$$

$$\Rightarrow (a - 10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a \right) \geq 0$$

$$\Rightarrow (a - 10)^2 - 4(33 - 4a) \geq 0$$

$$\Rightarrow a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, 4] \cup [8, \infty)$$

Thus, minimum value of ' a ' $\forall a \in \mathbf{Z}^+$ is 8.

25. If normal at P on the curve $y^2 - 3x^2 + y + 10 = 0$ passes through the point $\left(0, \frac{3}{2}\right)$ and the slope of tangent at P is n . The value of $|n|$ is equal to_____.

Answer: (4)

Solution:

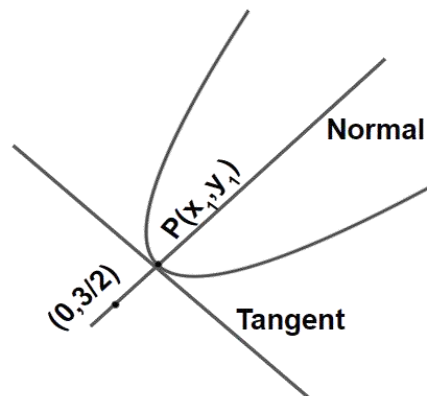
Let co-ordinate of P be (x_1, y_1)

Differentiating the curve w.r.t x

$$2yy' - 6x + y' = 0$$

Slope of tangent at P

$$\Rightarrow y' = \frac{6x_1}{1 + 2y_1}$$



$$\therefore m_{\text{normal}} = \left(\frac{y_1 - \frac{3}{2}}{x_1 - 0} \right)$$

$$\therefore m_{\text{normal}} \times m_{\text{tangent}} = -1$$

$$\Rightarrow \frac{\frac{3}{2} - y_1}{-x_1} \times \frac{6x_1}{1 + 2y_1} = -1$$

$$\Rightarrow y_1 = 1$$

$$\Rightarrow x_1 = \pm 2$$

$$\text{Slope of tangent} = \pm \frac{12}{3} = \pm 4$$

$$\Rightarrow |n| = 4$$