

JEE Main 2020 Paper

Date: 8th January 2020 (Shift 2)

Time: 2:30 P.M. to 5:30 P.M.

Subject: Mathematics

1. Solution set of $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$ contains
- | | |
|------------------------|---------------------------|
| a. exactly one element | b. at least four elements |
| c. two elements | d. infinite elements |

Answer: (a)

Solution:

$$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$$

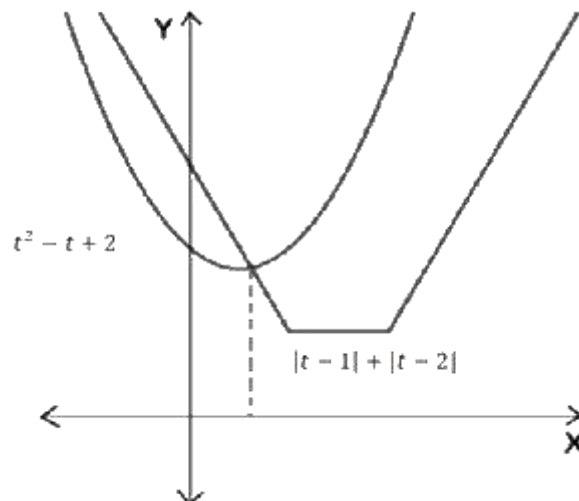
$$\text{Let } 3^x = t$$

$$t(t - 1) + 2 = |t - 1| + |t - 2|$$

$$\Rightarrow t^2 - t + 2 = |t - 1| + |t - 2|$$

We plot $t^2 - t + 2$ and $|t - 1| + |t - 2|$

As 3^x is always positive, therefore only positive values of t will be the solution.



Therefore, we have only one solution.

2. Which of the following is a tautology?
- | | |
|---|---|
| a. $\sim(p \wedge \sim q) \rightarrow (p \vee q)$ | b. $(\sim p \vee q) \rightarrow (p \vee q)$ |
| c. $\sim(p \vee \sim q) \rightarrow (p \vee q)$ | d. $\sim(p \vee \sim q) \rightarrow (p \wedge q)$ |

Answer: (c)

Solution:

$$= T$$

- $$\Rightarrow 2x + 5y = 100$$

- Page | 2

Solution:

For circle, $x^2 + y^2 = 1$

$$2x + 2yy' = 0 \Rightarrow y' = -\frac{x}{y}$$

Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -1$

$\Rightarrow$ Slope of tangent to $(x - 3)^2 + y^2 = 1$ is 1 $\Rightarrow m = 1$

Tangent to $(x - 3)^2 + y^2 = 1$ is $y = x + c$

Perpendicular distance of tangent $y = x + c$ from centre $(3, 0)$ is equal to radius = 1

$$\left| \frac{3 + c}{\sqrt{2}} \right| = 1$$

$$\Rightarrow c + 3 = \pm\sqrt{2}$$

$$\Rightarrow c^2 + 6c + 9 = 2$$

$$\Rightarrow c^2 + 6c + 7 = 0$$

5. If $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ is non-zero vector and $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$, $\vec{a} \cdot \vec{c} = 0$ then $\vec{b} \cdot \vec{c}$ is equal to

a. $\frac{1}{2}$

b. $-\frac{1}{3}$

c. $-\frac{1}{2}$

d. $\frac{1}{3}$

Answer: (c)

Solution:

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$$

$$\Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$$

$$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$

$$\Rightarrow -(\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$

$$\Rightarrow -4\vec{c} = 6(\hat{i} - \hat{j} + \hat{k}) - 4(\hat{i} - 2\hat{j} + \hat{k})$$

$$\Rightarrow \vec{c} = -\frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$$

$$\therefore \vec{b} \cdot \vec{c} = -\frac{1}{2}$$

6. If the coefficient of x^4 and x^2 in the expansion of $(x + \sqrt{x^2 - 1})^6 + (x - \sqrt{x^2 - 1})^6$ is α and β , then $\alpha - \beta$ is equal to
- a. 48 b. -60
c. 60 d. -132

Answer: (d)

Solution:

$$\begin{aligned} & \left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6 \\ &= 2[{}^6C_0x^6 + {}^6C_2x^4(x^2 - 1) + {}^6C_4x^2(x^2 - 1)^2 + {}^6C_6(x^2 - 1)^3] \\ &= 2[32x^6 - 48x^4 + 18x^2 - 1] \\ &\Rightarrow \alpha = -96, \quad \beta = 36 \\ &\Rightarrow \alpha - \beta = -132 \end{aligned}$$

7. Differential equation of $x^2 = 4b(y + b)$, where b is a parameter, is

- $x \left(\frac{dy}{dx} \right)^2 = 2y \left(\frac{dy}{dx} \right) + x$
- $x \left(\frac{dy}{dx} \right)^2 = 2y \left(\frac{dy}{dx} \right) + x^2$
- $x \left(\frac{dy}{dx} \right)^2 = y \left(\frac{dy}{dx} \right) + x^2$
- $x \left(\frac{dy}{dx} \right)^2 = y \left(\frac{dy}{dx} \right) + 2x^2$

Answer: (a)

Solution:

$$x^2 = 4b(y + b) \quad \dots (1)$$

Differentiating both the sides w.r.t. x , we get

$$\Rightarrow 2x = 4by'$$

$$\Rightarrow b = \frac{x}{2v'}$$

Putting the value of b in (1), we get

$$\Rightarrow x^2 = \frac{2x}{y'} \left(y + \frac{x}{2y'} \right)$$

$$\Rightarrow x^2 = \frac{2xy}{y'} + \frac{x^2}{y'^2}$$

$$\Rightarrow xy'^2 = 2yy' + x$$

$$\Rightarrow x \left(\frac{dy}{dx} \right)^2 = 2y \left(\frac{dy}{dx} \right) + x$$

8. Image of point $(1, 2, 3)$ w.r.t a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$ then which of the following points lie on this plane
- | | |
|------------------|-------------------|
| a. $(1, 1, -1)$ | b. $(-1, -1, 1)$ |
| c. $(-1, 1, -1)$ | d. $(-1, -1, -1)$ |

Answer: (a)

Solution:

Image of point $P(1, 2, 3)$ w.r.t. a plane $ax + by + cz + d = 0$ is $Q\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$

Direction ratios of PQ : $-\frac{10}{3}, -\frac{10}{3}, -\frac{10}{3} = 1, 1, 1$

Direction ratios of normal to plane is $1, 1, 1$

Mid-point of PQ lies on the plane

$\therefore$ The mid-point of $PQ = \left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

$\therefore$ Equation of plane is $x + \frac{2}{3} + y - \frac{1}{3} + z - \frac{4}{3} = 0$

$\Rightarrow x + y + z = 1$

$(1, 1, -1)$ satisfies the equation of the plane.

9. $\lim_{x \rightarrow 0} \frac{\int_0^x t \sin 10t \, dt}{x}$ is equal to

- | | |
|-------|------|
| a. 10 | b. 0 |
| c. 1 | d. 5 |

Answer: (b)

Solution:

$$\lim_{x \rightarrow 0} \frac{\int_0^x t \sin 10t \, dt}{x}$$

Applying L'Hospital's Rule:

$$= \lim_{x \rightarrow 0} \frac{x \sin 10x}{1} = 0$$

10. Let P be the set of points (x, y) such that $(x^2 \leq y \leq -2x + 3)$. Then area bounded by points in P is

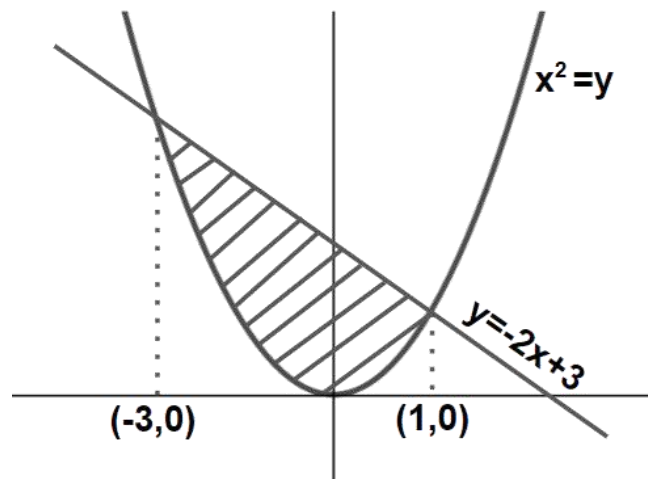
a. $\frac{16}{3}$
c. $\frac{20}{3}$

b. $\frac{29}{3}$
d. $\frac{32}{3}$

Answer: (d)

Solution:

We have $x^2 \leq y \leq -2x + 3$



For point of intersection of two curves -

$$x^2 + 2x - 3 = 0$$

$$\Rightarrow x = -3, 1$$

$$\Rightarrow \text{Area} = \int_{-3}^1 ((-2x + 3) - x^2) dx$$

$$= \left[-x^2 + 3x - \frac{x^3}{3} \right]_{-3}^1 = \frac{32}{3} \text{ sq. units.}$$

11. If $f(x) = \frac{x[x]}{x^2+1} : (1, 3) \rightarrow \mathbf{R}$, then the range of $f(x)$ is [where $[.]$ denotes greatest integer function]

a. $\left(0, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{7}{5}\right]$
c. $\left(0, \frac{1}{3}\right) \cup \left(\frac{2}{5}, \frac{4}{5}\right]$

b. $\left(\frac{2}{5}, 1\right) \cup \left(1, \frac{4}{5}\right]$
d. $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

Answer: (d)

Solution:

$$f(x) = \frac{x[x]}{x^2+1}$$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{x^2+1} : 1 < x < 2 \\ \frac{2x}{x^2+1} : 2 \leq x < 3 \end{cases}$$

$$\Rightarrow \text{Range of } f(x) \text{ is } \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right].$$

12. If $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then the value of $10A^{-1}$ is
- a. $A - 4I$ b. $A - 6I$
c. $6I - A$ d. $4I - A$

Answer: (b)

Solution:

$$A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{10} \begin{bmatrix} 4 & -2 \\ -9 & 2 \end{bmatrix}$$

$$\Rightarrow 10A^{-1} = \begin{bmatrix} -4 & 2 \\ 9 & -2 \end{bmatrix}$$

$$\Rightarrow 10A^{-1} = A - 6I$$

13. For 20 observations mean and variance is given as 10 and 4, later it was observed that by mistake 9 was taken in place of 11, then the correct variance is
- a. 3.98 b. 3.99
c. 4.01 d. 4.02

Answer: (b)

Solution:

$$\text{Mean} = 10 \Rightarrow \frac{\sum x_i}{20} = 10 \Rightarrow \sum x_i = 200$$

$$\text{Variance} = 4 \Rightarrow \frac{\sum x_i^2}{20} - 100 = 4 \Rightarrow \sum x_i^2 = 2080$$

$$\text{New mean} = \frac{200 - 9 + 11}{20} = \frac{202}{20} = 10.1$$

$$\begin{aligned}\text{New variance} &= \frac{2080 - 81 + 121}{20} - (10.1)^2 \\ &= 106 - 102.01 \\ &= 3.99\end{aligned}$$

14. The correct option for the system of linear equations
- $$\begin{aligned}\lambda x + 2y + 2z &= 5 \\ 2\lambda x + 3y + 5z &= 8 \\ 4x + \lambda y + 6z &= 10\end{aligned}$$
- a. Infinite solutions when $\lambda = 2$
- b. Infinite solutions when $\lambda = 8$

16. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$ and $a = (1 + \alpha) \sum_{k=0}^{100} \alpha^{2k}$, $b = \sum_{k=0}^{100} \alpha^{3k}$ where a and b are the roots of the quadratic equation then the quadratic equation will be

a. $x^2 - 102x + 101 = 0$

b. $x^2 + 102x + 100 = 0$

c. $x^2 - 101x + 100 = 0$

d. $x^2 + 101x + 100 = 0$

Answer: (a)

Solution:

$$\alpha = \frac{-1+i\sqrt{3}}{2} = \omega$$

$$a = (1 + \alpha) \sum_{k=0}^{100} \alpha^{2k}$$

$$\Rightarrow a(1 + \alpha)[1 + \alpha^2 + \alpha^4 + \dots + \alpha^{200}]$$

$$\Rightarrow a = (1 + \alpha) \left[\frac{1 - (\alpha^2)^{101}}{1 - \alpha^2} \right]$$

$$\Rightarrow a = \left[\frac{1 - (\omega^2)^{101}}{1 - \omega} \right] = \left[\frac{1 - \omega}{1 - \omega} \right] = 1$$

$$b = \sum_{k=0}^{100} a^{3k} = 1 + a^3 + a^6 + \cdots .. + a^{300}$$

$$\Rightarrow b = 1 + \omega^3 + \omega^6 + \dots + \omega^{300}$$

$$\Rightarrow b = 101$$

Required equation is $x^2 - 102x + 101 = 0$

17. If $f(x)$ is a three-degree polynomial for which $f'(-1) = 0, f''(1) = 0, f(-1) = 10, f(1) = 6$ then the local minima of $f(x)$ will be at

a. $x = -1$

b. $x = 1$

c. $x = 2$

d. $x = 3$

Answer: (d)

Solution:

Let the polynomial be

$$f(x) = ax^3 + bx^2 + cx + d$$

$$\Rightarrow f'(x) = 3ax^2 + 2bx + c$$

$$\Rightarrow f''(x) = 6ax + 2b$$

$$f''(1) = 0 \Rightarrow 6a + 2b = 0 \Rightarrow b = -3a$$

$$f'(-1) = 0 \Rightarrow 3a - 2b + c = 0$$

$$\Rightarrow c = -9a$$

$$f(-1) = 10 \Rightarrow -a + b - c + d = 10$$

$$\Rightarrow -a - 3a + 9a + d = 10$$

$$d = -5a + 10$$

$$f(1) = 6 \Rightarrow a + b + c + d = 6$$

$$\Rightarrow a - 3a - 9a - 5a + 10 = 6$$

$$\Rightarrow a = \frac{1}{4}$$

$$\therefore f'(x) = \frac{3}{4}x^2 - \frac{6}{4}x - \frac{9}{4} = \frac{3}{4}(x^2 - 2x - 3)$$

$$\text{For } f'(x) = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = 3, -1$$

Minima exists at $x = 3$

18. Let $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$ then

a. $\frac{1}{9} < I^2 < \frac{1}{8}$

c. $\frac{1}{3} < I^2 < \frac{1}{2}$

b. $\frac{1}{9} < I < \frac{1}{8}$

d. $\frac{1}{3} < I < \frac{1}{2}$

Answer: (a)

Solution:

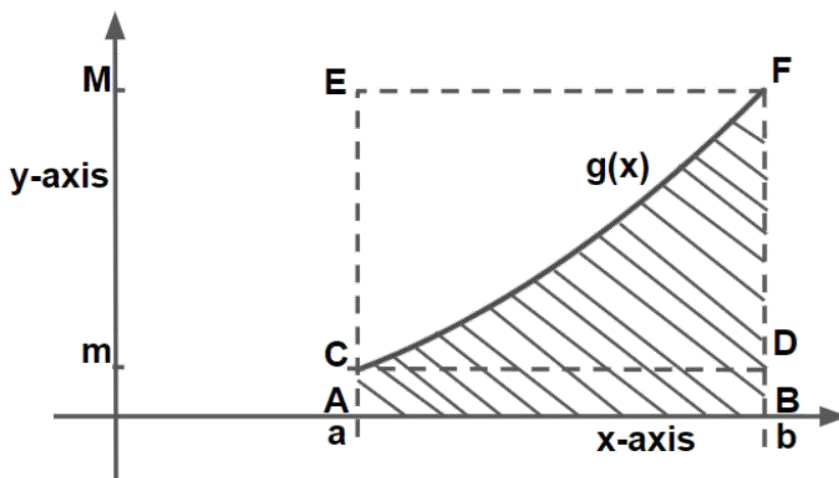
$$f(x) = \frac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$$

Differentiating w.r.t x

$$\begin{aligned} f'(x) &= -\frac{1}{2} \times \frac{(6x^2 - 18x + 12)}{(2x^3 - 9x^2 + 12x + 4)^{3/2}} \\ &= \frac{-6(x-1)(x-2)}{2(2x^3 - 9x^2 + 12x + 4)^{3/2}} \end{aligned}$$

Here $f(x)$ is increasing in (1,2)

$$\text{At } x = 1, f(1) = \frac{1}{3} \quad \text{and } x = 2, f(2) = \frac{1}{\sqrt{8}}$$



Let $g(x)$ be a function such that it is increasing in (a, b) and $m \leq g(x) \leq M$, then

$$\text{ar}(ABCD) < \int_a^b g(x) dx < \text{ar}(ABEF)$$

$$m(b-a) < \int_a^b g(x) dx < M(b-a)$$

$$\text{Thus, } \frac{1}{3} < \int_1^2 f(x) dx < \frac{1}{\sqrt{8}}$$

$$\frac{1}{3} < I < \frac{1}{\sqrt{8}},$$

$$\text{or } \frac{1}{9} < I^2 < \frac{1}{8}$$

19. Normal at $(2, 2)$ to curve $x^2 + 2xy - 3y^2 = 0$ is L . Then perpendicular distance from origin to line L is

- | | |
|----------------|------|
| a. $2\sqrt{2}$ | b. 4 |
| c. $4\sqrt{2}$ | d. 2 |

Answer: (a)

Solution:

$$\text{Given curve: } x^2 + 2xy - 3y^2 = 0$$

$$\Rightarrow x^2 + 3xy - xy - 3y^2 = 0$$

$$\Rightarrow (x + 3y)(x - y) = 0$$

Equating we get,

$$x + 3y = 0 \text{ or } x - y = 0$$

$$(2, 2) \text{ lies on } x - y = 0$$

$\therefore$ Equation of normal will be $x + y = \lambda$

It passes through $(2, 2)$

$$\therefore \lambda = 4$$

$$L : x + y = 4$$

$$\text{Distance of } L \text{ from the origin} = \left| \frac{-4}{\sqrt{2}} \right| = 2\sqrt{2}$$

20. If A and B are two events such that $P(\text{exactly one}) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{2}$ then $P(A \cap B)$ is

a. $\frac{1}{8}$

b. $\frac{1}{10}$

c. $\frac{1}{12}$

d. $\frac{2}{9}$

Answer: (b)

Solution:

$$P(\text{exactly one of } A \text{ or } B) = \frac{2}{5}$$

$$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A \cup B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A \cap B) = \frac{1}{10}$$

21. The number of four-letter words that can be made from the letters of word "EXAMINATION" is

Answer: (2454)

Solution:

Word "EXAMINATION" consists of 2A, 2I, 2N, E, X, M, T, O

Case I: All different letters are selected

$$\text{Number of words formed} = {}^8C_4 \times 4! = 1680$$

Case II: 2 letters are same and 2 are different

$$\text{Number of words formed} = {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$$

Case III: 2 pair of letters are same

$$\text{Number of words formed} = {}^3C_2 \times \frac{4!}{2! \times 2!} = 18$$

Total number of words formed = $1680 + 756 + 18 = 2454$

22. Line $y = mx$ intersects the curve $y^2 = x$ at point P . The tangent to $y^2 = x$ at P intersects x -axis at Q . If area $\Delta OPQ = 4$, find m , ($m > 0$)

Answer: (0.5)

Solution:

Let the co-ordinates of P be (t^2, t)

Equation of tangent at $P(t^2, t)$ is $y - t = \frac{1}{2t}(x - t^2)$

Therefore, co-ordinates of Q will be $(-t^2, 0)$

Area of $\Delta OPQ = 4$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = \pm 4$$

$$\Rightarrow t^3 = 8 \Rightarrow t = \pm 2 \Rightarrow t = 2 \text{ as } t > 0$$

$$m = \frac{1}{t} = \frac{1}{2}$$

23. $\sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4}$ is equal to

Answer: (504)

Solution:

$$\begin{aligned} & \sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4} \\ &= \frac{1}{4} \sum_{n=1}^7 (2n^3 + 3n^2 + n) \\ &= \frac{1}{4} \left[2 \sum_{n=1}^7 n^3 + 3 \sum_{n=1}^7 n^2 + \sum_{n=1}^7 n \right] \\ &= \frac{1}{4} \left[2 \times \left(\frac{7 \times 8}{2} \right)^2 + 3 \times \frac{7 \times 8 \times 15}{6} + \frac{7 \times 8}{2} \right] \\ &= \frac{1}{4} [2 \times 784 + 420 + 28] = 504 \end{aligned}$$

24. Let $\frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, where $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$. Then $\tan(\alpha + 2\beta)$ is equal to

Answer: (1)

Solution:

$$\frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7} \Rightarrow \frac{\sqrt{2} \sin \alpha}{\sqrt{2} \cos \alpha} = \frac{1}{7} \Rightarrow \tan \alpha = \frac{1}{7}$$

$$\sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}} \Rightarrow \frac{\sqrt{2} \sin \beta}{\sqrt{2}} = \frac{1}{\sqrt{10}} \Rightarrow \sin \beta = \frac{1}{\sqrt{10}} \Rightarrow \tan \beta = \frac{1}{3}$$

$$\tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta} = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \times \frac{3}{4}} = 1$$