

JEE Main 2020 Paper

Date of Exam: 9th January 2020 (Shift 1)

Time: 9:30 A.M. to 12:30 P.M.

Subject: Mathematics

1. A sphere of 10 cm radius has a uniform thickness of ice around it. If the ice is melting at the rate of $50 \text{ cm}^3/\text{min}$ when thickness is 5 cm, then the rate of change of thickness is

a. $\frac{1}{12\pi}$
c. $\frac{1}{9\pi}$

b. $\frac{1}{18\pi}$
d. $\frac{1}{36\pi}$

Answer: (b)

Solution:

Let thickness of ice be x cm.

Therefore, net radius of sphere = $(10 + x)$ cm

$$\text{Volume of sphere } V = \frac{4}{3}\pi(10 + x)^3$$

$$\Rightarrow \frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$$

$$\text{At } x = 5, \frac{dV}{dt} = 50 \text{ cm}^3/\text{min}$$

$$\Rightarrow 50 = 4\pi \times 225 \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{18\pi} \text{ cm/min}$$

2. The number of real roots of $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is

a. 1
c. 3

b. 2
d. 4

Answer: (a)

Solution:

$$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$$

$$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\text{Let } e^x + \frac{1}{e^x} = u$$

$$\text{Then, } u^2 + u - 6 = 0$$

$$\Rightarrow u = 2, -3$$

$$u \neq -3 \text{ as } u > 0 (\because e^x > 0)$$

$$\Rightarrow e^x + \frac{1}{e^x} = 2 \Rightarrow (e^x - 1)^2 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$$

Hence, only one real solution is possible.

3. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f(0) = 0$, then the value of $f(1)$ is

a. $\frac{\pi-1}{4}$
c. $\frac{\pi+1}{2}$

b. $\frac{\pi+1}{4}$
d. 0

Answer: (b)

Solution:

$$f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right)$$

$$f'(x) = \tan^{-1}\left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}\right)$$

$$f'(x) = \tan^{-1}\left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)}\right]$$

$$f'(x) = \tan^{-1}\left[\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}\right]$$

$$f'(x) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$f'(x) = \frac{\pi}{4} + \frac{x}{2}$$

$$\Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$f(0) = 0 \Rightarrow c = 0$$

$$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi + 1}{4}$$

4. The number of solutions of $\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}} |\cos x|$, $x \in [0, 2\pi]$ is

a. 2
c. 8

b. 4
d. 6

Answer: (c)

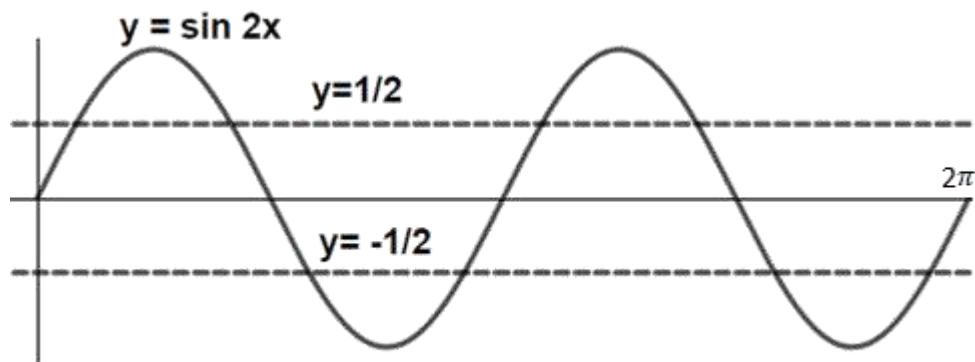
Solution:

$$\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0, 2\pi]$$

$$\Rightarrow \log_{\frac{1}{2}} |\sin x| |\cos x| = 2$$

$$\Rightarrow |\sin x \cos x| = \frac{1}{4}$$

$$\therefore \sin 2x = \pm \frac{1}{2}$$



$\therefore$ We have 8 solutions for $x \in [0, 2\pi]$

5. If e_1 and e_2 are the eccentricities of $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and $\frac{x^2}{9} - \frac{y^2}{4} = 1$, respectively. If the points (e_1, e_2) lies on the ellipse $15x^2 + 3y^2 = k$. Then the value of k is
- a. 16
b. 14
c. 15
d. 17

Answer: (a)

Solution:

$$e_1 = \sqrt{1 - \frac{4}{18}} = \frac{\sqrt{7}}{3} \text{ \& } e_2 = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$$

$\therefore (e_1, e_2)$ lies on the ellipse $15x^2 + 3y^2 = k$

$$\therefore 15e_1^2 + 3e_2^2 = k$$

$$\Rightarrow 15 \times \frac{7}{9} + 3 \times \frac{13}{9} = k \Rightarrow k = 16$$

6. The value of $\int \frac{dx}{(x-3)^{\frac{6}{7}} \times (x+4)^{\frac{8}{7}}}$ is -

a. $7 \left(\frac{x-3}{x+4} \right)^{\frac{1}{7}} + c$

b. $7 \left(\frac{x-3}{x+4} \right)^{\frac{6}{7}} + c$

c. $\left(\frac{x-3}{x+4} \right)^{\frac{1}{7}} + c$

d. $7 \left(\frac{x+4}{x-3} \right)^{\frac{6}{7}} + c$

Answer: (c)

Solution:

$$I = \int \frac{dx}{(x-3)^{\frac{6}{7}} \times (x+4)^{\frac{8}{7}}}$$

$$\Rightarrow I = \int \frac{(x+4)^{\frac{6}{7}} dx}{(x-3)^{\frac{6}{7}} \times (x+4)^2} = \int \left(\frac{x-3}{x+4} \right)^{-\frac{6}{7}} \times \frac{dx}{(x+4)^2}$$

$$\text{Put } \frac{x-3}{x+4} = t \Rightarrow dt = 7 \left(\frac{1}{(x+4)^2} \right) dx$$

$$\Rightarrow I = \frac{\int t^{-\frac{6}{7}} dt}{7} = t^{\frac{1}{7}} + c = \left(\frac{x-3}{x+4} \right)^{\frac{1}{7}} + c$$

7. If $\left| \frac{z-i}{z+2i} \right| = 1$, $|z| = \frac{5}{2}$ then the value of $|z + 3i|$ is

a. $\sqrt{10}$

b. $\sqrt{5}$

c. $\frac{7}{2}$

d. $\sqrt{3}$

Answer: (c)

Solution:

$$\text{If } \left| \frac{z-i}{z+2i} \right| = 1 \text{ \& } |z| = \frac{5}{2}$$

$$\Rightarrow |z - i| = |z + 2i|$$

$$\Rightarrow x^2 + (y - 1)^2 = x^2 + (y + 2)^2$$

$$\Rightarrow y - 1 = \pm(y + 2)$$

$$\Rightarrow y - 1 = -y - 2$$

$$\Rightarrow y = -\frac{1}{2}$$

$$\Rightarrow |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$$

$$\Rightarrow x^2 + \frac{1}{4} = \frac{25}{4}$$

$$\Rightarrow x = \pm\sqrt{6}$$

$$|z + 3i| = \sqrt{x^2 + (y + 3)^2}$$

$$\Rightarrow |z + 3i| = \frac{7}{2}$$

8. The value of $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \dots \infty$ is

a. 2

b. 1

c. $\sqrt{2}$

d. $2^{\frac{1}{4}}$

Answer: (c)

Solution:

$$2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \dots \infty = 2^{\frac{1}{4}} \times 2^{\frac{2}{16}} \times 2^{\frac{4}{48}} \dots \infty$$

$$\Rightarrow 2^{\frac{1}{4}} \times 2^{\frac{1}{8}} \times 2^{\frac{1}{16}} \dots \infty = 2^{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \infty}$$

$$\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1 - \frac{1}{2}}\right)} = \sqrt{2}$$

9. The value of $\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8}$ is -

a. $\frac{1}{2}$

b. $-\frac{1}{2}$

c. $\frac{1}{\sqrt{2}}$

d. $\frac{1}{2\sqrt{2}}$

Answer: (d)

Solution:

$$\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8} = \cos^3 \frac{\pi}{8} \left[4 \cos^3 \frac{\pi}{8} - 3 \cos \frac{\pi}{8} \right] + \sin^3 \frac{\pi}{8} \left[3 \sin \frac{\pi}{8} - 4 \sin^3 \frac{\pi}{8} \right]$$

$$= 4 \left[\cos^6 \frac{\pi}{8} - \sin^6 \frac{\pi}{8} \right] + 3 \left[\sin^4 \frac{\pi}{8} - \cos^4 \frac{\pi}{8} \right]$$

$$= 4 \left[\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right] \left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} \sin^2 \frac{\pi}{8} \right] - 3 \left[\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right]$$

$$= \left[\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right] \left[4 \left(1 - \cos^2 \frac{\pi}{8} \sin^2 \frac{\pi}{8} \right) - 3 \right]$$

$$= \cos \frac{\pi}{4} \left[1 - \sin^2 \frac{\pi}{4} \right] = \frac{1}{2\sqrt{2}}$$

10. The value of $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ is

a. $4\pi^2$

b. $2\pi^2$

c. π^2

d. $3\pi^2$

Answer: (c)

Solution:

$$\text{Let } I = \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx \quad \dots (1)$$

$$\begin{aligned} I &= \int_0^{2\pi} \frac{(2\pi-x) \sin^8(2\pi-x)}{\sin^8(2\pi-x) + \cos^8(2\pi-x)} dx \\ &= \int_0^{2\pi} \frac{(2\pi-x) \sin^8 x}{\sin^8 x + \cos^8 x} dx \quad \dots (2) \end{aligned}$$

Adding (1) & (2), we get:

$$\begin{aligned} \Rightarrow 2I &= 2\pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx \\ I &= \pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx \\ I &= 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx \quad \dots (3) \end{aligned}$$

$$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8(\frac{\pi}{2}-x)}{\sin^8(\frac{\pi}{2}-x) + \cos^8(\frac{\pi}{2}-x)} dx = 4\pi \int_0^{\frac{\pi}{2}} \frac{\cos^8 x}{\sin^8 x + \cos^8 x} dx \quad \dots (4)$$

Adding (3) & (4), we get -

$$I = 2\pi \int_0^{\frac{\pi}{2}} 1 dx = 2\pi \times \frac{\pi}{2} = \pi^2$$

11. If $f(x) = a + bx + cx^2$ where $a, b, c \in \mathbf{R}$ then the value of $\int_0^1 f(x)dx$ is -

- | | |
|--|--|
| a. $\frac{1}{6} \left(f(1) + f(0) - 4f\left(\frac{1}{2}\right) \right)$ | b. $\frac{1}{3} \left(f(1) + f(0) + 2f\left(\frac{1}{2}\right) \right)$ |
| c. $\frac{1}{6} \left(f(1) + f(0) + 4f\left(\frac{1}{2}\right) \right)$ | d. $\frac{1}{6} \left(f(1) - f(0) - 4f\left(\frac{1}{2}\right) \right)$ |

Answer: (c)

Solution:

$$f(x) = a + bx + cx^2$$

$$f(0) = a, f(1) = a + b + c$$

$$f\left(\frac{1}{2}\right) = \frac{c}{4} + \frac{b}{2} + a$$

$$\int_0^1 f(x)dx = \int_0^1 (a + bx + cx^2)dx = a + \frac{b}{2} + \frac{c}{3}$$

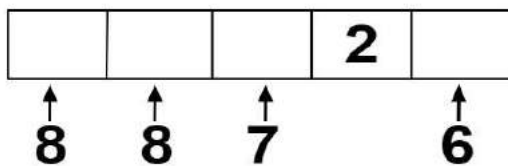
$$= \frac{1}{6} (6a + 3b + 2c) = \frac{1}{6} (a + (a + b + c) + (4a + 2b + c))$$

$$= \frac{1}{6} \left(f(0) + f(1) + 4f\left(\frac{1}{2}\right) \right)$$

12. If the number of ways of forming 5 digit numbers (without repeating any digit), such that the tenth place of the number must be occupied by 2 is $336k$, then the value of k is
- a. 5 b. 6
c. 7 d. 8

Answer: (d)

Solution:



Total numbers that can be formed are

$$= 8 \times 8 \times 7 \times 6$$

$$= 8 \times 336$$

$$\therefore k = 8$$

13. If D is the centroid of the $\triangle ABC$ having vertices $A(3, -1)$, $B(1, 3)$, $C(2, 4)$ and P is the point of intersection of lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$, then which of the following point lies on the line joining D and P ?
- a. $(-9, -6)$ b. $(9, -6)$
c. $(9, 6)$ d. $(-9, -7)$

Answer: (a)

Solution:

Coordinates of D are $\left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right) = (2, 2)$

Point of intersection of two lines

$$x + 3y - 1 = 0 \text{ and } 3x - y + 1 = 0$$

is $P\left(\frac{-1}{5}, \frac{2}{5}\right)$

Equation of line DP is $8x - 11y + 6 = 0$

Point $(-9, -6)$ lies on DP

14. If $f(x)$ is twice differentiable and continuous function in $x \in [a, b]$. Also $f'(x) > 0$ and $f''(x) < 0$ and $c \in (a, b)$, then $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than

- a. 1
 b. $\frac{a+b}{b-c}$
 c. $\frac{b-c}{c-a}$
 d. $\frac{c-a}{b-c}$

Answer: (d)

Solution:

$\therefore c \in (a, b)$ and f is twice differentiable and continuous function (a, b)

$\therefore$ LMVT is applicable

$$\text{For } p \in (a, c), \quad f'(p) = \frac{f(c)-f(a)}{c-a}$$

$$\text{For } q \in (c, b), \quad f'(q) = \frac{f(b)-f(c)}{b-c}$$

$\therefore f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$$f'(p) > f'(q)$$

$$\Rightarrow \frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$$

$$\Rightarrow \frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c} \quad (\text{as } f'(x) > 0 \Rightarrow f(x) \text{ is increasing})$$

15. If three planes

$$x + 4y - 2z = 1$$

$$x + 7y - 5z = \beta$$

$$x + 5y + \alpha z = 5$$

intersect in a line, then $\alpha + \beta =$

- a. -10
 b. 0
 c. 2
 d. 10

Answer: (d)

Solution:

The given planes intersect in a line

$$\therefore D = D_x = D_y = D_z = 0$$

$$D = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$$

$$\Rightarrow \alpha = -3$$

$$D_z = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 35 - 5\beta - 20 + 4\beta - 2 = 0$$

$$\Rightarrow \beta = 13$$

$$\therefore \alpha + \beta = 10$$

16. $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If mean and variance of observations $(x_1 - 3), (x_2 - 3) \dots (x_{10} - 3)$ is λ and μ respectively, then ordered pair (λ, μ) is
- a. (1,1) b. (1,3)
 c. (3,1) d. (3,3)

Answer: (d)

Solution:

$$\sum_{i=1}^{10}(x_i - 5) = 10 \Rightarrow \sum_{i=1}^{10} x_i - 50 = 10$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 60$$

$$\lambda = \frac{\sum_{i=1}^{10}(x_i - 3)}{10} = \frac{\sum_{i=1}^{10} x_i - 30}{10} = 3$$

Variance is unchanged, if a constant is added or subtracted from each observation

$$\begin{aligned} \mu &= Var(x_i - 3) = Var(x_i - 5) = \frac{\sum_{i=1}^{10}(x_i - 5)^2}{10} - \left(\frac{\sum (x_i - 5)}{10} \right)^2 \\ &= \frac{40}{10} - \left(\frac{10}{10} \right)^2 = 3 \end{aligned}$$

17. 20 cards are placed in a bag with 10 named as A and another 10 named as B. If cards are drawn one by one (with replacement), then the probability that second A comes before third B is
- a. $\frac{11}{16}$ b. $\frac{7}{16}$
 c. $\frac{9}{16}$ d. $\frac{13}{16}$

Answer: (a)

Solution:

Here $P(A) = P(B) = \frac{1}{2}$

Then, these following cases are possible $\rightarrow AA, BAA, ABA, ABBA, BBAA, BABA$

So, the required probability is $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

18. The negation of ' $\sqrt{5}$ is an integer or 5 is an irrational number' is

- $\sqrt{5}$ is an integer and 5 is not an irrational number.
- $\sqrt{5}$ is not an integer and 5 is not an irrational number.
- $\sqrt{5}$ is not an integer or 5 is not an irrational number.
- $\sqrt{5}$ is not an integer and 5 is an irrational number.

Answer: (b)

Solution:

p : $\sqrt{5}$ is an integer

q : 5 is an irrational number

Given statement : $p \vee q$

Required negation statement: $\sim(p \vee q) = \sim p \wedge \sim q$

' $\sqrt{5}$ is not an integer and 5 is not an irrational number'

19. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj}(A)$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|}$ is
- 2
 - 4
 - 8
 - 16

Answer: (c)

Solution:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 13 + 1 - 8 = 6$$

$$B = \text{adj}(A) \Rightarrow |\text{adj } B| = |\text{adj}(\text{adj } A)| = |A|^4 = 6^4$$

$$|C| = |3A| = 3^3 |A| = 3^3 \times 6$$

$$\frac{|\text{adj } B|}{|C|} = \frac{6^4}{3^3 \times 6} = \frac{2^3 \times 3^3}{3^3} = 8$$

20. If a circle touches y-axis at (0,4) and passes through (2,0), then which of the following can be the tangent to the circle?

a. $3x + 4y - 24 = 0$

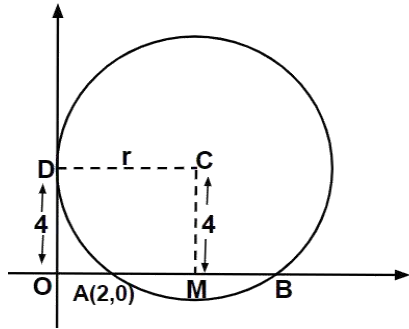
c. $4x + 3y - 6 = 0$

b. $4x - 3y - 17 = 0$

d. $3x + 4y - 6 = 0$

Answer: (d)

Solution:



$$OD^2 = OA \times OB \Rightarrow 16 = 2 \times OB \Rightarrow OB = 8$$

$$\therefore AB = 6$$

$$\therefore AM = 3, CM = 4 \Rightarrow CA = 5$$

$$\therefore OM = 5$$

Centre will be (5,4) and radius is 5

Now checking option (d)

$$3x + 4y - 6 = 0$$

$$\frac{15 + 16 - 6}{\sqrt{3^2 + 4^2}} = 5 \quad (p = r)$$

21. $(1+x) \frac{dy}{dx} = [(1+x)^2 + (y-3)]$. If $y(2) = 0$, then the value of $y(3)$ is

Answer: (3)

Solution:

$$(1+x) \frac{dy}{dx} = [(1+x)^2 + (y-3)]$$

$$\Rightarrow (1+x) \frac{dy}{dx} - y = (1+x)^2 - 3$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{(1+x)} y = 1+x - \frac{3}{1+x}$$

$$\text{I. F.} = e^{-\int \frac{1}{1+x} dx} = \frac{1}{1+x}$$

$$y \times \frac{1}{1+x} = \int 1 - \frac{3}{(1+x)^2} dx$$

$$\frac{y}{1+x} = x + \frac{3}{1+x} + c$$

$$\Rightarrow y = x(1+x) + 3 + c(1+x)$$

At $x = 2, y = 0$, we get

$$0 = 6 + 3 + 3c$$

$$\Rightarrow c = -3$$

$$\Rightarrow \text{At } x = 3,$$

$$y = x^2 - 2x = 9 - 6 = 3$$

$$\Rightarrow y(3) = 3$$

22. Function $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}, & x < 0 \\ b, & x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{x^{\frac{4}{3}}}, & x > 0 \end{cases}$ is continuous at $x = 0$. The value of $a + 2b$ is

Answer: (0)

Solution:

$f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = b = \lim_{x \rightarrow 0^+} f(x)$$

$$b = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{(h+3h^2)^{\frac{1}{3}} - h^{\frac{1}{3}}}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h \rightarrow 0} \frac{h^{\frac{1}{3}} \left[(1+3h)^{\frac{1}{3}} - 1 \right]}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h \rightarrow 0} \frac{(1+3h)^{\frac{1}{3}} - 1}{h}$$

$$\Rightarrow b = \lim_{h \rightarrow 0} \frac{1}{3} (1+3h)^{-\frac{2}{3}} \times 3$$

$$\text{or, } b = 1$$

$$\lim_{x \rightarrow 0^-} f(x) = 1 \Rightarrow \lim_{h \rightarrow 0} \frac{\sin(a+2)(-h)) + \sin(-h)}{-h} = 1$$

$$\Rightarrow a + 3 = 1 \Rightarrow a = -2$$

$$\Rightarrow a + 2b = 0$$

23. The coefficient of x^4 in $(1 + x + x^2)^{10}$ is

Answer: (615)

Solution:

General term of the given expression is given by $\frac{10!}{p!q!r!} x^{q+2r}$

Here, $q + 2r = 4$

For $p = 6, q = 4, r = 0$, coefficient = $\frac{10!}{6! \times 4!} = 210$

For $p = 7, q = 2, r = 1$, coefficient = $\frac{10!}{7! \times 2! \times 1!} = 360$

For $p = 8, q = 0, r = 2$, coefficient = $\frac{10!}{8! \times 2!} = 45$

Therefore, sum = 615

24. If $\vec{P} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$

$$\vec{Q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$$

$$\vec{R} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$$

and $\vec{P}, \vec{Q}, \vec{R}$ are coplanar vectors and $3(\vec{P} \cdot \vec{Q})^2 - \lambda |\vec{R} \times \vec{Q}|^2 = 0$, then value of λ is

Answer: (1)

Solution:

As $\vec{P}, \vec{Q}, \vec{R}$ are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$(3a + 1) \begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1$$

$$(3a + 1) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$$3a + 1 = 0$$

$$\Rightarrow a = -\frac{1}{3}$$

$$\vec{P} = \frac{1}{3}(2\hat{i} - \hat{j} - \hat{k}), \quad \vec{Q} = \frac{1}{3}(-\hat{i} + 2\hat{j} - \hat{k}), \quad \vec{R} = \frac{1}{3}(-\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{R} \times \vec{Q} = \frac{1}{9} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$$

$$\vec{R} \times \vec{Q} = \frac{1}{9}(-3\hat{i} - 3\hat{j} - 3\hat{k}) = -\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$$

$$|\vec{R} \times \vec{Q}|^2 = \frac{1}{3}$$

$$\vec{P} \cdot \vec{Q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$$

$$3(\vec{P} \cdot \vec{Q})^2 - \lambda |\vec{R} \times \vec{Q}|^2 = 0$$

$$\Rightarrow \frac{1}{3} - \lambda \times \frac{1}{3} = 0 \Rightarrow \lambda = 1$$

25. Points $A(2, 4, 0), B(3, 1, 8), C(3, 1, -3), D(7, -3, 4)$ are four points. The projection of line segment AB on line CD is

Answer: (8)

Solution:

$$\overrightarrow{AB} = \hat{i} - 3\hat{j} + 8\hat{k}$$

$$\overrightarrow{CD} = 4\hat{i} - 4\hat{j} + 7\hat{k}$$

$$\text{Projection of } \overrightarrow{AB} \text{ on } \overrightarrow{CD} \text{ is } = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{CD}|} = \frac{4+12+56}{\sqrt{4^2+4^2+7^2}} = \frac{72}{9} = 8$$