

JEE Main 2020 Paper

Date: 9th January 2020 (Shift 2)

Time: 2:30 P.M. to 5:30 P.M.

Subject: Mathematics

1. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ and $a - 2b + c = 1$ then

a. $f(-50) = -1$

b. $f(50) = 1$

c. $f(50) = -501$

d. $f(50) = 501$

Answer: (a)

Solution:

$$\text{Given } f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$$a - 2b + c = 1$$

$$\text{Applying } R_1 \rightarrow R_1 - 2R_2 + R_3$$

$$f(x) = \begin{vmatrix} a - 2b + c & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$$\text{Using } a - 2b + c = 1$$

$$\therefore f(x) = (x+3)^2 - (x+2)(x+4)$$

$$\Rightarrow f(x) = 1$$

$$\Rightarrow f(50) = 1$$

$$\Rightarrow f(-50) = 1$$

2. If $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x - \frac{1}{2}\right)^2$ then find the area bounded by $f(x)$ and $g(x)$ from $x = \frac{1}{2}$ to $x = \frac{\sqrt{3}}{2}$.

a. $\frac{\sqrt{3}}{2} - \frac{1}{3}$

b. $\frac{\sqrt{3}}{4} + \frac{1}{3}$

c. $2\sqrt{3}$

d. $3\sqrt{3}$

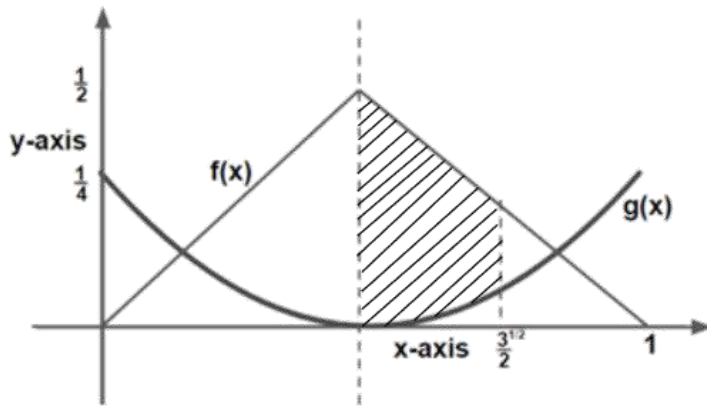
Answer: (a)

Solution:

$$\text{Given } f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1 - x & \frac{1}{2} < x < 1 \end{cases}$$

$$g(x) = \left(x - \frac{1}{2}\right)^2$$

The area between $f(x)$ and $g(x)$ from $x = \frac{1}{2}$ to $x = \frac{\sqrt{3}}{2}$:



Points of intersection of $f(x)$ and $g(x)$:

$$1 - x = \left(x - \frac{1}{2}\right)^2$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

$$\begin{aligned} \text{Required area} &= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} (f(x) - g(x)) dx \\ &= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left(1 - x - \left(x - \frac{1}{2}\right)^2\right) dx \\ &= \left[x - \frac{x^2}{2} - \frac{1}{3}\left(x - \frac{1}{2}\right)^3\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \\ &= \frac{\sqrt{3}}{4} - \frac{1}{3} \end{aligned}$$

- [illegible]

Solution:

Truth table:

p	q	$\sim q$	$(p \wedge \sim q)$	$p \rightarrow (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

4. $\int \frac{d\theta}{\cos^2 \theta (\sec 2\theta + \tan 2\theta)} = \lambda \tan \theta + 2 \log f(x) + c$ then ordered pair $(\lambda, f(x))$ is

a. $(1, 1 + \tan \theta)$

b. $(1, 1 - \tan \theta)$

c. $(-1, 1 + \tan \theta)$

d. $(-1, 1 - \tan \theta)$

Solution:

$$I = \int \frac{\sec^2 \theta d\theta}{\left(\frac{1+\tan^2 \theta}{1-\tan^2 \theta}\right) + \left(\frac{2 \tan \theta}{1-\tan^2 \theta}\right)}$$

$$I = \int \frac{(1 - \tan^2 \theta)(\sec^2 \theta) d\theta}{(1 + \tan \theta)^2}$$

$$\text{Let } \tan \theta = k \Rightarrow \sec^2 \theta \, d\theta = dk$$

$$I = \int \frac{(1-k^2)}{(1+k)^2} dk = \int \frac{(1-k)}{(1+k)} dk$$

$$I = \left(\frac{2}{1+k} - 1 \right) dk$$

$$I = 2 \ln|1 + k| - k + c$$

$$I = 2 \ln|1 + \tan \theta| - \tan \theta + c$$

$$\therefore \lambda = -1, f(x) = |1 + \tan \theta|$$

- Answer:** (a)

a_n is a positive term of GP.

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \dots + a_{201}$$

$$200 = \frac{ar^2(r^{200}-1)}{r^2-1} \quad \dots (1)$$

$$100 = ar + ar^3 + \dots + ar^{199}$$

$$100 = \frac{ar(r^{200}-1)}{r^2-1} \quad \dots (2)$$

And $\sum_{n=1}^{100} a_{2n+1} + \sum_{n=1}^{100} a_{2n} = 300$

$$\Rightarrow ar + ar^2 + ar^3 + \dots + ar^{200} = 300$$

$$\Rightarrow r(a + ar + ar^2 + \dots + ar^{199}) = 300$$

$$\Rightarrow 2(a_1 + a_2 + a_3 + \dots + a_{200}) = 300$$

$$\sum_{n=1}^{200} a_n = 150$$

- a. $\sqrt{8}$

b. $\sqrt{7}$

c. $\sqrt{\frac{17}{2}}$

d. $\sqrt{10}$

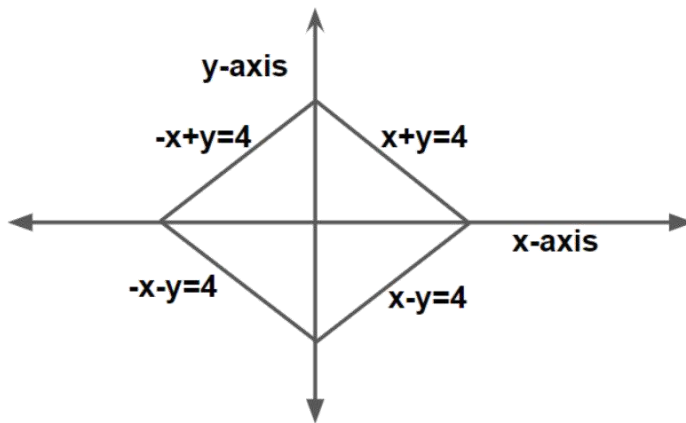
Answer: (b)

Solution:

$$|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 4$$

$$\text{Let } z = x + iy$$

$$\Rightarrow |x| + |y| = 4$$



$\therefore z$ lies on the rhombus.

Maximum value of $|z| = 4$ when $z = 4, -4, 4i, -4i$

Minimum value of $|z| = 2\sqrt{2}$ when $z = 2 \pm 2i, \pm 2 + 2i$

$$|z| \in [2\sqrt{2}, 4]$$

$$|z| \in [\sqrt{8}, \sqrt{16}]$$

$$|z| \neq \sqrt{7}$$

7. $f(x) : [0, 5] \rightarrow \mathbb{R}, F(x) = \int_0^x x^2 g(x) dx, f(1) = 3, g(x) = \int_1^x f(t) dt$ then correct choice is

- a. $F(x)$ has no critical point
- b. $F(x)$ has local minimum at $x = 1$
- c. $F(x)$ has local maximum at $x = 1$
- d. $F(x)$ has point of inflection at $x = 1$

Answer: (b)

Solution:

$$F(x) = x^2 g(x)$$

$$\text{Put } x = 1$$

$$\Rightarrow F(1) = g(1) = 0 \quad \dots (1)$$

$$\text{Now } F''(x) = 2xg(x) + g'(x)x^2$$

$$F''(1) = 2g(1) + g'(1) \quad \{\because g'(x) = f(x)\}$$

$$F''(1) = f(1) = 3 \quad \dots (2)$$

From (1) and (2), $F(x)$ has local minimum at $x = 1$

8. Let $x = 2 \sin \theta - \sin 2\theta$ and $y = 2 \cos \theta - \cos 2\theta$, then the value of $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is

a. $\frac{3}{8}$

b. $\frac{5}{8}$

c. $\frac{7}{8}$

d. $\frac{3}{2}$

Answer: (a)

Solution:

$$\frac{dx}{d\theta} = 2 \cos \theta - 2 \cos 2\theta$$

$$\frac{dy}{d\theta} = -2 \sin \theta + 2 \sin 2\theta$$

$$\frac{dy}{dx} = \frac{2 \cos \frac{3\theta}{2} \sin \frac{\theta}{2}}{2 \sin \frac{3\theta}{2} \sin \frac{\theta}{2}}$$

$$\frac{dy}{dx} = \cot \frac{3\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{2} \operatorname{cosec}^2 \frac{3\theta}{2} \frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \left(-\frac{3}{2} \operatorname{cosec}^2 \frac{3\theta}{2} \right) \frac{1}{(2 \cos \theta - 2 \cos 2\theta)}$$

$$\left. \frac{d^2y}{dx^2} \right|_{\theta=\pi} = \frac{3}{8}$$

9. If $f(x)$ and $g(x)$ are continuous functions, $f \circ g$ is identity function, $g'(b) = 5$ and $g(b) = a$, then $f'(a)$ is

a. $\frac{3}{5}$

b. 5

c. $\frac{2}{5}$

d. $\frac{1}{5}$

Answer: (d)

Solution:

$$f(g(x)) = x$$

$$f'(g(x))g'(x) = 1$$

$$\text{Put } x = b$$

$$f'(g(b))g'(b) = 1$$

$$f'(a) \times 5 = 1$$

$$f'(a) = \frac{1}{5}$$

10. Let $x + 6y = 8$ is tangent to standard ellipse where minor axis is $\frac{4}{\sqrt{3}}$, then eccentricity of ellipse is

a. $\frac{1}{4} \sqrt{\frac{11}{12}}$

b. $\frac{1}{4} \sqrt{\frac{11}{3}}$

c. $\sqrt{\frac{5}{6}}$

d. $\sqrt{\frac{11}{12}}$

Answer: (d)

Solution:

$$\text{If } 2b = \frac{4}{\sqrt{3}}$$

$$b = \frac{2}{\sqrt{3}}$$

$$\text{Comparing } y = -\frac{x}{6} + \frac{8}{6} \text{ with } y = mx \pm \sqrt{a^2m^2 + b^2}$$

$$m = -\frac{1}{6} \text{ and } a^2m^2 + b^2 = \frac{16}{9}$$

$$\frac{a^2}{36} + \frac{4}{3} = \frac{16}{9}$$

$$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3}$$

$$\Rightarrow a^2 = 16$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{\frac{11}{12}}$$

11. If one end of focal chord of parabola $y^2 = 8x$ is $\left(\frac{1}{2}, -2\right)$, then the equation of tangent at the other end of this focal chord is

a. $x + 2y + 8 = 0$

b. $x - 2y = 8$

c. $x - 2y + 8 = 0$

d. $x + 2y = 8$

Answer: (c)

Solution:

Let PQ be the focal chord of the parabola $y^2 = 8x$

$$\Rightarrow P(t_1) = (2t_1^2, 4t_1) \text{ \& } Q(t_2) = (2t_2^2, 4t_2)$$

$$\Rightarrow t_1 t_2 = -1$$

$\therefore \left(\frac{1}{2}, -2\right)$ is one of the ends of the focal chord of the parabola

$$\text{Let } \left(\frac{1}{2}, -2\right) = (2t_2^2, 4t_2)$$

$$\Rightarrow t_2 = -\frac{1}{2}$$

$\Rightarrow$ Other end of focal chord will have parameter $t_1 = 2$

$\Rightarrow$ The co-ordinate of the other end of the focal chord will be $(8, 8)$

$\therefore$ The equation of the tangent will be given as $\rightarrow 8y = 4(x + 8)$

$$\Rightarrow 2y - x = 8$$

12. If $7x + 6y - 2z = 0$, $3x + 4y + 2z = 0$ & $x - 2y - 6z = 0$, then the system of equations has

a. No solution

b. Infinite non-trivial solution for $(x = 2z)$

c. Infinite non-trivial solution for $(y = 2z)$

d. Only trivial solution

Answer: (b)

Solution:

$$7x + 6y - 2z = 0$$

$$3x + 4y + 2z = 0$$

$$x - 2y - 6z = 0$$

As the system of equations are Homogeneous $\Rightarrow$ the system is consistent.

$$\Rightarrow \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0$$

$\Rightarrow$ Infinite solutions exist (both trivial and non-trivial solutions)

When $y = 2z$

Let's take $y = 2, z = 1$

When $(x, 2, 1)$ is substituted in the system of equations

$$\Rightarrow 7x + 10 = 0$$

$$3x + 10 = 0$$

$$x - 10 = 0 \text{ (which is not possible)}$$

$\therefore y = 2z \Rightarrow$ Infinite non-trivial solutions does not exist.

For $x = 2z$, let's take $x = 2, z = 1, y = y$

Substitute $(2, y, 1)$ in system of equations

$$\Rightarrow y = -2$$

$\therefore$ For each pair of (x, z) , we get a value of y .

Therefore, for $x = 2z$ infinite non-trivial solution exists.

13. If both the roots of the equation $ax^2 - 2bx + 5 = 0$ are α and of the equation $x^2 - 2bx - 10 = 0$ are α and β . Then the value of $\alpha^2 + \beta^2$

a. 15

b. 20

c. 25

d. 30

Answer: (c)

Solution:

$ax^2 - 2bx + 5 = 0$ has both roots as α

$$\Rightarrow 2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

$$\text{And } \alpha^2 = \frac{5}{a}$$

$$\Rightarrow b^2 = 5a(a \neq 0) \quad \dots (1)$$

$$\Rightarrow \alpha + \beta = 2b \text{ \& } \alpha\beta = -10$$

$$\alpha = \frac{b}{a} \text{ is also a root of } x^2 - 2bx - 10 = 0$$

$$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$$

$$\because b^2 = 5a \Rightarrow 5a - 10a^2 - 10a^2 = 0$$

$$\Rightarrow a = \frac{1}{4} \Rightarrow b^2 = \frac{5}{4}$$

$$\Rightarrow \alpha^2 = 20, \beta^2 = 5$$

$$\Rightarrow \alpha^2 + \beta^2 = 25$$

14. If $A = \{x: |x| < 2$ and $B = \{x: |x - 2| \geq 3\}$ then

a. $A \cap B = [-2, -1]$

c. $A - B = [-1, 2)$

b. $B - A = \mathbf{R} - (-2, 5)$

d. $A \cup B = \mathbf{R} - (2, 5)$

Answer: (b)

Solution:

$$A = \{x: x \in (-2, 2)\}$$

$$B = \{x: x \in (-\infty, -1] \cup [5, \infty)\}$$

$$A \cap B = \{x: x \in (-2, -1]\}$$

$$B - A = \{x: x \in (-\infty, -2] \cup [5, \infty)\}$$

$$A - B = \{x: x \in (-1, 2)\}$$

$$A \cup B = \{x: x \in (-\infty, 2) \cup [5, \infty)\}$$

15. The value of $P(x_i > 2)$ for the given probability distribution is

x_i	1	2	3	4	5
P_i	k^2	$2k$	k	$2k$	$5k^2$

a. $\frac{1}{36}$

b. $\frac{23}{36}$

c. $\frac{1}{6}$

d. $\frac{7}{12}$

Answer: (b)

Solution:

We know that $\sum_{x_i=1}^5 P_i = 1$

$$\Rightarrow k^2 + 2k + k + 2k + 5k^2 = 1$$

$$\Rightarrow k = -1, \frac{1}{6} \therefore k = \frac{1}{6}$$

$$P(x_i > 2) = P(x_i = 3) + P(x_i = 4) + P(x_i = 5)$$

$$= k + 2k + 5k^2 = \frac{23}{36}$$

16. Let the distance between the plane passing through lines $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{8}$ and $\frac{x+3}{2} = \frac{y+2}{1} = \frac{z-1}{\lambda}$ and the plane $23x - 10y - 2z + 48 = 0$ is $\frac{k}{\sqrt{633}}$, then the value of k is

a. 4

b. 3

c. 2

d. 1

Answer: (b)

Solution:

We find the point of intersection of the two lines, and the distance of given plane from the two lines is the distance of plane from the point of intersection.

$$\therefore (2p - 1, 2p + 3, 8p - 1) = (2q - 3, q - 2, \lambda q + 1)$$

$$p = -2 \text{ and } q = -1$$

$$\lambda = 18$$

Point of intersection is $(-5, -3, -17)$

$$\therefore \frac{k}{\sqrt{633}} = \left| \frac{-115 + 30 + 34 + 48}{\sqrt{633}} \right| \Rightarrow k = 3$$

17. Let $x = \sum_{n=0}^{\infty} (-1)^n (\tan \theta)^{2n}$ and $y = \sum_{n=0}^{\infty} (\cos \theta)^{2n}$, where $\theta \in \left(0, \frac{\pi}{4}\right)$, then

a. $y(x - 1) = 1$

b. $y(1 - x) = 1$

c. $x(y + 1) = 1$

d. $y(1 + x) = 1$

Answer: (b)

Solution:

$$y = 1 + \cos^2 \theta + \cos^4 \theta + \dots$$

$$\Rightarrow y = \frac{1}{1 - \cos^2 \theta} \Rightarrow \frac{1}{y} = \sin^2 \theta$$

$$x = 1 - \tan^2 \theta + \tan^4 \theta - \dots$$

$$\Rightarrow x = \frac{1}{1 - (-\tan^2 \theta)} = \cos^2 \theta$$

$$\therefore x + \frac{1}{y} = 1 \Rightarrow y(1 - x) = 1$$

18. If $\lim_{x \rightarrow 0} x \left[\frac{4}{x} \right] = A$, then the value of x at which $f(x) = [x^2] \sin \pi x$ is discontinuous at (where $[.]$ denotes greatest integer function)

a. $\sqrt{A + 5}$

b. $\sqrt{A + 1}$

c. $\sqrt{A + 21}$

d. $\sqrt{A}$

Answer: (b)

Solution:

$$f(x) = [x^2] \sin \pi x$$

It is continuous $\forall x \in \mathbf{Z}$ as $\sin \pi x \rightarrow 0$ as $x \rightarrow \mathbf{Z}$.

$f(x)$ is discontinuous at points where $[x^2]$ is discontinuous i.e. $x^2 \in \mathbf{Z}$ with an exception that $f(x)$ is continuous as x is an integer.

$\therefore$ Points of discontinuity for $f(x)$ would be at

$$x = \pm\sqrt{2}, \pm\sqrt{3}, \pm\sqrt{5}, \dots$$

Also, it is given that $\lim_{x \rightarrow 0} x \left[\frac{4}{x} \right] = A$ (indeterminate form $(0 \times \infty)$)

$$\Rightarrow \lim_{x \rightarrow 0} x \left(\frac{4}{x} - \left\{ \frac{4}{x} \right\} \right) = A$$

$$\Rightarrow 4 - \lim_{x \rightarrow 0} \left\{ \frac{4}{x} \right\} = A$$

$$\Rightarrow A = 4$$

$$\sqrt{A+5} = 3$$

$$\sqrt{A+1} = \sqrt{5}$$

$$\sqrt{A+21} = 5$$

$$\sqrt{A} = 2$$

$\therefore$ Points of discontinuity for $f(x)$ is $x = \sqrt{5}$

19. Circles $(x - 0)^2 + (y - 4)^2 = k$ and $(x - 3)^2 + (y - 0)^2 = 1^2$ touch each other. The maximum value of k is _____.

Answer: (36)

Solution:

Two circles touch each other if $C_1 C_2 = |r_1 \pm r_2|$

$$\sqrt{k} + 1 = 5 \text{ or } |\sqrt{k} - 1| = 5$$

$$\Rightarrow k = 16 \text{ or } 36$$

Maximum value of k is 36

20. If ${}^{25}C_0 + 5{}^{25}C_1 + 9{}^{25}C_2 + \dots + 101{}^{25}C_{25} = 2^{25}k$, then the value of k is _____.

Answer: (51)

Solution:

$$S = {}^{25}C_0 + 5{}^{25}C_1 + 9{}^{25}C_2 + \dots + 97{}^{25}C_{24} + 101{}^{25}C_{25} = 2^{25}k \quad (1)$$

Reverse and apply property ${}^nC_r = {}^nC_{n-r}$ in all coefficients

$$S = 101{}^{25}C_0 + 97{}^{25}C_1 + \dots + 5{}^{25}C_{24} + {}^{25}C_{25} \quad (2)$$

Adding (1) and (2), we get

$$2S = 102[{}^{25}C_0 + {}^{25}C_1 + \dots + {}^{25}C_{25}]$$

$$S = 51 \times 2^{25}$$

$$\Rightarrow k = 51$$

21. Number of common terms in both the sequences 3, 7, 11, ... 407 and 2, 9, 16, ... 905 is _____.

Answer: (14)

Solution:

First common term is 23

Common difference = LCM(7, 4) = 28

$$23 + (n - 1)28 \leq 407$$

$$n - 1 \leq 13.71$$

$$n = 14$$

22. Let $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b} \cdot \vec{c} = 10$ and angle between $\vec{b}$ and $\vec{c}$ is equal to $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to $\vec{b} \times \vec{c}$, then the value of $|\vec{a} \times (\vec{b} \times \vec{c})|$ is

Answer: (30)

Solution:

$$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}| |\vec{b} \times \vec{c}| \sin \theta \quad \text{where } \theta \text{ is the angle between } \vec{a} \text{ and } \vec{b} \times \vec{c}$$

$$\theta = \frac{\pi}{2} \quad \text{given}$$

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} |\vec{b} \times \vec{c}| = \sqrt{3} |\vec{b}| |\vec{c}| \sin \frac{\pi}{3}$$

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} \times 5 \times |\vec{c}| \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \frac{15}{2} |\vec{c}|$$

$$\text{Now, } |\vec{b}| |\vec{c}| \cos \theta = 10$$

$$5 |\vec{c}| \frac{1}{2} = 10$$

$$|\vec{c}| = 4$$

23. If minimum value of term free from x for $\left(\frac{x}{\sin \theta} + \frac{1}{x \cos \theta}\right)^{16}$ is L_1 in $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ and L_2 in $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$, the value of $\frac{L_2}{L_1}$ is

Answer: (16)

Solution:

$$T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin \theta} \right)^{16-r} \left(\frac{1}{x \cos \theta} \right)^r$$

For term independent of x ,

$$16 - 2r = 0 \Rightarrow r = 8$$

$$T_9 = {}^{16}C_8 \left(\frac{1}{\sin \theta \cos \theta} \right)^8 = {}^{16}C_8 2^8 \left(\frac{1}{\sin 2\theta} \right)^8$$

$$L_1 = {}^{16}C_8 2^8 \quad \text{at } \theta = \frac{\pi}{4}$$

$$L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}} \right)^8} = {}^{16}C_8 2^{12} \quad \text{at } \theta = \frac{\pi}{8}$$

$$\frac{L_2}{L_1} = 16$$