

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019**(Held On Thursday 10th JANUARY, 2019) TIME : 2 : 30 PM To 5 : 30 PM****MATHEMATICS**

1. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I[z]$

respectively denote the real and imaginary parts of z , then :

- (1) $R(z) > 0$ and $I(z) > 0$
 (2) $R(z) < 0$ and $I(z) > 0$
 (3) $R(z) = -3$
 (4) $I(z) = 0$

Ans. (4)

Sol. $z = \left(\frac{\sqrt{3}+i}{2}\right)^5 + \left(\frac{\sqrt{3}-i}{2}\right)^5$

$$\begin{aligned} z &= \left(e^{i\pi/6}\right)^5 + \left(e^{-i\pi/6}\right)^5 \\ &= e^{i5\pi/6} + e^{-i5\pi/6} \\ &= \cos \frac{5\pi}{6} + i \frac{\sin 5\pi}{6} + \cos \left(\frac{-5\pi}{6}\right) + i \sin \left(\frac{-5\pi}{6}\right) \\ &= 2 \cos \frac{5\pi}{6} < 0 \end{aligned}$$

$$I(z) = 0 \text{ and } \operatorname{Re}(z) < 0$$

Option (4)

2. Let $a_1, a_2, a_3, \dots, a_{10}$ be in G.P. with $a_i > 0$ for $i = 1, 2, \dots, 10$ and S be the set of pairs (r, k) , $r, k \in \mathbb{N}$ (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S , is :

- (1) Infinitely many (2) 4
 (3) 10 (4) 2

Ans. (1)

Sol. Apply

$$C_3 \rightarrow C_3 - C_2$$

$$C_2 \rightarrow C_2 - C_1$$

We get $D = 0$

Option (1)

3. The positive value of λ for which the co-efficient of x^2 in the expression

$$x^2 \left(\sqrt{x} + \frac{\lambda}{x^2} \right)^{10} \text{ is } 720, \text{ is :}$$

- (1) $\sqrt{5}$ (2) 4
 (3) $2\sqrt{2}$ (4) 3

Ans. (2)

Sol. $x^2 \left({}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{\lambda}{x^2} \right)^r \right)$

$$x^2 \left[{}^{10}C_r (x)^{\frac{10-r}{2}} (\lambda)^r (x)^{-2r} \right]$$

$$x^2 \left[{}^{10}C_r \lambda^r x^{\frac{10-5r}{2}} \right]$$

$$\therefore r = 2$$

$$\text{Hence, } {}^{10}C_2 \lambda^2 = 720$$

$$\lambda^2 = 16$$

$$\lambda = \pm 4$$

Option (2)

4. The value of $\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \cdot \dots \cdot \cos \frac{\pi}{2^{10}} \cdot \sin \frac{\pi}{2^{10}}$ is :

- (1) $\frac{1}{256}$ (2) $\frac{1}{2}$
 (3) $\frac{1}{512}$ (4) $\frac{1}{1024}$

Ans. (3)

Sol. $2 \sin \frac{\pi}{2^{10}} \cos \frac{\pi}{2^{10}} \cdot \dots \cdot \cos \frac{\pi}{2^2}$

$$\frac{1}{2^9} \sin \frac{\pi}{2} = \frac{1}{512}$$

Option (3)

5. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$, where $[t]$

denotes the greatest integer less than or equal to t , is :

(1) $\frac{1}{12}(7\pi+5)$ (2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{1}{12}(7\pi-5)$ (4) $\frac{3}{20}(4\pi-3)$

Ans. (4)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$

$$= \int_{-\pi/2}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^0 \frac{dx}{-1-1+4}$$

$$+ \int_0^1 \frac{dx}{0+0+4} + \int_1^{\pi/2} \frac{dx}{1+0+4}$$

$$\int_{-\pi/2}^{-1} \frac{dx}{1} + \int_{-1}^0 \frac{dx}{2} + \int_0^1 \frac{dx}{4} + \int_1^{\pi/2} \frac{dx}{5}$$

$$\left(-1 + \frac{\pi}{2}\right) + \frac{1}{2}(0+1) + \frac{1}{4} + \frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$-1 + \frac{1}{2} + \frac{1}{4} - \frac{1}{5} + \frac{\pi}{2} + \frac{\pi}{10}$$

$$\frac{-20+10+5-4}{20} + \frac{6\pi}{10}$$

$$\frac{-9}{20} + \frac{3\pi}{5}$$

Option (4)

6. If the probability of hitting a target by a shooter, in any shot, is $1/3$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target

at least once is greater than $\frac{5}{6}$, is :

- (1) 6 (2) 5
(3) 4 (4) 3

Ans. (2)

Sol. $1 - {}^nC_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n > \frac{5}{6}$

$$\frac{1}{6} > \left(\frac{2}{3}\right)^n \Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$$

$$n_{\min} = 5 \Rightarrow \text{Option (2)}$$

7. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, \dots, x_5$ and -50 is equal to :

- (1) 582.5 (2) 507.5
(3) 586.5 (4) 509.5

Ans. (2)

Sol. $\bar{x} = 10 \Rightarrow \sum_{i=1}^5 x_i = 50$

$$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^5 x_i^2}{5} - (\bar{x})^2} = 8$$

$$\Rightarrow \sum_{i=1}^5 (x_i)^2 = 109$$

$$\begin{aligned} \text{variance} &= \frac{\sum_{i=1}^5 (x_i)^2 + (-50)^2}{6} - \left(\frac{\sum_{i=1}^5 x_i - 50}{6}\right)^2 \\ &= 507.5 \end{aligned}$$

Option (2)

8. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

- (1) $2\sqrt{11}$ (2) $3\sqrt{2}$
(3) $6\sqrt{3}$ (4) $8\sqrt{2}$

Ans. (3)

Sol. $x^2 = 4y$

$$x - \sqrt{2}y + 4\sqrt{2} = 0$$

Solving together we get

$$x^2 = 4 \left(\frac{x + 4\sqrt{2}}{\sqrt{2}} \right)$$

$$\sqrt{2}x^2 + 4x + 16\sqrt{2}$$

$$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$$

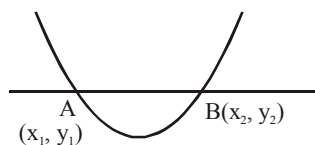
$$x_1 + x_2 = 2\sqrt{2}; \quad x_1x_2 = \frac{-16\sqrt{2}}{\sqrt{2}} = -16$$

Similarly,

$$(\sqrt{2}y - 4\sqrt{2})^2 = 4y$$

$$2y^2 + 32 - 16y = 4y$$

$$2y^2 - 20y + 32 = 0 \begin{cases} y_1 + y_2 = 10 \\ y_1 y_2 = 16 \end{cases}$$



$$\begin{aligned} \ell_{AB} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2\sqrt{2})^2 + 64 + (10)^2 - 4(16)} \\ &= \sqrt{8 + 64 + 100 - 64} \\ &= \sqrt{108} = 6\sqrt{3} \end{aligned}$$

Option (3)

9. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the

minimum value of $\frac{\det(A)}{b}$ is :

- (1) $\sqrt{3}$ (2) $-\sqrt{3}$
(3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

Ans. (4)

Sol. $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{bmatrix} \quad (b > 0)$

$$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1)$$

$$|A| = 2(b^2 + 2) - b^2 - 1$$

$$|A| = b^2 + 3$$

$$\frac{|A|}{b} = b + \frac{3}{b} \Rightarrow \frac{b + \frac{3}{b}}{2} \geq \sqrt{3}$$

$$b + \frac{3}{b} \geq 2\sqrt{3}$$

Option (4)

10. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1,e) also passes through the point :

- (1) $\left(\frac{4}{3}, 2e\right)$ (2) $(2, 3e)$
(3) $\left(\frac{5}{3}, 2e\right)$ (4) $(3, 6e)$

Ans. (1)

Sol. $y = xe^{x^2}$

$$\frac{dy}{dx} \Big|_{(1,e)} = \left(e \cdot e^{x^2} \cdot 2x + e^{x^2} \right) \Big|_{(1,e)} = 2 \cdot e + e = 3e$$

$$T : y - e = 3e(x - 1)$$

$$y = 3ex - 3e + e$$

$$y = (3e)x - 2e$$

$$\left(\frac{4}{3}, 2e\right) \text{ lies on it}$$

Option (1)

11. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$$x + 3y + 7z = 0$$

$$-x + 4y + 7z = 0$$

$$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$$

has a non-trivial solution, is :

- (1) One (2) Three
(3) Four (4) Two

Ans. (4)

Sol.
$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$$\begin{aligned} (8 - 7 \cos 2\theta) - 3(-2 - 7 \sin 3\theta) \\ + 7(-\cos 2\theta - 4 \sin 3\theta) &= 0 \\ 14 - 7 \cos 2\theta + 21 \sin 3\theta - 7 \cos 2\theta \\ - 28 \sin 3\theta &= 0 \\ 14 - 7 \sin 3\theta - 14 \cos 2\theta &= 0 \\ 14 - 7(3 \sin \theta - 4 \sin^3 \theta) - 14(1 - 2 \sin^2 \theta) &= 0 \\ -21 \sin \theta + 28 \sin^3 \theta + 28 \sin^2 \theta &= 0 \\ 7 \sin \theta [-3 + 4 \sin^2 \theta + 4 \sin \theta] &= 0 \\ \sin \theta, \\ 4 \sin^2 \theta + 6 \sin \theta - 2 \sin \theta - 3 &= 0 \\ 2 \sin \theta(2 \sin \theta + 3) - 1(2 \sin \theta + 3) &= 0 \end{aligned}$$

$$\sin \theta = \frac{-3}{2}; \quad \sin \theta = \frac{1}{2}$$

Hence, 2 solutions in $(0, \pi)$

Option (4)

12. If $\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt$, then $f'(1/2)$ is :

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$

(3) $\frac{18}{25}$ (4) $\frac{4}{5}$

Ans. (2)

Sol.
$$\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt \quad f'\left(\frac{1}{2}\right) = ?$$

Differentiate w.r.t. 'x'

$$f(x) = 2x + 0 - x^2 f(x)$$

$$f(x) = \frac{2x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$$

$$f'(x) = \frac{2x^2 - 4x^2 + 2}{(1+x^2)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{2 - 2\left(\frac{1}{4}\right)}{\left(1 + \frac{1}{4}\right)^2} = \frac{\left(\frac{3}{2}\right)}{\frac{25}{16}} = \frac{48}{50} = \frac{24}{25}$$

Option (2)

13. Let $f : (-1, 1) \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = \max\{-|x|, -\sqrt{1-x^2}\}.$$

If K be the set of all points at which f is not differentiable, then K has exactly :

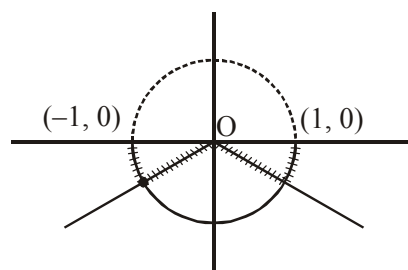
(1) Three elements (2) One element

(3) Five elements (4) Two elements

Ans. (1)

Sol. $f : (-1, 1) \rightarrow \mathbb{R}$

$$f(x) = \max\{-|x|, -\sqrt{1-x^2}\}$$



Non-derivable at 3 points in $(-1, 1)$

Option (1)

14. Let $S = \left\{ (x, y) \in \mathbb{R}^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1 \right\}$, where

$r \neq \pm 1$. Then S represents :

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$,

where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$,

where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$,

when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$,

when $r > 1$

Ans. (4)

Sol. $\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$

for $r > 1$, $\frac{y^2}{1+r} + \frac{x^2}{r-1} = 1$

$$e = \sqrt{1 - \left(\frac{r-1}{r+1}\right)}$$

$$= \sqrt{\frac{(r+1) - (r-1)}{(r+1)}}$$

$$= \sqrt{\frac{2}{r+1}} = \sqrt{\frac{2}{r+1}}$$

Option (4)

- 15.** If $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K \left({}^{50}C_{25} \right)$, then K is equal to :

- (1) $2^{25} - 1$ (2) $(25)^2$ (3) 2^{25} (4) 2^{24}

Ans. (3)

Sol. $\sum_{r=0}^{25} {}^{50}C_r \cdot {}^{50-r}C_{25-r}$

$$= \sum_{r=0}^{25} \frac{50!}{r! (50-r)!} \times \frac{(50-r)!}{(25)! (25-r)!}$$

$$= \sum_{r=0}^{25} \frac{50!}{25! 25!} \times \frac{25!}{(25-r)! (r)!}$$

$$= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = \left(2^{25}\right) {}^{50}C_{25}$$

$\therefore K = 2^{25}$

Option (3)

- 16.** Let N be the set of natural numbers and two functions f and g be defined as $f, g : N \rightarrow N$

such that : $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$

and $g(n) = n - (-1)^n$. The fog is :

- (1) Both one-one and onto
(2) One-one but not onto
(3) Neither one-one nor onto
(4) onto but not one-one

Ans. (4)

Sol. $f(x) = \begin{cases} \frac{n+1}{2} & n \text{ is odd} \\ n/2 & n \text{ is even} \end{cases}$

$$g(x) = n - (-1)^n \begin{cases} n+1 ; & n \text{ is odd} \\ n-1 ; & n \text{ is even} \end{cases}$$

$$f(g(n)) = \begin{cases} \frac{n}{2}; & n \text{ is even} \\ \frac{n+1}{2}; & n \text{ is odd} \end{cases}$$

$\therefore$ many one but onto

Option (4)

- 17.** The values of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is :

- (1) 2 (2) $\frac{4}{9}$
(3) $\frac{15}{8}$ (4) 1

Ans. (1)

Sol. $\alpha + \beta = \lambda - 3$

$$\alpha\beta = 2 - \lambda$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (\lambda - 3)^2 - 2(2 - \lambda)$$

$$= \lambda^2 + 9 - 6\lambda - 4 + 2\lambda$$

$$= \lambda^2 - 4\lambda + 5$$

$$= (\lambda - 2)^2 + 1$$

$$\therefore \lambda = 2$$

Option (1)

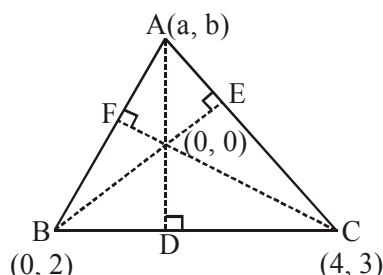
- 18.** Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

- (1) Fourth
(2) Second
(3) Third
(4) First

Ans. (2)

Sol. $m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right) \times \left(\frac{b-0}{a-0}\right) = -1$

$\Rightarrow b + 4a = 0 \dots\dots(i)$



$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{b-2}{a-0}\right) \times \left(\frac{3}{4}\right) = -1$

$\Rightarrow 3b - 6 = -4a \Rightarrow 4a + 3b = 6 \dots\dots(ii)$

From (i) and (ii)

$a = \frac{-3}{4}, b = 3$

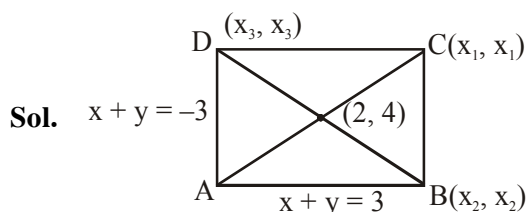
$\therefore$ IInd quadrant.

Option (2)

- 19.** Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2,4), then one of its vertex is :

- (1) (2,6) (2) (2,1)
(3) (3,5) (4) (3,6)

Ans. (4)



Sol. $x + y = -3$

Solving $x + y = 3$ and $x - y = -3$ $\Rightarrow A(0, 3)$

$\frac{x_1 + 0}{2} = 2; x_1 = 4$ similarly $y_1 = 5$

$C \Rightarrow (4, 5)$

Now equation of BC is $x - y = -1$

and equation of CD is $x + y = 9$

Solving $x + y = 9$ and $x - y = -3$

Point D is (3, 6)

Option (4)

- 20.** Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be

two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors

$\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

- (1) -3 (2) 4
(3) 3 (4) -4

Ans. (4)

Sol. $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$

$\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$

$\frac{\lambda - 2}{4\lambda - 2} = \frac{1}{3}$

$3\lambda - 6 = 4\lambda - 2$

$\boxed{\lambda = -4}$

$\therefore$ Option (4)

- 21.** The value of $\cot\left(\sum_{n=1}^{19} \cot^{-1}\left(1 + \sum_{p=1}^n 2p\right)\right)$ is :

- (1) $\frac{22}{23}$ (2) $\frac{23}{22}$ (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

Ans. (3)

Sol. $\cot\left(\sum_{n=1}^{19} \cot^{-1}(1 + n(n+1))\right)$

$\cot\left(\sum_{n=1}^{19} \cot^{-1}(n^2 + n + 1)\right) = \cot\left(\sum_{n=1}^{19} \tan^{-1} \frac{1}{1 + n(n+1)}\right)$

$\sum_{n=1}^{19} (\tan^{-1}(n+1) - \tan^{-1} n)$

$\cot(\tan^{-1} 20 - \tan^{-1} 1) = \frac{\cot A \cot \beta + 1}{\cot \beta - \cot A}$

(Where $\tan A = 20$, $\tan B = 1$) $\frac{1\left(\frac{1}{20}\right) + 1}{1 - \frac{1}{20}} = \frac{21}{19}$

$\therefore$ Option (3)

- 22.** With the usual notation, in ΔABC , if

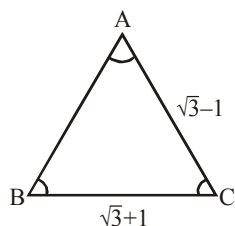
$\angle A + \angle B = 120^\circ$, $a = \sqrt{3} + 1$ and $b = \sqrt{3} - 1$,

then the ratio $\angle A : \angle B$, is :

- (1) 7 : 1 (2) 5 : 3
(3) 9 : 7 (4) 3 : 1

Ans. (1)

Sol. $A + B = 120^\circ$



$$\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \left(\frac{C}{2} \right)$$

$$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})} \cot(30^\circ) = \frac{1}{\sqrt{3}} \cdot \sqrt{3} = 1$$

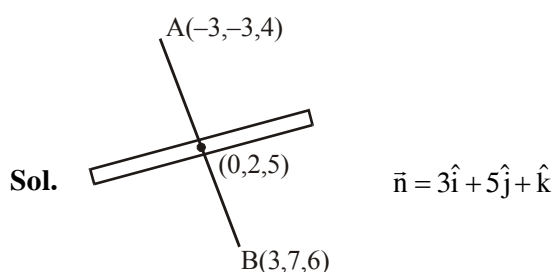
$$\frac{A-B}{2} = 45^\circ \quad \Rightarrow \quad \begin{aligned} A-B &= 90^\circ \\ A+B &= 120^\circ \\ \hline 2A &= 210^\circ \\ A &= 105^\circ \\ B &= 15^\circ \end{aligned}$$

$\therefore$ Option (1)

23. The plane which bisects the line segment joining the points $(-3, -3, 4)$ and $(3, 7, 6)$ at right angles, passes through which one of the following points ?

- (1) $(4, -1, 7)$ (2) $(4, 1, -2)$
 (3) $(-2, 3, 5)$ (4) $(2, 1, 3)$

Ans. (2)



Sol.

$$p : 3(x - 0) + 5(y - 2) + 1(z - 5) = 0$$

$$3x + 5y + z = 15$$

$\therefore$ Option (2)

24. Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ?

- (1) $(P \wedge Q) \vee (\sim R)$
 (2) $(\sim P) \wedge (\sim Q \wedge R)$
 (3) $(\sim P) \vee (Q \wedge R)$
 (4) $P \vee (\sim Q \wedge R)$

Ans. (4)

Sol. It is obvious

$\therefore$ Option (4)

25. On which of the following lines lies the point

of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$

and the plane, $x + y + z = 2$?

- (1) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$
 (2) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$
 (3) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$
 (4) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

Ans. (3)

Sol. General point on the given line is

$$x = 2\lambda + 4$$

$$y = 2\lambda + 5$$

$$z = \lambda + 3$$

Solving with plane,

$$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$$

$$5\lambda + 12 = 2$$

$$5\lambda = -10$$

$$\boxed{\lambda = -2}$$

$\therefore$ Option (3)

26. Let f be a differentiable function such that

$$f'(x) = 7 - \frac{3f(x)}{4x}, (x > 0) \text{ and } f(1) \neq 4.$$

$$\text{Then } \lim_{x \rightarrow 0^+} xf\left(\frac{1}{x}\right):$$

(1) Exists and equals 4

(2) Does not exist

(3) Exist and equals 0

(4) Exists and equals $\frac{4}{7}$

Ans. (1)

Sol. $f'(x) = 7 - \frac{3f(x)}{4x} \quad (x > 0)$

Given $f(1) \neq 4 \quad \lim_{x \rightarrow 0^+} xf\left(\frac{1}{x}\right) = ?$

$$\frac{dy}{dx} + \frac{3y}{4x} = 7 \quad (\text{This is LDE})$$

$$\text{IF} = e^{\int \frac{3}{4x} dx} = e^{\frac{3}{4} \ln|x|} = x^{\frac{3}{4}}$$

$$y \cdot x^{\frac{3}{4}} = \int 7 \cdot x^{\frac{3}{4}} dx$$

$$y \cdot x^{\frac{3}{4}} = 7 \cdot \frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C$$

$$f(x) = 4x + C \cdot x^{-\frac{3}{4}}$$

$$f\left(\frac{1}{x}\right) = \frac{4}{x} + C \cdot x^{\frac{3}{4}}$$

$$\lim_{x \rightarrow 0^+} xf\left(\frac{1}{x}\right) = \lim_{x \rightarrow 0^+} \left(4 + C \cdot x^{\frac{7}{4}}\right) = 4$$

$\therefore$ Option (1)

27. A helicopter is flying along the curve given by $y - x^{3/2} = 7, (x \geq 0)$. A soldier positioned at the

point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter

when it is nearest to him. Then this nearest distance is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$

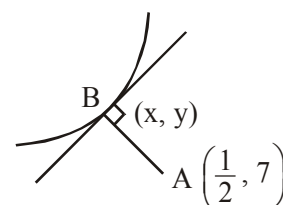
(3) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (4) $\frac{\sqrt{5}}{6}$

Ans. (3)

Sol. $y - x^{3/2} = 7 \quad (x \geq 0)$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$$

$$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right) = -1$$



$$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right) = -1$$

$$\frac{3}{2} \cdot x^2 = \frac{1}{2} - x$$

$$3x^2 = 1 - 2x$$

$$3x^2 + 2x - 1 = 0$$

$$3x^2 + 3x - x - 1 = 0$$

$$(x+1)(3x-1) = 0$$

$$\therefore x = -1 \text{ (rejected)}$$

$$x = \frac{1}{3}$$

$$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$$

$$\ell_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$$

$$= \sqrt{\frac{3+4}{9 \times 12}}$$

$$= \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}$$

Option (3)

28. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then $f(x)$ is equal to :

- (1) $-4x^3 - 1$ (2) $4x^3 + 1$
 (3) $-2x^3 - 1$ (4) $-2x^3 + 1$

Ans. (1)

Sol. $\int x^5 \cdot e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + c$

Put $x^3 = t$

$3x^2 dx = dt$

$\int x^3 \cdot e^{-4x^3} \cdot x^2 dx$

$\frac{1}{3} \int t \cdot e^{-4t} dt$

$\frac{1}{3} \left[t \cdot \frac{e^{-4t}}{-4} - \int \frac{e^{-4t}}{-4} dt \right]$

$-\frac{e^{-4t}}{48} [4t + 1] + c$

$-\frac{e^{-4x^3}}{48} [4x^3 + 1] + c$

$\therefore f(x) = -1 - 4x^3$

Option (1)

(From the given options (1) is most suitable)

29. The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xy dy = 0$ which passes through (1,1) is :

- (1) A circle with centre on the y-axis
 (2) A circle with centre on the x-axis
 (3) An ellipse with major axis along the y-axis
 (4) A hyperbola with transverse axis along the x-axis

Ans. (2)

Sol. $(x^2 - y^2) dx + 2xy dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

Solving we get,

$\int \frac{2v}{v^2 + 1} dv = \int -\frac{dx}{x}$

$\ln(v^2 + 1) = -\ln x + C$

$(y^2 + x^2) = Cx$

$1 + 1 = C \Rightarrow C = 2$

$\boxed{y^2 + x^2 = 2x}$

$\therefore$ Option (2)

30. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

- (1) 20 (2) 25
(3) 13 (4) -25

Ans. (2)

Sol. $3\left(\frac{1}{2}r^2 \cdot \sin 120^\circ\right) = 27\sqrt{3}$

$$\frac{r^2}{2} \cdot \frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{3}$$

$$r^2 = \frac{108}{3} = 36$$

$$\text{Radius} = \sqrt{25 + 36 - C} = \sqrt{36}$$

$$\boxed{C = 25}$$

$\therefore$ Option (2)

