

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let f be a differentiable function from $\mathbb{R}$ to $\mathbb{R}$ such that $|f(x) - f(y)| \leq 2|x - y|^{\frac{3}{2}}$, for all $x, y \in \mathbb{R}$. If

$f(0) = 1$ then $\int_0^1 f^2(x) dx$ is equal to

- (1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1

Ans. (4)

Sol. $|f(x) - f(y)| \leq 2|x - y|^{\frac{3}{2}}$

divide both sides by $|x - y|$

$$\left| \frac{f(x) - f(y)}{x - y} \right| \leq 2|x - y|^{\frac{1}{2}}$$

apply limit $x \rightarrow y$

$$|f'(y)| \leq 0 \Rightarrow f'(y) = 0 \Rightarrow f(y) = c \Rightarrow f(x) = 1$$

$$\int_0^1 1 \cdot dx = 1$$

2. If $\int_0^{\frac{\pi}{3}} \frac{\tan \theta}{\sqrt{2k \sec \theta}} d\theta = 1 - \frac{1}{\sqrt{2}}$, ($k > 0$), then the

value of k is :

- (1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1

Ans. (1)

Sol.
$$\frac{1}{\sqrt{2k}} \int_0^{\frac{\pi}{3}} \frac{\tan \theta}{\sqrt{\sec \theta}} d\theta = \frac{1}{\sqrt{2k}} \int_0^{\frac{\pi}{3}} \frac{\sin \theta}{\sqrt{\cos \theta}} d\theta$$

$$= -\frac{1}{\sqrt{2k}} 2\sqrt{\cos \theta} \Big|_0^{\frac{\pi}{3}} = -\frac{\sqrt{2}}{\sqrt{k}} \left(\frac{1}{\sqrt{2}} - 1 \right)$$

given it is $1 - \frac{1}{\sqrt{2}} \Rightarrow k = 2$

3. The coefficient of t^4 in the expansion of

$$\left(\frac{1-t^6}{1-t} \right)^3$$
 is

- (1) 12 (2) 15 (3) 10 (4) 14

Ans. (2)

Sol. $(1 - t^6)^3 (1 - t)^{-3}$

$$(1 - t^{18} - 3t^6 + 3t^{12}) (1 - t)^{-3}$$

$\Rightarrow$ coefficient of t^4 in $(1 - t)^{-3}$ is

$${}^{3+4-1}C_4 = {}^6C_2 = 15$$

4. For each $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x . Then

$$\lim_{x \rightarrow 0^+} \frac{x([x] + |x|) \sin[x]}{|x|}$$
 is equal to

- (1) $-\sin 1$ (2) 0 (3) 1 (4) $\sin 1$

Ans. (1)

Sol.
$$\lim_{x \rightarrow 0^+} \frac{x([x] + |x|) \sin[x]}{|x|}$$

$$x \rightarrow 0^+$$

$$[x] = -1 \Rightarrow \lim_{x \rightarrow 0^+} \frac{x(-x-1) \sin(-1)}{-x} = -\sin 1$$

$$|x| = -x$$

5. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval $[1, 5]$, then m lies in the interval:

- (1) (4, 5) (2) (3, 4) (3) (5, 6) (4) $(-5, -4)$

Ans. (Bonus/1)

Sol. $x^2 - mx + 4 = 0$

$$\alpha, \beta \in [1, 5]$$

$$(1) D > 0 \Rightarrow m^2 - 16 > 0$$

$$\Rightarrow m \in (-\infty, -4) \cup (4, \infty)$$

$$(2) f(1) \geq 0 \Rightarrow 5 - m \geq 0 \Rightarrow m \in (-\infty, 5]$$

$$(3) f(5) \geq 0 \Rightarrow 29 - 5m \geq 0 \Rightarrow m \in \left(-\infty, \frac{29}{5}\right]$$

$$(4) 1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2, 10)$$

$$\Rightarrow m \in (4, 5)$$

No option correct : Bonus

* If we consider $\alpha, \beta \in (1, 5)$ then option (1) is correct.

6. If

$$A = \begin{bmatrix} e^t & e^{-t} \cos t & e^{-t} \sin t \\ e^t & -e^{-t} \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\ e^t & 2e^{-t} \sin t & -2e^{-t} \cos t \end{bmatrix}$$

Then A is-

- (1) Invertible only if $t = \frac{\pi}{2}$
 (2) not invertible for any $t \in \mathbb{R}$
 (3) invertible for all $t \in \mathbb{R}$
 (4) invertible only if $t = \pi$

Ans. (3)

Sol. $|A| = e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2 \sin t & -2 \cos t \end{vmatrix}$

$$= e^{-t} [5 \cos^2 t + 5 \sin^2 t] \quad \forall t \in \mathbb{R}$$

$$= 5e^{-t} \neq 0 \quad \forall t \in \mathbb{R}$$

7. The area of the region

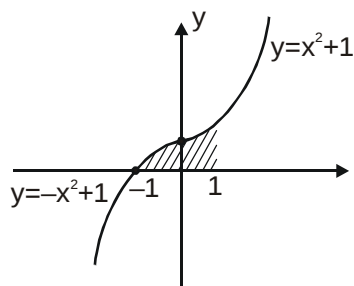
$$A = \{(x, y) : 0 \leq y \leq x|x| + 1 \text{ and } -1 \leq x \leq 1\}$$

in sq. units, is :

- (1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

Ans. (3)

Sol. The graph is as follows



$$\int_{-1}^0 (-x^2 + 1) dx + \int_0^1 (x^2 + 1) dx = 2$$

8. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then $\arg z$ is equal to:

- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) 0 (4) $\frac{\pi}{6}$

Ans. (1)

Sol. $z_0 = \omega$ or ω^2 (where ω is a non-real cube root of unity)

$$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$$

$$z = 3 + 3i$$

$$\Rightarrow \arg z = \frac{\pi}{4}$$

9. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

- (1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

Ans. (4)

Sol. Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = |\vec{a}|$

$$\Rightarrow b_1 + b_2 = 2 \quad \dots(1)$$

$$\text{and } (\vec{a} + \vec{b}) \perp \vec{c} \Rightarrow (\vec{a} + \vec{b}) \cdot \vec{c} = 0$$

$$\Rightarrow 5b_1 + b_2 = -10 \quad \dots(2)$$

$$\text{from (1) and (2)} \Rightarrow b_1 = -3 \text{ and } b_2 = 5$$

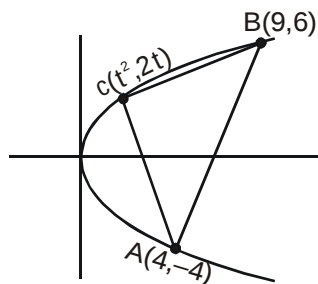
$$\text{then } |\vec{b}| = \sqrt{b_1^2 + b_2^2 + 2} = 6$$

10. Let A(4, -4) and B(9, 6) be points on the parabola, $y^2 + 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of $\triangle ACB$ is maximum. Then, the area (in sq. units) of $\triangle ACB$, is:

- (1) $31\frac{3}{4}$ (2) 32 (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

Ans. (4)

Sol.



$$\text{Area} = 5|t^2 - t - 6| = 5 \left| \left(t - \frac{1}{2}\right)^2 - \frac{25}{4} \right|$$

is maximum if $t = \frac{1}{2}$

11. The logical statement

$[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)]$ is equivalent to:

- (1) $(p \wedge r) \wedge \sim q$ (2) $(\sim p \wedge \sim q) \wedge r$
 (3) $\sim p \vee r$ (4) $(p \wedge \sim q) \vee r$

Ans. (1)

Sol. $s[\sim(\sim p \vee q) \wedge (p \wedge r)] \cap (\sim q \wedge r)$

$$\equiv [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$$

$$\equiv [p \wedge (\sim q \vee r)] \wedge (\sim q \wedge r)$$

$$\equiv p \wedge (\sim q \wedge r)$$

$$\equiv (p \wedge r) \wedge \sim q$$

12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

- (1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

Ans. (2)

Sol. E_1 : Event of drawing a Red ball and placing a green ball in the bag

E_2 : Event of drawing a green ball and placing a red ball in the bag

E : Event of drawing a red ball in second draw

$$P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$$

$$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49}$$

13. If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is

- (1) 2 (2) 1
 (3) 3 (4) 4

Ans. (1)

Sol. $\sin x - \sin 2x + \sin 3x = 0$

$$\Rightarrow (\sin x + \sin 3x) - \sin 2x = 0$$

$$\Rightarrow 2 \sin x \cdot \cos x - \sin 2x = 0$$

$$\Rightarrow \sin 2x (2 \cos x - 1) = 0$$

$$\Rightarrow \sin 2x = 0 \text{ or } \cos x = \frac{1}{2}$$

$$\Rightarrow x = 0, \frac{\pi}{3}$$

14. The equation of the plane containing the straight

line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{2} \text{ and } \frac{x}{4} = \frac{y}{2} = \frac{z}{3} \text{ is:}$$

- (1) $x + 2y - 2z = 0$ (2) $x - 2y + z = 0$
 (3) $5x + 2y - 4z = 0$ (4) $3x + 2y - 3z = 0$

Ans. (2)

Sol. Vector along the normal to the plane containing the lines

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{2} \text{ and } \frac{x}{4} = \frac{y}{2} = \frac{z}{3}$$

$$\text{is } (8\hat{i} - \hat{j} - 10\hat{k})$$

vector perpendicular to the vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$

$$\text{and } 8\hat{i} - \hat{j} - 10\hat{k} \text{ is } 26\hat{i} - 52\hat{j} + 26\hat{k}$$

so, required plane is

$$26x - 52y + 26z = 0$$

$$x - 2y + z = 0$$

15. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at $(1,1)$, then the equation of its third side is :

$$(1) 122y - 26x - 1675 = 0$$

$$(2) 26x + 61y + 1675 = 0$$

$$(3) 122y + 26x + 1675 = 0$$

$$(4) 26x - 122y - 1675 = 0$$

Ans. (4)

Sol. Equation of AB is

$$3x - 2y + 6 = 0$$

equation of AC is

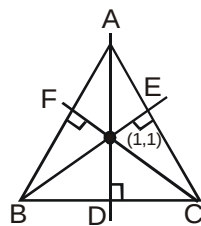
$$4x + 5y - 20 = 0$$

Equation of BE is

$$2x + 3y - 5 = 0$$

$$\text{Equation of CF is } 5x - 4y - 1 = 0$$

$$\Rightarrow \text{Equation of BC is } 26x - 122y = 1675$$



16. If $x = 3 \tan t$ and $y = 3 \sec t$, then the value of

$$\frac{d^2y}{dx^2} \text{ at } t = \frac{\pi}{4}, \text{ is:}$$

$$(1) \frac{3}{2\sqrt{2}} \quad (2) \frac{1}{3\sqrt{2}} \quad (3) \frac{1}{6} \quad (4) \frac{1}{6\sqrt{2}}$$

Ans. (4)

$$\text{Sol. } \frac{dx}{dt} = 3 \sec^2 t$$

$$\frac{dy}{dt} = 3 \sec t \tan t$$

$$\frac{dy}{dx} = \frac{\tan t}{\sec t} = \sin t$$

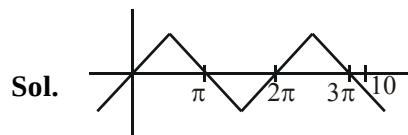
$$\frac{d^2y}{dx^2} = \cos t \frac{dt}{dx}$$

$$= \frac{\cos t}{3 \sec^2 t} = \frac{\cos^3 t}{3} = \frac{1}{3 \cdot 2\sqrt{2}} = \frac{1}{6\sqrt{2}}$$

17. If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to:

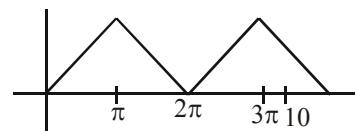
$$(1) \pi \quad (2) 7\pi \quad (3) 0 \quad (4) 10$$

Ans. (1)



Sol.

$$x = \sin^{-1}(\sin 10) = 3\pi - 10$$



$$y = \cos^{-1}(\cos 10) = 4\pi - 10$$

$$y - x = \pi$$

18. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then:

$$(1) cc' + a + a' = 0$$

$$(2) aa' + c + c' = 0$$

$$(3) ab' + bc' + 1 = 0$$

$$(4) bb' + cc' + 1 = 0$$

Ans. (2)

$$\text{Sol. Line } x = ay + b, z = cy + d \Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

$$\text{Line } x = a'z + b', y = c'z + d'$$

$$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$$

Given both the lines are perpendicular

$$\Rightarrow aa' + c' + c = 0$$

19. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is :

$$(1) 2 \quad (2) 5 \quad (3) 3 \quad (4) 4$$

Ans. (3)

$$\text{Sol. } 6x^2 - 11x + \alpha = 0$$

given roots are rational

$$\Rightarrow D \text{ must be perfect square}$$

$$\Rightarrow 121 - 24\alpha = \lambda^2$$

$$\Rightarrow \text{maximum value of } \alpha \text{ is } 5$$

$$\alpha = 1 \Rightarrow \lambda \notin I$$

$$\alpha = 2 \Rightarrow \lambda \notin I$$

$$\alpha = 3 \Rightarrow \lambda \in I$$

$$\alpha = 4 \Rightarrow \lambda \in I$$

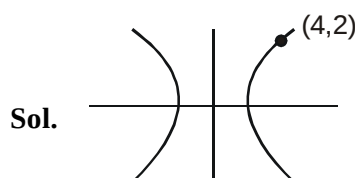
$$\alpha = 5 \Rightarrow \lambda \in I$$

$$\Rightarrow 3 \text{ integral values}$$

20. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

- (1) $\frac{2}{\sqrt{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{3}$ (4) 2

Ans. (1)



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$2a = 4 \quad a = 2$$

$$\frac{x^2}{4} - \frac{y^2}{b^2} = 1$$

Passes through (4,2)

$$4 - \frac{4}{b^2} = 1 \Rightarrow b^2 = \frac{4}{3} \Rightarrow e = \frac{2}{\sqrt{3}}$$

21. Let $A = \{x \in \mathbb{R} : x \text{ is not a positive integer}\}$

Define a function $f : A \rightarrow \mathbb{R}$ as $f(x) = \frac{2x}{x-1}$ then f is

- (1) injective but not surjective
(2) not injective
(3) surjective but not injective
(4) neither injective nor surjective

Ans. (1)

Sol. $f(x) = 2\left(1 + \frac{1}{x-1}\right)$

$$f'(x) = -\frac{2}{(x-1)^2}$$

$\Rightarrow f$ is one-one but not onto

22. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx, (x \geq 0)$ and $f(0) = 0$, then the value of $f(1)$ is :

- (1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. (4)

Sol.
$$\int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$$

$$= \int \frac{5x^{-6} + 7x^{-8}}{\left(\frac{1}{x^7} + \frac{1}{x^5} + 2\right)^2} dx = \frac{1}{2 + \frac{1}{x^5} + \frac{1}{x^7}} + C$$

As $f(0) = 0, f(x) = \frac{x^7}{2x^7 + x^2 + 1}$

$$f(1) = \frac{1}{4}$$

23. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x-4)^2 + (y-7)^2 = 36$ intersect at two distinct points, then:

- (1) $0 < r < 1$ (2) $1 < r < 11$
(3) $r > 11$ (4) $r = 11$

Ans. (2)

Sol. $x^2 + y^2 - 16x - 20y + 164 = r^2$

$$A(8,10), R_1 = r$$

$$(x-4)^2 + (y-7)^2 = 36$$

$$B(4,7), R_2 = 6$$

$$|R_1 - R_2| < AB < R_1 + R_2$$

$$\Rightarrow 1 < r < 11$$

24. Let S be the set of all triangles in the xy -plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is:

- (1) 9 (2) 18 (3) 32 (4) 36

Ans. (4)

Sol. Let $A(\alpha,0)$ and $B(0,\beta)$

be the vectors of the given triangle AOB

$$\Rightarrow |\alpha\beta| = 100$$

$\Rightarrow$ Number of triangles

$$= 4 \times (\text{number of divisors of } 100)$$

$$= 4 \times 9 = 36$$

25. The sum of the following series

$$1 + 6 + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9}$$

$$+ \frac{15(1^2 + 2^2 + \dots + 5^2)}{11} + \dots \text{ up to 15 terms, is:}$$

- (1) 7820 (2) 7830 (3) 7520 (4) 7510

Ans. (1)

Sol. $T_n = \frac{(3 + (n-1) \times 3)(1^2 + 2^2 + \dots + n^2)}{(2n+1)}$

$$T_n = \frac{3 \cdot \frac{n(n+1)(2n+1)}{6}}{2n+1} = \frac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2} \sum_{n=1}^{15} (n^3 + n^2) = \frac{1}{2} \left[\left(\frac{15(15+1)}{2} \right)^2 + \frac{15 \times 16 \times 31}{6} \right]$$

$$= 7820$$

- 26.** Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:

- (1) $\frac{1}{2}$ (2) 4
(3) 2 (4) $\frac{7}{13}$

Ans. (2)

Sol. $a = A + 6d$

$$b = A + 10d$$

$$c = A + 12d$$

a, b, c are in G.P.

$$\Rightarrow (A + 10d)^2 = (A + 6d)(A + 12d)$$

$$\Rightarrow \frac{A}{d} = -14$$

$$\frac{a}{c} = \frac{A + 6d}{A + 12d} = \frac{6 + \frac{A}{d}}{12 + \frac{A}{d}} = \frac{6 - 14}{12 - 14} = 4$$

- 27.** If the system of linear equations

$$x - 4y + 7z = g$$

$$3y - 5z = h$$

$$-2x + 5y - 9z = k$$

is consistent, then :

- (1) $g + h + k = 0$
(2) $2g + h + k = 0$
(3) $g + h + 2k = 0$
(4) $g + 2h + k = 0$

Ans. (2)

Sol. $P_1 \equiv x - 4y + 7z - g = 0$

$$P_2 \equiv 3x - 5y - h = 0$$

$$P_3 \equiv -2x + 5y - 9z - k = 0$$

Here $\Delta = 0$

$$2P_1 + P_2 + P_3 = 0 \text{ when } 2g + h + k = 0$$

- 28.** Let $f: [0, 1] \rightarrow \mathbb{R}$ be such that $f(xy) = f(x)f(y)$ for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the

differential equation, $\frac{dy}{dx} = f(x)$ with

$$y(0) = 1, \text{ then } y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) \text{ is equal to}$$

- (1) 4 (2) 3 (3) 5 (4) 2

Ans. (2)

Sol. $f(xy) = f(x)f(y)$

$$f(0) = 1 \text{ as } f(0) \neq 0$$

$$\Rightarrow f(x) = 1$$

$$\frac{dy}{dx} = f(x) = 1$$

$$\Rightarrow y = x + c$$

$$\text{At, } x = 0, y = 1 \Rightarrow c = 1$$

$$y = x + 1$$

$$\Rightarrow y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4} + 1 + \frac{3}{4} + 1 = 3$$

- 29.** A data consists of n observations:

$$x_1, x_2, \dots, x_n. \text{ If } \sum_{i=1}^n (x_i + 1)^2 = 9n \text{ and}$$

$$\sum_{i=1}^n (x_i - 1)^2 = 5n, \text{ then the standard deviation of}$$

this data is :

- (1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

Ans. (2)

Sol. $\sum (x_i + 1)^2 = 9n \quad \dots(1)$

$\sum (x_i - 1)^2 = 5n \quad \dots(2)$

$(1) + (2) \Rightarrow \sum (x_i^2 + 1) = 7n$

$\Rightarrow \frac{\sum x_i^2}{n} = 6$

$(1) - (2) \Rightarrow 4\sum x_i = 4n$

$\Rightarrow \sum x_i = n$

$\Rightarrow \frac{\sum x_i}{n} = 1$

$\Rightarrow \text{variance} = 6 - 1 = 5$

$\Rightarrow \text{Standard deviation} = \sqrt{5}$

- 30.** The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to :

- (1) 250 (2) 374 (3) 372 (4) 375

Ans. (2)

Sol.

a_1	a_2	a_3
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Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3
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2 ways for a_4

Number of numbers = 2×5^3

Required number = $5^3 + 2 \times 5^3 - 1$
= 374