

JEE(Advanced) – 2017 TEST PAPER WITH SOLUTION

(HELD ON SUNDAY 21st MAY, 2017)

MATHEMATICS

SECTION–1 : (Maximum Marks : 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :
Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks : +1 For darkening a bubble corresponding to **each correct option**, Provided NO incorrect option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : –2 In all other cases.
- for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened

37. Which of the following is(are) NOT the square of a 3×3 matrix with real entries ?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Ans. (A,B)

38. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k) , then which of the following is(are) possible value(s) of p , h and k ?

(A) $p = 5, h = 4, k = -3$

(B) $p = -1, h = 1, k = -3$

(C) $p = -2, h = 2, k = -4$

(D) $p = 2, h = 3, k = -4$

Ans. (D)

Sol. Equation of chord with mid point (h, k) :

$$k \cdot y - 16 \left(\frac{x+h}{2} \right) = k^2 - 16h$$

$$\Rightarrow 8x - ky + k^2 - 8h = 0$$

Comparing with $2x + y - p = 0$, we get

$$k = -4; 2h - p = 4$$

only (D) satisfies above relation.

39. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ?

- (A) $-1 - \sqrt{1-y^2}$ (B) $1 + \sqrt{1+y^2}$ (C) $1 - \sqrt{1+y^2}$ (D) $-1 + \sqrt{1-y^2}$

Ans. (A,D)

Sol. $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$ and $z = x + iy$

$$\therefore \operatorname{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$$

$$\Rightarrow \operatorname{Im}\left(\frac{(ax+b+iaiy)(x+1-iy)}{(x+1)^2+y^2}\right) = y$$

$$\Rightarrow -y(ax+b) + ay(x+1) = y((x+1)^2+y^2)$$

$$\Rightarrow (a-b)y = y((x+1)^2+y^2)$$

$$\because y \neq 0 \text{ and } a-b=1$$

$$\Rightarrow (x+1)^2+y^2=1$$

$$\Rightarrow x = -1 \pm \sqrt{1-y^2}$$

40. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then

(A) $P(X'|Y) = \frac{1}{2}$

(B) $P(X \cap Y) = \frac{1}{5}$

(C) $P(X \cup Y) = \frac{2}{5}$

(D) $P(Y) = \frac{4}{15}$

Ans. (A,D)

Sol. $P(x) = \frac{1}{3}$; $\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$; $\frac{P(Y \cap X)}{P(X)} = \frac{2}{5}$

from this information, we get

$$P(X \cap Y) = \frac{2}{15}; P(Y) = \frac{4}{15}$$

$$\therefore P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$$

$$P(\bar{X}/Y) = \frac{P(\bar{X} \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)}$$

$$\Rightarrow P(\bar{X}/Y) = 1 - \frac{2/15}{4/15} = \frac{1}{2}$$

41. Let $[x]$ be the greatest integer less than or equal to x . Then, at which of the following point(s) the function $f(x) = x \cos(\pi(x + [x]))$ is discontinuous ?

(A) $x = -1$ (B) $x = 0$ (C) $x = 2$ (D) $x = 1$

Ans. (A,C,D)

Sol. $f(x) = x \cos(\pi x + [x]\pi)$

$$\Rightarrow f(x) = (-1)^{[x]} x \cos \pi x.$$

Discontinuous at all integers except zero.

42. If $2x - y + 1 = 0$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be sides of a right angled triangle ?

(A) $2a, 4, 1$ (B) $2a, 8, 1$ (C) $a, 4, 1$ (D) $a, 4, 2$

Ans. (B,C,D)

Sol. The line $y = mx + c$ is tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $c^2 = a^2 m^2 - b^2$

$$\therefore (1)^2 = 4a^2 - 16 \Rightarrow a^2 = \frac{17}{4}$$

$$\Rightarrow a = \frac{\sqrt{17}}{2}$$

For option (A), sides are $\sqrt{17}, 4, 1$ ($\Rightarrow$ Right angled triangle)

For option (B), sides are $\sqrt{17}, 8, 1$ ($\Rightarrow$ Triangle is not possible)

For option (C), sides are $\frac{\sqrt{17}}{2}, 4, 1$ ($\Rightarrow$ Triangle is not possible)

For option (D), sides are $\frac{\sqrt{17}}{2}, 4, 2$ ($\Rightarrow$ Triangle exist but not right angled)

43. Let $f: \mathbb{R} \rightarrow (0,1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval $(0, 1)$?

(A) $e^x - \int_0^x f(t) \sin t dt$ (B) $x^9 - f(x)$

(C) $f(x) + \int_0^{\frac{\pi}{2}-x} f(t) \sin t dt$ (D) $x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t dt$

Ans. (B,D)

Sol. For option (A),

$$\text{Let } g(x) = e^x - \int_0^x f(t) \sin t \, dt$$

$$\therefore g'(x) = e^x - (f(x) \cdot \sin x) > 0 \quad \forall x \in (0,1)$$

$\Rightarrow g(x)$ is strictly increasing function.

$$\text{Also, } g(0) = 1$$

$$\Rightarrow g(x) > 1 \quad \forall x \in (0,1)$$

$\therefore$ option (A) is not possible.

For option (B), let

$$k(x) = x^9 - f(x)$$

$$\text{Now, } k(0) = -f(0) < 0 \quad (\text{As } f \in (0,1))$$

$$\text{Also, } k(1) = 1 - f(1) > 0 \quad (\text{As } f \in (0,1))$$

$$\Rightarrow k(0) \cdot k(1) < 0$$

So, option(B) is correct.

For option (C), let

$$T(x) = f(x) + \int_0^{\frac{\pi}{2}} f(t) \cdot \sin t \, dt$$

$$\Rightarrow T(x) > 0 \quad \forall x \in (0,1) \quad (\text{As } f \in (0,1))$$

so, option(C) is not possible.

For option (D),

$$\text{Let } M(x) = x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$$

$$\therefore M(0) = 0 - \int_0^{\pi/2} f(t) \cdot \cos t \, dt < 0$$

$$\text{Also, } M(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t) \cdot \cos t \, dt > 0$$

$$\Rightarrow M(0) \cdot M(1) < 0$$

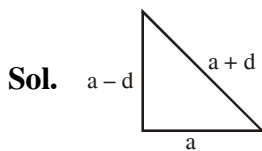
$\therefore$ option (D) is correct.

SECTION-2 : (Maximum Marks : 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :
Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 In all other cases.

- 44.** The sides of the right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

Ans. 6



where $d > 0$, $a > 0$

$\Rightarrow$ length of smallest side = $a - d$

$$\text{Now } (a + d)^2 = a^2 + (a - d)^2$$

$$\Rightarrow a(a - 4d) = 0$$

$$\therefore a = 4d \quad \dots(1)$$

(As $a = 0$ is rejected)

$$\text{Also, } \frac{1}{2}a(a - d) = 24$$

$$\Rightarrow a(a - d) = 48 \quad \dots(2)$$

$\therefore$ From (1) and (2), we get $a = 8$, $d = 2$

Hence, length of smallest side

$$\Rightarrow (a - d) = (8 - 2) = 6$$

- 45.** For how many values of p , the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ?

Ans. 2

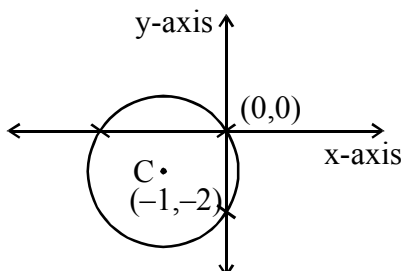
Sol. We shall consider 3 cases.

Case I : When $p = 0$

(i.e. circle passes through origin)

Now, equation of circle becomes

$$x^2 + y^2 + 2x + 4y = 0$$



Case II : When circle intersects x-axis at 2 distinct points and touches y-axis

$$\text{Now } (g^2 - c) > 0 \quad \& \quad f^2 - c = 0$$

$$\Rightarrow 1 - (-p) > 0 \quad \& \quad 4 - (-p) = 0 \quad \Rightarrow \quad p = -4$$

$$\Rightarrow p > -1$$

$\therefore$ Not possible.

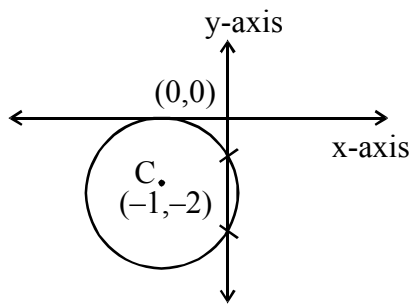
Case III : When circle intersects y-axis at 2 distinct points & touches x-axis.

$$\text{Now, } g^2 - c = 0 \quad \& \quad f^2 - c > 0$$

$$\Rightarrow 1 - (-p) = 0 \quad \& \quad 4 - (-p) > 0$$

$$\Rightarrow p = -1 \quad \Rightarrow \quad p > -4$$

$\therefore p = -1$ is possible.



$\therefore$ Finally we conclude that $p = 0, -1$

$\Rightarrow$ Two possible values of p .

46. For a real number α , if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Ans. 1

$$\text{Sol. } \Delta = 0 \Rightarrow 1(1 - \alpha^2) - \alpha(\alpha - \alpha^3) + \alpha^2(\alpha^2 - \alpha^2) = 0$$

$$(1 - \alpha^2) - \alpha^2 + \alpha^4 = 0$$

$$(\alpha^2 - 1)^2 = 0 \Rightarrow \alpha = \pm 1$$

but at $\alpha = 1$ No solution so rejected

at $\alpha = -1$ all three equation become

$$x - y + z = 1 \text{ (coincident planes)}$$

$$\therefore 1 + \alpha + \alpha^2 = 1$$

47. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$

Ans. 5

Sol. $x = 10!$

$$y = {}^{10}C_1 {}^9C_8 \frac{10!}{2!}$$

$$\frac{y}{9x} = \frac{5.9.10!}{9.10!} = 5$$

48. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] dt$$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x \rightarrow 0} g(x) =$

Ans. 2

Sol. $g(x) = \int_x^{\pi/2} (f'(t) \operatorname{cosec} t - f(t) \operatorname{cosec} t \cot t) dt$

$$= \int_x^{\pi/2} (f(t) \operatorname{cosec} t)' dt$$

$$= f\left(\frac{\pi}{2}\right) \operatorname{cosec}\left(\frac{\pi}{2}\right) - \frac{f(x)}{\sin x} = 3 - \frac{f(x)}{\sin x}$$

$$\therefore \lim_{x \rightarrow 0} g(x) = 3 - \lim_{x \rightarrow 0} \frac{f(x)}{\sin x}; \text{ as } f'(0) = 1$$

$$\Rightarrow \lim_{x \rightarrow 0} g(x) = 3 - 1 = 2$$

SECTION-3 : (Maximum Marks : 18)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : -1 In all other cases

Column 1,2 and 3 contain conics, equation of tangents to the conics and points of contact, respectively.		
Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2 + 1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2 + 1}}, \frac{a}{\sqrt{m^2 + 1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2 - 1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2 + 1}}, \frac{1}{\sqrt{a^2m^2 + 1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2 + 1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2 - 1}}, \frac{-1}{\sqrt{a^2m^2 - 1}}\right)$

49. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only **CORRECT** combination ?

(A) (II) (iii) (R) (B) (IV) (iv) (S) (C) (IV) (iii) (S) (D) (II) (iv) (R)

Ans. (D)

Sol. $P\left(\sqrt{3}, \frac{1}{2}\right)$; tangent $\sqrt{3}x + 2y = 4$

$$\Rightarrow (\sqrt{3})x + 4\left(\frac{1}{2}\right)y = 4 \text{ comparing with (II)}$$

$$\Rightarrow a = 2 \therefore y = mx + \sqrt{a^2m^2 + 1} \text{ is tangent for } m = -\frac{\sqrt{3}}{2} \text{ i.e (ii)}$$

$$\therefore \text{ point of contact for } a = 2, m = -\frac{\sqrt{3}}{2} \text{ is R}$$

50. If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is $(8, 16)$, then which of the following options is the only **CORRECT** combination ?

(A) (III) (i) (P) (B) (III) (ii) (Q) (C) (II) (iv) (R) (D) (I) (ii) (Q)

Ans. (A)

Sol. $y = x + 8$ is tangent $\Rightarrow m = 1$; $P(8, 16)$

Comparing tangent with (i) of column 2, $m = 1$ satisfied and $a = 8$ obtained which matches for point of contact (P) of column 3 and (III) of column I.

51. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact $(-1, 1)$, then which of the following options is the only **CORRECT** combination for obtaining its equation ?

(A) (II) (ii) (Q) (B) (III) (i) (P) (C) (I) (i) (P) (D) (I) (ii) (Q)

Ans. (D)

Sol. For $a = \sqrt{2}$ and point $(-1, 1)$ only I of column-1 satisfies. Hence equation of tangent is $-x + y = 2$ or $y = x + 2 \Rightarrow m = 1$ which matches with (ii) of column 2 and also with Q of column 3

Let $f(x) = x + \log_e x - x \log_e x$, $x \in (0, \infty)$.

- * Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
- * Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
- * Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim_{x \rightarrow \infty} f(x) = 0$	(P) f is increasing in $(0, 1)$
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x \rightarrow \infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x \rightarrow \infty} f'(x) = -\infty$	(R) f' is increasing in $(0, 1)$
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x \rightarrow \infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

52. Which of the following options is the only **CORRECT** combination ?

(A) (IV) (i) (S) (B) (I) (ii) (R) (C) (III) (iv) (P) (D) (II) (iii) (S)

Ans. (D)

53. Which of the following options is the only **CORRECT** combination ?

(A) (III) (iii) (R) (B) (I) (i) (P) (C) (IV) (iv) (S) (D) (II) (ii) (Q)

Ans. (D)

54. Which of the following options is the only **INCORRECT** combination ?

(A) (II) (iii) (P) (B) (II) (iv) (Q) (C) (I) (iii) (P) (D) (III) (i) (R)

Ans. (D)

Sol. 52. to 54.

$$f(x) = x + \ln x - x \ln x, x > 0$$

$$f'(x) = 1 + \frac{1}{x} - \ln x$$

$$f''(x) = -\frac{1}{x^2} - \frac{1}{x} = \frac{-(x+1)}{x^2}$$

- (I) $f(1) f(e^2) < 0$ so true
 (II) $f'(1) f'(e) < 0$ so true
 (III) Graph of $f'(x)$ so (III) is false
 (IV) Is false

$$\text{As } \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} x \left[1 + \frac{\ln x}{x} - \ln x \right] = -\infty$$

$\therefore$ (i) is false (ii) is true

$$\lim_{x \rightarrow \infty} f'(x) = -\infty \text{ so (iii) is true}$$

$$\lim_{x \rightarrow \infty} f''(x) = 0 \text{ so (iv) is true.}$$

(P) $f'(x)$ is positive in $(0,1)$ so true

(Q) $f'(x) < 0$ for in (e, e^2) so true

As $f'(x) < 0 \forall x > 0$ therefor R is false, S is true.

Alternate :

$$f(x) = x + \ln x - x \ln x$$

$$f'(x) = \frac{1}{x} - \ln x = 0 \text{ at } x = x_0 \text{ where } x_0 \in (1, e)$$

$$f''(x) = -\frac{1}{x^2} - \frac{1}{x} < 0 \forall x > 0 \Rightarrow f(x) \text{ concave down}$$

