

JEE(Advanced) – 2018 TEST PAPER - 1 WITH SOLUTIONS

(Exam Date: 21-05-2018)

PART-1 : MATHEMATICS

SECTION-1

1. For a non-zero complex number z , let $\arg(z)$ denotes the principal argument with $-\pi < \arg(z) \leq \pi$. Then, which of the following statement(s) is (are) **FALSE** ?

(A) $\arg(-1 - i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f : \mathbb{R} \rightarrow (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2 , $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3 , the locus of the point z satisfying the condition

$$\arg\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi, \text{ lies on a straight line}$$

Ans. (A,B,D)

Sol. (A) $\arg(-1 - i) = -\frac{3\pi}{4}$,

$$(B) f(t) = \arg(-1 + it) = \begin{cases} \pi - \tan^{-1}(t), & t \geq 0 \\ -\pi + \tan^{-1}(t), & t < 0 \end{cases}$$

Discontinuous at $t = 0$.

$$(C) \arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$$

$$= \arg z_1 - \arg(z_2) + 2n\pi - \arg(z_1) + \arg(z_2) = 2n\pi.$$

$$(D) \arg\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi$$

$$\Rightarrow \frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)} \text{ is real.}$$

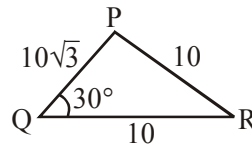
$$\Rightarrow z, z_1, z_2, z_3 \text{ are concyclic.}$$

2. In a triangle PQR, let $\angle PQR = 30^\circ$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE ?

- (A) $\angle QPR = 45^\circ$
 (B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120^\circ$
 (C) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$
 (D) The area of the circumcircle of the triangle PQR is 100π .

Ans. (B,C,D)

Sol. $\cos 30^\circ = \frac{(10\sqrt{3})^2 + (10)^2 - (PR)^2}{2 \times 10\sqrt{3} \times 10}$



$$\Rightarrow PR = 10$$

$$\because QR = PR \Rightarrow \angle PQR = \angle QPR$$

$$\angle QPR = 30^\circ$$

$$(B) \text{ area of } \Delta PQR = \frac{1}{2} \times 10\sqrt{3} \times 10 \times \sin 30^\circ = \frac{1}{2} \times 10 \times 10\sqrt{3} \times \frac{1}{2}$$

$$= 25\sqrt{3}$$

$$\angle QRP = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$$

$$(C) \quad r = \frac{\Delta}{S} = \frac{25\sqrt{3}}{\left(\frac{10+10+10\sqrt{3}}{2}\right)} = \frac{25\sqrt{3}}{10+5\sqrt{3}}$$

$$= 5\sqrt{3} \cdot (2 - \sqrt{3}) = 10\sqrt{3} - 15$$

$$(D) \quad R = \frac{a}{2 \sin A} = \frac{10}{2 \sin 30^\circ} = 10$$

$$\therefore \text{Area} = \pi R^2 = 100\pi$$

3. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE ?

- (A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, -1
 (B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
 (C) The acute angle between P_1 and P_2 is 60°
 (D) If P_3 is the plane passing through the point (4, 2, -2) and perpendicular to the line of intersection of P_1 and P_2 , then the distance of the point (2, 1, 1) from the plane P_2 is $\frac{2}{\sqrt{3}}$

Ans. (C,D)

Sol. D.C. of line of intersection (a, b, c)

$$\Rightarrow 2a + b - c = 0$$

$$a + 2b + c = 0$$

$$\frac{a}{1+2} = \frac{b}{-1-2} = \frac{c}{4-1}$$

$\therefore$ D.C. is (1, -1, 1)

$$(B) \frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$$

$$\Rightarrow \frac{x-4/3}{3} = \frac{y-1/3}{-3} = \frac{z}{3}$$

$\Rightarrow$ lines are parallel.

$$(C) \text{ Acute angle between } P_1 \text{ and } P_2 = \cos^{-1} \left(\frac{2 \times 1 + 1 \times 2 - 1 \times 1}{\sqrt{6}\sqrt{6}} \right)$$

$$= \cos^{-1} \left(\frac{3}{6} \right) = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ$$

$$(D) \text{ Plane is given by } (x-4) - (y-2) + (z+2) = 0$$

$$\Rightarrow x - y + z = 0$$

$$\text{Distance of } (2, 1, 1) \text{ from plane} = \frac{2-1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

4. For every twice differentiable function $f : \mathbb{R} \rightarrow [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE ?

- (A) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s)
 (B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$
 (C) $\lim_{x \rightarrow \infty} f(x) = 1$
 (D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

Ans. (A,B,D)

Sol. $f(x)$ can't be constant throughout the domain. Hence we can find $x \in (r, s)$ such that $f(x)$ is one-one
option (A) is true.

$$\text{Option (B): } |f'(x_0)| = \left| \frac{f(0) - f(-4)}{4} \right| \leq 1 \quad (\text{LMVT})$$

Option (C): $f(x) = \sin(\sqrt{85}x)$ satisfies given condition

$$\text{but } \lim_{x \rightarrow \infty} \sin(\sqrt{85}x) \text{ D.N.E.}$$

$\Rightarrow$ Incorrect

$$\text{Option (D): } g(x) = f^2(x) + (f'(x))^2$$

$$|f'(x_1)| \leq 1 \quad (\text{by LMVT})$$

$$|f(x_1)| \leq 2 \quad (\text{given})$$

$$\Rightarrow g(x_1) \leq 5 \quad \exists x_1 \in (-4, 0)$$

$$\text{Similarly } g(x_2) \leq 5 \quad \exists x_2 \in (0, 4)$$

$$g(0) = 85 \Rightarrow g(x) \text{ has maxima in } (x_1, x_2) \text{ say at } \alpha.$$

$$g'(\alpha) = 0 \text{ \& } g(\alpha) \geq 85$$

$$2f'(\alpha) (f(\alpha) + f''(\alpha)) = 0$$

$$\text{If } f'(\alpha) = 0 \Rightarrow g(\alpha) = f^2(\alpha) \geq 85 \text{ Not possible}$$

$$\Rightarrow f(\alpha) + f''(\alpha) = 0 \quad \exists \alpha \in (x_1, x_2) \in (-4, 4)$$

option (D) correct.

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be two non-constant differentiable functions. If $f'(x) = (e^{(f(x) - g(x))})g'(x)$ for all $x \in \mathbb{R}$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE ?

$$(A) f(2) < 1 - \log_e 2$$

$$(B) f(2) > 1 - \log_e 2$$

$$(C) g(1) > 1 - \log_e 2$$

$$(D) g(1) < 1 - \log_e 2$$

Ans. (B,C)

$$\text{Sol. } f'(x) = e^{(f(x) - g(x))} g'(x) \quad \forall x \in \mathbb{R}$$

$$\Rightarrow e^{-f(x)} \cdot f'(x) - e^{-g(x)} g'(x) = 0$$

$$\Rightarrow \int (e^{-f(x)} f'(x) - e^{-g(x)} g'(x)) dx = C$$

$$\Rightarrow -e^{-f(x)} + e^{-g(x)} = C$$

$$\Rightarrow -e^{-f(1)} + e^{-g(1)} = -e^{-f(2)} + e^{-g(2)}$$

$$\Rightarrow -\frac{1}{e} + e^{-g(1)} = -e^{-f(2)} + \frac{1}{e}$$

$$\Rightarrow e^{-f(2)} + e^{-g(1)} = \frac{2}{e}$$

$$\therefore e^{-f(2)} < \frac{2}{e} \text{ and } e^{-g(1)} < \frac{2}{e}$$

$$\Rightarrow -f(2) < \ln 2 - 1 \text{ and } -g(1) < \ln 2 - 1$$

$$\Rightarrow f(2) > 1 - \ln 2 \text{ and } g(1) > 1 - \ln 2$$

6. Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a continuous function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt$

for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE ?

(A) The curve $y = f(x)$ passes through the point (1, 2)

(B) The curve $y = f(x)$ passes through the point (2, -1)

(C) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1-x^2}\}$ is $\frac{\pi-2}{4}$

(D) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1-x^2}\}$ is $\frac{\pi-1}{4}$

Ans. (B,C)

Sol. $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt$

$$\Rightarrow e^{-x} f(x) = e^{-x} (1 - 2x) + \int_0^x e^{-t} f(t) dt$$

Differentiate w.r.t. x.

$$-e^{-x} f(x) + e^{-x} f'(x) = -e^{-x} (1 - 2x) + e^{-x} (-2) + e^{-x} f(x)$$

$$\Rightarrow -f(x) + f'(x) = -(1 - 2x) - 2 + f(x).$$

$$\Rightarrow f'(x) - 2f(x) = 2x - 3$$

Integrating factor = e^{-2x} .

$$f(x) \cdot e^{-2x} = \int e^{-2x} (2x - 3) dx$$

$$= (2x - 3) \int e^{-2x} dx - \int \left((2) \int e^{-2x} dx \right) dx$$

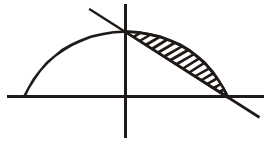
$$= \frac{(2x - 3)e^{-2x}}{-2} - \frac{e^{-2x}}{2} + c$$

$$f(x) = \frac{2x - 3}{-2} - \frac{1}{2} + ce^{2x}$$

$$f(0) = \frac{3}{2} - \frac{1}{2} + c = 1 \Rightarrow c = 0$$

$$\therefore f(x) = 1 - x$$

$$\text{Area} = \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}$$



SECTION-2

7. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is —

Ans. (8)

$$\text{Sol. } \log_2 9^{\frac{2}{\log_2(\log_2 9)}} \times 7^{\frac{1/2}{\log_4 7}}$$

$$= (\log_2 9)^{2 \log_2^2 9} \times 7^{\frac{1}{2} \log_7 4}$$

$$= 4 \times 2 = 8$$

8. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is —

Ans. (625)

Sol. Option for last two digits are (12), (24), (32), (44) are (52).

$\therefore$ Total No. of digits

$$= 5 \times 5 \times 5 \times 5 = 625$$

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11,, and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23,, Then, the number of elements in the set $X \cup Y$ is —

Ans. (3748)

Sol. X : 1, 6, 11,, 10086

Y : 9, 16, 23,, 14128

$X \cap Y$: 16, 51, 86,

Let $m = n(X \cap Y)$

$$\therefore 16 + (m - 1) \times 35 \leq 10086$$

$$\Rightarrow m \leq 288.71$$

$$\Rightarrow m = 288$$

$$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$= 2018 + 2018 - 288 = 3748$$

10. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty} \left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty} (-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is —

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume value in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

Ans. (2)

Sol.
$$\sum_{i=1}^{\infty} x^{i+1} = \frac{x^2}{1-x}$$

$$\sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i = \frac{x}{2-x}$$

$$\sum_{i=1}^{\infty} \left(-\frac{x}{2}\right)^i = \frac{-x}{2+x}$$

$$\sum_{i=1}^{\infty} (-x)^i = \frac{-x}{1+x}$$

To have real solutions

$$\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i = \sum_{i=1}^{\infty} \left(\frac{-x}{2}\right)^i - \sum_{i=1}^{\infty} (-x)^i$$

$$\frac{x^2}{1-x} - \frac{x^2}{2-x} = \frac{-x}{2+x} + \frac{x}{1+x}$$

$$x(x^3 + 2x^2 + 5x - 2) = 0$$

$$\therefore x = 0 \text{ and let } f(x) = x^3 + 2x^2 + 5x - 2$$

$$f\left(\frac{1}{2}\right) \cdot f\left(-\frac{1}{2}\right) < 0$$

Hence two solutions exist

11. For each positive integer n , let

$$y_n = \frac{1}{n}(n+1)(n+2)\dots(n+n)^{1/n}$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x . If $\lim_{n \rightarrow \infty} y_n = L$, then the value of $[L]$

is —

Ans. (1)

Sol. $y_n = \left\{ \left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{n}\right) \dots \left(1 + \frac{n}{n}\right) \right\}^{\frac{1}{n}}$

$$y_n = \prod_{r=1}^n \left(1 + \frac{r}{n}\right)^{1/n}$$

$$\log y_n = \frac{1}{n} \sum_{r=1}^n \ell n \left(1 + \frac{r}{n}\right)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \log y_n = \lim_{x \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \ell n \left(1 + \frac{r}{n}\right)$$

$$\Rightarrow \log L = \int_0^1 \ell n(1+x) dx$$

$$\Rightarrow \log L = \log \frac{4}{e}$$

$$\Rightarrow L = \frac{4}{e}$$

$$\Rightarrow [L] = 1$$

- 12.** Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a} \cdot \vec{b} = 0$. For some $x, y \in \mathbb{R}$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2 \alpha$ is —

Ans. (3)

Sol. $\vec{c} = x\vec{a} + y\vec{b} + \vec{a} \times \vec{b}$

$$\vec{c} \cdot \vec{a} = x \text{ and } x = 2\cos\alpha$$

$$\vec{c} \cdot \vec{b} = y \text{ and } y = 2\cos\alpha$$

$$\text{Also, } |\vec{a} \times \vec{b}| = 1$$

$$\therefore \vec{c} = 2\cos(\vec{a} + \vec{b}) + \vec{a} \times \vec{b}$$

$$\vec{c}^2 = 4\cos^2 \alpha (\vec{a} + \vec{b})^2 + (\vec{a} \times \vec{b})^2 + 2\cos\alpha (\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b})$$

$$4 = 8\cos^2 \alpha + 1$$

$$8\cos^2 \alpha = 3$$

- 13.** Let a, b, c be three non-zero real numbers such that the equation

$$\sqrt{3}a \cos x + 2b \sin x = c, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then the value of $\frac{b}{a}$ is —

Ans. (0.5)

Sol. $\sqrt{3} \cos x + \frac{2b}{a} \sin x = \frac{c}{a}$

Now, $\sqrt{3} \cos \alpha + \frac{2b}{a} \sin \alpha = \frac{c}{a} \quad \dots (1)$

$\sqrt{3} \cos \beta + \frac{2b}{a} \sin \beta = \frac{c}{a} \quad \dots (2)$

$\sqrt{3} [\cos \alpha - \cos \beta] + \frac{2b}{a} (\sin \alpha - \sin \beta) = 0$

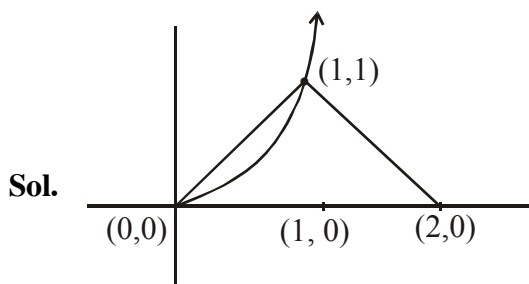
$\sqrt{3} \left[-2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right) \right] + \frac{2b}{a} \left[2 \cos \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right) \right] = 0$

$-\sqrt{3} + 2\sqrt{3} \cdot \frac{b}{a} = 0$

$\frac{b}{a} = \frac{1}{2} = 0.5$

- 14.** A farmer F_1 has a land in the shape of a triangle with vertices at $P(0, 0)$, $Q(1, 1)$ and $R(2, 0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n$ ($n > 1$). If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR , then the value of n is —

Ans. (4)



$$\text{Area} = \int_0^1 (x - x^n) dx = \frac{3}{10}$$

$$\left[\frac{x^2}{2} - \frac{x^{n+1}}{n+1} \right]_0^1 = \frac{3}{10}$$

$$\frac{1}{2} - \frac{1}{n+1} = \frac{3}{10} \quad \therefore n + 1 = 5$$

$$\Rightarrow n = 4$$

SECTION-3

Paragraph "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(There are two question based on Paragraph "X", the question given below is one of them)

15. Let E_1E_2 and F_1F_2 be the chord of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slop -1 . Let the tangents to S at E_1 and E_2 meet at E_3 , the tangents of S at F_1 and F_2 meet at F_3 , and the tangents to S at G_1 and G_2 meet at G_3 . Then, the points E_3 , F_3 and G_3 lie on the curve

(A) $x + y = 4$

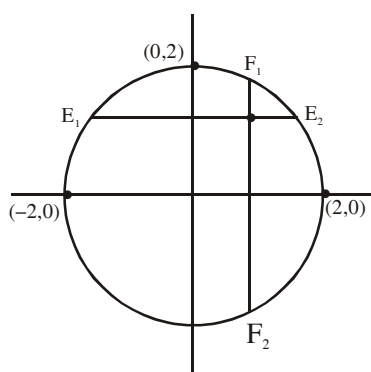
(B) $(x - 4)^2 + (y - 4)^2 = 16$

(C) $(x - 4)(y - 4) = 4$

(D) $xy = 4$

Ans. (A)

Sol.



co-ordinates of E_1 and E_2 are obtained by solving $y = 1$ and $x^2 + y^2 = 4$

$$\therefore E_1(-\sqrt{3}, 1) \text{ and } E_2(\sqrt{3}, 1)$$

co-ordinates of F_1 and F_2 are obtained by solving

$$x = 1 \text{ and } x^2 + y^2 = 4$$

$$F_1(1, \sqrt{3}) \text{ and } F_2(1, -\sqrt{3})$$

$$\text{Tangent at } E_1: -\sqrt{3}x + y = 4$$

$$\text{Tangent at } E_2: \sqrt{3}x + y = 4$$

$$\therefore E_3(0, 4)$$

$$\text{Tangent at } F_1: x + \sqrt{3}y = 4$$

$$\text{Tangent at } F_2: x - \sqrt{3}y = 4$$

$$\therefore F_3(4, 0)$$

and similarly $G_3(2, 2)$

$(0, 4)$, $(4, 0)$ and $(2, 2)$ lies on $x + y = 4$

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$

(There are two questions based on Paragraph "X", the question given below is one of them)

16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve -

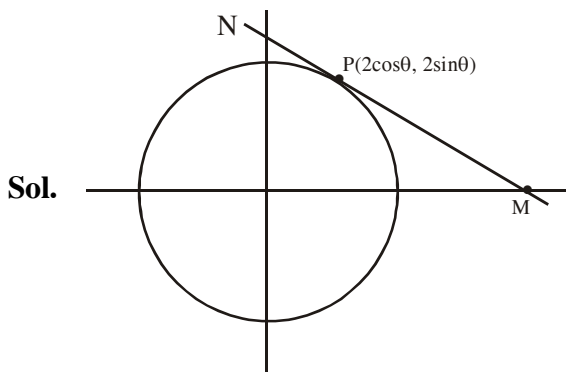
(A) $(x + y)^2 = 3xy$

(B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$

(D) $x^2 + y^2 = x^2y^2$

Ans. (D)



Tangent at $P(2\cos\theta, 2\sin\theta)$ is $x\cos\theta + y\sin\theta = 2$

$M(2\sec\theta, 0)$ and $N(0, 2\csc\theta)$

Let midpoint be (h, k)

$h = \sec\theta, k = \csc\theta$

$$\frac{1}{h^2} + \frac{1}{k^2} = 1$$

$$\frac{1}{x^2} + \frac{1}{y^2} = 1$$

PARAGRAPH "A"

There are five students S_1, S_2, S_3 and S_4 and S_5 in a music class and for them there are five sets R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student $S_i, i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on Paragraph "A". the question given below is one of them)

17. The probability that, on the examination day, the student S_i gets the previously allotted seat R_i and **NONE** of the remaining students gets the seat previously allotted to him/her is -

(A) $\frac{3}{40}$

(B) $\frac{1}{8}$

(C) $\frac{7}{40}$

(D) $\frac{1}{5}$

Ans. (A)

Sol. Required probability = $\frac{4! \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right)}{5!} = \frac{9}{120} = \frac{3}{40}$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student $S_i, i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on Paragraph "A", the question given below is one of them)

- 18.** For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is-

- (A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

Ans. (C)

Sol. $n(T_1 \cap T_2 \cap T_3 \cap T_4) = \text{Total} - n(\bar{T}_1 \cup \bar{T}_2 \cup \bar{T}_3 \cup \bar{T}_4)$

$$= 5! - \left({}^4C_1 4! 2! - \left({}^3C_1 3! 2! + {}^3C_1 3! 2! 2! \right) + \left({}^2C_1 2! 2! + {}^4C_1 2! 2! \right) - 2 \right)$$

$$= 14$$

$$\text{Probability} = \frac{14}{5!} = \frac{7}{60}$$