

Ex 11.1

Q1(i)

We have,

$$\sin \theta = \frac{1}{2}$$

$$\Rightarrow \sin \theta = \sin \frac{\pi}{6} \quad \left[\because \sin \frac{\pi}{6} = \frac{1}{2} \right]$$

$\Rightarrow$ the general solution is

$$\theta = n\pi + (-1)^n \frac{\pi}{6}; n \in \mathbb{Z} \quad \left[\because \text{if } \sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha \right]$$

Q1(ii)

We have,

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos \theta = \cos \left(\pi + \frac{\pi}{6} \right)$$

$$\Rightarrow \cos \theta = \cos \frac{7\pi}{6} \quad \left[\because \cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2} \right]$$

$\therefore$ the general solution is

$$\theta = 2n\pi \pm \frac{7\pi}{6}, n \in \mathbb{Z}$$

Q1(iii)

$$\operatorname{cosec} \theta = -\sqrt{2}$$

$$\Rightarrow \frac{1}{\sin \theta} = -\sqrt{2}$$

$$\Rightarrow \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin \theta = \sin \left(\pi + \frac{\pi}{4} \right)$$

$$\Rightarrow \sin \theta = \sin \frac{5\pi}{4} \text{ or } \sin \theta = \sin \left(-\frac{\pi}{4} \right)$$

$$\because \sin(-\theta) = -\sin \theta.$$

$$\therefore \theta = n\pi + (-1)^{n+1} \frac{\pi}{4}, n \in \mathbb{Z}$$

Q1(iv)

We have,

$$\begin{aligned} \sec \theta &= \sqrt{2} \\ \Rightarrow \frac{1}{\cos \theta} &= \sqrt{2} \\ \Rightarrow \cos \theta &= \frac{1}{\sqrt{2}} \quad \Rightarrow \cos \theta = \cos \frac{\pi}{4} \\ &\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4}, n \in \mathbb{Z} \end{aligned}$$

Q1(v)

We have,

$$\begin{aligned} \tan \theta &= \frac{-1}{\sqrt{3}} \\ \Rightarrow \tan \theta &= \tan \left(-\frac{\pi}{6} \right) \\ \Rightarrow \tan \theta &= \tan \left(-\frac{\pi}{6} \right) \quad [\because \tan(-\theta) = -\tan \theta] \\ \Rightarrow \theta &= n\pi + \left(-\frac{\pi}{6} \right), n \in \mathbb{Z} \\ \text{or } \theta &= n\pi - \frac{\pi}{6}, n \in \mathbb{Z} \end{aligned}$$

Q1(vi)

We have,

$$\begin{aligned} \sqrt{3} \sec \theta &= 2 \\ \Rightarrow \frac{1}{\cos \theta} &= \frac{2}{\sqrt{3}} \\ \Rightarrow \cos \theta &= \frac{\sqrt{3}}{2} \\ \Rightarrow \cos \theta &= \cos \left(\frac{\pi}{6} \right) \\ \Rightarrow \theta &= 2n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z} \end{aligned}$$

Q2(i)

We have,

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin 2\theta = \sin\left(\frac{\pi}{3}\right)$$

$$\Rightarrow 2\theta = n\pi + (-1)^n \frac{\pi}{3}, n \in \mathbb{Z}$$

$$\theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$$

Q2(ii)

We have,

$$\cos 3\theta = \frac{1}{2}$$

$$\Rightarrow \cos 3\theta = \cos\left(\frac{\pi}{3}\right)$$

$$\Rightarrow 3\theta = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

$$\Rightarrow \theta = 2n\frac{\pi}{3} \pm \frac{\pi}{9}, n \in \mathbb{Z}$$

Q2(iii)

$$\sin 9\theta = \sin \theta$$

$$\sin 9\theta - \sin \theta = 0$$

Apply $\sin A - \sin B$ formula

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\sin 9\theta - \sin \theta = 2 \cos 5\theta \sin 4\theta = 0$$

$$\cos 5\theta \sin 4\theta = 0$$

$$\Rightarrow \cos 5\theta = 0 \text{ (or) } \sin 4\theta = 0$$

$$5\theta = \frac{(2n+1)\pi}{2} \text{ (or) } 4\theta = n\pi$$

$$\theta = \left\{ \frac{(2n+1)\pi}{10} \right\} \text{ (or) } \theta = \left\{ \frac{n\pi}{4} \right\} \text{ where } n \in \mathbb{Z}$$

Q2(iv)

We have,

$$\begin{aligned}
 & \sin 2\theta = \cos 3\theta \\
 \Rightarrow & \cos 3\theta = \sin 2\theta \\
 \Rightarrow & \cos 3\theta = \cos \left(\frac{\pi}{2} - 2\theta \right) \quad \left[\because \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta \right] \\
 \Rightarrow & 3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta \right), n \in \mathbb{Z} \\
 \Rightarrow & \text{either} \\
 & \quad 5\theta = 2n\pi + \frac{\pi}{2}, n \in \mathbb{Z} \text{ or } \theta = 2n\pi - \frac{\pi}{2}, n \in \mathbb{Z} \\
 \Rightarrow & 5\theta = (4n+1)\frac{\pi}{2}, n \in \mathbb{Z} \text{ or } \theta = (4n-1)\frac{\pi}{2} \\
 \Rightarrow & \theta = (4n+1)\frac{\pi}{10}, n \in \mathbb{Z} \text{ or } \theta = (4n-1)\frac{\pi}{2}, n \in \mathbb{Z}
 \end{aligned}$$

Q2(v)

We have,

$$\begin{aligned}
 & \tan \theta + \cot 2\theta = 0 \\
 & \tan \theta = -\cot 2\theta \\
 \Rightarrow & \cot 2\theta = -\tan \theta \\
 \Rightarrow & \tan 2\theta = -\cot \theta \\
 \Rightarrow & \tan 2\theta = -\tan \left(\frac{\pi}{2} - \theta \right) \\
 \Rightarrow & \tan 2\theta = \tan \left(\theta - \frac{\pi}{2} \right) \\
 \Rightarrow & 2\theta = n\pi + \left(\theta - \frac{\pi}{2} \right), n \in \mathbb{Z} \\
 \Rightarrow & \theta = n\pi - \frac{\pi}{2}, n \in \mathbb{Z}
 \end{aligned}$$

Q2(vi)

We have,

$$\tan 3\theta = \cot \theta$$

$$\Rightarrow \tan 3\theta = \tan \left(\frac{\pi}{2} - \theta \right) \quad \left[\because \tan \left(\frac{\pi}{2} - \theta \right) = \cot \theta \right]$$

$$\Rightarrow 3\theta = n\pi + \frac{\pi}{2} - \theta, n \in \mathbb{Z}$$

$$\Rightarrow 4\theta = n\pi + \frac{\pi}{2}, n \in \mathbb{Z}$$

$$\Rightarrow \theta = \frac{n\pi}{4} + \frac{\pi}{8}, n \in \mathbb{Z}$$

Q2(vii)

We have,

$$\tan 2\theta \cdot \tan \theta = 1$$

$$\Rightarrow \tan 2\theta = \frac{1}{\tan \theta}$$

$$\Rightarrow \tan 2\theta = \cot \theta$$

$$\Rightarrow \tan 2\theta = \tan \left(\frac{\pi}{2} - \theta \right)$$

$$\Rightarrow 2\theta = n\pi + \frac{\pi}{2} - \theta, n \in \mathbb{Z}$$

$$\Rightarrow 3\theta = n\pi + \frac{\pi}{2}, n \in \mathbb{Z}$$

$$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{6}, n \in \mathbb{Z}$$

Q2(viii)

$$\tan m\theta + \cot n\theta = 0$$

$$\sin m\theta \sin n\theta + \cos m\theta \cos n\theta = 0$$

$$\cos(m-n)\theta = 0$$

$$(m-n)\theta = \left(\frac{2k+1}{2} \right) \pi$$

$$\theta = \left(\frac{2k+1}{2(m-n)} \right) \pi, k \in \mathbb{Z}$$

Q2(ix)

We have,

$$\tan p\theta = \cot q\theta$$

$$\Rightarrow \tan p\theta = \tan\left(\frac{\pi}{2} - q\theta\right)$$

$$\Rightarrow p\theta = n\pi \pm \left(\frac{\pi}{2} - q\theta\right), n \in \mathbb{Z}$$

$$\Rightarrow (p+q)\theta = n\pi + \frac{\pi}{2}, n \in \mathbb{Z}$$

$$\Rightarrow (p+q)\theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$$

$$\Rightarrow \theta = \frac{(2n+1)\pi}{(p+q)2}, n \in \mathbb{Z}$$

Q2(x)

$$\sin 2x + \cos x = 0$$

$$2\sin x \cos x + \cos x = 0$$

$$\cos x(2\sin x + 1) = 0$$

$$\cos x = 0 \text{ or } 2\sin x + 1 = 0$$

$$x = (4m-1)\frac{\pi}{2} \text{ or } \sin x = -\frac{1}{2}$$

$$x = (4m-1)\frac{\pi}{2} \text{ or } x = (4n-1)\frac{\pi}{6}, m, n \in \mathbb{Z}$$

Q2(xi)

We have,

$$\sin \theta = \tan \theta$$

$$\Rightarrow \sin \theta = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \sin \theta = \frac{\sin \theta}{\cos \theta} = 0$$

$$\Rightarrow \sin \theta (\cos \theta - 1) = 0$$

$$\Rightarrow \text{either } \sin \theta = 0 \quad \text{or } \cos \theta - 1 = 0$$

$$\Rightarrow \theta = n\pi, n \in \mathbb{Z} \quad \text{or } \cos \theta = 1$$

$$\Rightarrow \cos \theta = \cos 0^\circ$$

$$\theta = 2m\pi, m \in \mathbb{Z}$$

Thus,

$$\theta = n\pi, n \in \mathbb{Z} \quad \text{or } \theta = 2m\pi, m \in \mathbb{Z}$$

Q2(xii)

$$\cos(2x) = -\sin(3x)$$

$$= -\cos\left(\frac{\pi}{2} - 3x\right)$$

$$= \cos\left(\frac{\pi}{2} + 3x\right)$$

$$\Rightarrow 2n\pi + 2x = \frac{\pi}{2} + 3x$$

$$x = (4m-1)\frac{\pi}{2}, m \in \mathbb{Z}$$

or

$$\Rightarrow 2n\pi - 2x = \frac{\pi}{2} + 3x$$

$$x = (4n-1)\frac{\pi}{10}, n \in \mathbb{Z}$$

Q3(i)

We have,

$$\sin^2 \theta - \cos \theta = \frac{1}{4}$$

$$\Rightarrow 1 - \cos^2 \theta - \cos \theta = \frac{1}{4} \quad \left[\because \sin^2 \theta = 1 - \cos^2 \theta \right]$$

$$\Rightarrow \cos^2 \theta + \cos \theta - \frac{3}{4} = 0$$

$$\Rightarrow 4\cos^2 \theta + 4\cos \theta - 3 = 0$$

$$\Rightarrow 4\cos^2 \theta + 6\cos \theta - 2\cos \theta - 3 = 0 \quad [\text{factorize it}]$$

$$\Rightarrow 2\cos \theta (2\cos \theta + 3) - 1(\cos \theta + 3) = 0$$

$$\Rightarrow (2\cos \theta - 1)(2\cos \theta + 3) = 0$$

$\Rightarrow$ either

$$2\cos \theta - 1 = 0 \quad \text{or} \quad 2\cos \theta + 3 = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2} \quad \text{or} \quad \cos \theta = -\frac{3}{2} \quad \left[\text{This is not possible as } -1 < \cos \theta < 1 \right]$$

$$\Rightarrow \cos \theta = \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

Q3(ii)

We have,

$$2\cos^2 \theta - 5\cos \theta + 2 = 0$$

$$\Rightarrow 2\cos^2 \theta - 4\cos \theta - \cos \theta + 2 = 0 \quad [\text{use factorization}]$$

$$\Rightarrow 2\cos \theta (\cos \theta - 2) - 1(\cos \theta - 2) = 0$$

$$\Rightarrow (2\cos \theta - 1)(\cos \theta - 2) = 0$$

$\Rightarrow$ either

$$2\cos \theta - 1 = 0 \quad \text{or} \quad \cos \theta - 2 = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2} \quad \text{or} \quad \cos \theta = 2$$

$$\Rightarrow \cos \theta = \cos \frac{\pi}{3} \quad [\text{This is not possible as } -1 < \cos \theta < 1]$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

Thus,

$$\theta = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

Q3(iii)

We have,

$$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$$

$$\Rightarrow 2(1 - \cos^2 x) + \sqrt{3}\cos x + 1 = 0$$

$$\Rightarrow 2\cos^2 x - \sqrt{3}\cos x - 3 = 0$$

factorise it, we get,

$$\Rightarrow 2\cos^2 x - 2\sqrt{3}\cos x + \sqrt{3}\cos x - 3 = 0$$

$$\Rightarrow 2\cos x (\cos x - \sqrt{3}) + \sqrt{3}(\cos x - \sqrt{3}) = 0$$

$$\Rightarrow (2\cos x + \sqrt{3})(\cos x - \sqrt{3}) = 0$$

$\Rightarrow$ either

$$\cos x = -\frac{\sqrt{3}}{2} \quad \text{or} \quad \cos x = \sqrt{3} \quad [\text{This is not possible as } -1 < \cos x < 1]$$

$$\Rightarrow \cos x = \cos \left(\pi - \frac{\pi}{6} \right)$$

$$\Rightarrow \cos x = \cos \frac{5\pi}{6}$$

$$\Rightarrow x = 2n\pi \pm \frac{5\pi}{6}, n \in \mathbb{Z}$$

Q3(iv)

We have,

$$4 \sin^2 \theta - 8 \cos \theta + 1 = 0$$

$$\Rightarrow 4(1 - \cos^2 \theta) - 8 \cos \theta + 1 = 0$$

$$\Rightarrow 4 \cos^2 \theta + 8 \cos \theta - 5 = 0$$

factorise it, we get,

$$\Rightarrow 4 \cos^2 \theta + 10 \cos \theta - 2 \cos \theta - 5 = 0$$

$$\Rightarrow 2 \cos \theta (2 \cos \theta + 5) - 1 (2 \cos \theta + 5) = 0$$

$$\Rightarrow (2 \cos \theta - 1)(2 \cos \theta + 5) = 0$$

$$\text{either } 2 \cos \theta - 1 = 0 \quad \text{or } 2 \cos \theta + 5 = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2} \quad \text{or } \cos \theta = -\frac{5}{2} \quad \left[\text{This is not possible as } -1 < \cos \theta < 1 \right]$$

$$\Rightarrow \cos \theta = \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

Q3(v)

We have,

$$\tan^2 x + (1 - \sqrt{3}) \tan x - \sqrt{3} = 0$$

$$\Rightarrow \tan^2 x + \tan x - \sqrt{3} \tan x - \sqrt{3} = 0$$

$$\Rightarrow \tan x (\tan x + 1) - \sqrt{3} (\tan x + 1) = 0$$

$$\Rightarrow (\tan x - \sqrt{3})(\tan x + 1) = 0$$

$\Rightarrow$ either

$$\tan x = \sqrt{3} \quad \text{or } \tan x = -1$$

$$\Rightarrow \tan x = \tan \frac{\pi}{3} \quad \text{or } \tan x = -\tan \frac{\pi}{4}$$

$$\Rightarrow x = n\pi + \frac{\pi}{3}, n \in \mathbb{Z} \quad \text{or } x = m\pi - \frac{\pi}{4}, m \in \mathbb{Z}$$

$$\therefore x = n\pi + \frac{\pi}{3} \quad \text{or } m\pi - \frac{\pi}{4}, n, m \in \mathbb{Z}$$

Q3(vi)

$$3\cos^2 \theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2 \theta = 0$$

$$\sqrt{3}\cos^2 \theta - 2\sin\theta\cos\theta - \sqrt{3}\sin^2 \theta = 0 \quad (\text{Dividing by } \sqrt{3})$$

$$\sqrt{3}\cos^2 \theta + \sin\theta\cos\theta - 3\sin\theta\cos\theta - \sqrt{3}\sin^2 \theta = 0$$

$$\cos\theta(\sqrt{3}\cos\theta + \sin\theta) - \sqrt{3}\sin\theta(\sqrt{3}\cos\theta + \sin\theta) = 0$$

$$(\sqrt{3}\cos\theta + \sin\theta)(\cos\theta - \sqrt{3}\sin\theta) = 0$$

$$\sqrt{3}\cos\theta + \sin\theta = 0 \quad \text{or} \quad \cos\theta - \sqrt{3}\sin\theta = 0$$

$$\tan\theta = -\sqrt{3} = -\tan\frac{\pi}{3} \quad \text{or} \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$$

$$\theta = n\pi - \frac{\pi}{3} \quad \text{or} \quad \theta = m\pi + \frac{\pi}{6}$$

$$n, m \in \mathbb{Z}$$

Q3(vii)

We have,

$$\cos 4\theta = \cos 2\theta$$

$$\Rightarrow \cos 4\theta - \cos 2\theta = 0$$

$$\Rightarrow 2\sin\theta \cdot \sin 3\theta = 0$$

$$\Rightarrow \text{either}$$

$$\sin\theta = 0 \quad \text{or} \quad \sin 3\theta = 0$$

$$\Rightarrow \theta = n\pi, n \in \mathbb{Z} \quad \text{or} \quad 3\theta = m\pi, m \in \mathbb{Z}$$

Thus,

$$\theta = n\pi \text{ or } m\frac{\pi}{3}, n, m \in \mathbb{Z}$$

Q4(i)

$$\cos \theta + \cos 2\theta + \cos 3\theta = 0$$

$$\Rightarrow \cos 2\theta + 2\cos 2\theta \cdot \cos \theta = 0 \quad \left[\because \cos \theta + \cos 3\theta = 2\cos 2\theta \cdot \cos \theta \right]$$

$$\Rightarrow \cos 2\theta (1 + 2\cos \theta) = 0$$

either

$$\cos 2\theta = 0 \quad \text{or} \quad 1 + 2\cos \theta = 0$$

$$\Rightarrow 2\theta = (2n+1)\frac{\pi}{4}, n \in \mathbb{Z} \quad \text{or} \quad \cos \theta = -\frac{1}{2}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{4}, n \in \mathbb{Z} \quad \text{or} \quad \cos \theta = +\cos\left(\pi - \frac{\pi}{3}\right)$$

$$\text{or} \quad \cos \theta = \cos 2\frac{\pi}{3}$$

$$\text{or} \quad \theta = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{Z}$$

Thus,

$$\theta = (2n+1)\frac{\pi}{4}, \quad \text{or} \quad \left(2n\pi \pm \frac{2\pi}{3}\right), n \in \mathbb{Z}$$

Q4(ii)

$$\cos \theta + \cos 3\theta - \cos 2\theta = 0$$

$$\Rightarrow 2\cos 2\theta \cdot \cos \theta - \cos 2\theta = 0$$

$$\Rightarrow \cos 2\theta (2\cos \theta - 1) = 0$$

either

$$\cos 2\theta = 0 \quad \text{or} \quad 2\cos \theta = 1$$

$$\Rightarrow 2\theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \quad \text{or} \quad \cos \theta = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{4}, n \in \mathbb{Z} \quad \text{or} \quad \theta = 2m\pi \pm \frac{\pi}{3}, m \in \mathbb{Z}$$

Q4(iii)

$$\sin \theta + \sin 5\theta = \sin 3\theta$$

$$\Rightarrow 2 \sin 3\theta \cdot \cos 2\theta - \sin 3\theta = 0 \quad \left[\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2} \right]$$

$$\Rightarrow \sin 3\theta [2 \cos 2\theta - 1] = 0$$

$\Rightarrow$ either

$$\sin 3\theta = 0 \quad \text{or} \quad 2 \cos 2\theta - 1 = 0$$

$$\Rightarrow 3\theta = n\pi, n \in \mathbb{Z} \quad \text{or} \quad \cos 2\theta = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{3}, n \in \mathbb{Z} \quad \text{or} \quad 2\theta = 2m\pi \pm \frac{\pi}{3}, m \in \mathbb{Z}$$

$$\text{or} \quad \theta = m\pi \pm \frac{\pi}{6}$$

Thus,

$$\theta = \frac{n\pi}{3} \quad \text{or} \quad m\pi \pm \frac{\pi}{6}, n, m \in \mathbb{Z}$$

Q4(iv)

We have,

$$\cos \theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4}$$

$$\Rightarrow 2 \cos \theta \cdot \cos 3\theta \cdot \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow (\cos 4\theta + \cos 2\theta) \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow (2 \cos^2 2\theta - 1 + \cos 2\theta) \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow 2 \cos^3 2\theta + \cos^2 2\theta - \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow 4 \cos^2 2\theta + 2 \cos^2 2\theta - 2 \cos 2\theta - 1 = 0$$

$$\Rightarrow 2 \cos^2 2\theta (2 \cos \theta + 1) - 1 (2 \cos 2\theta + 1) = 0$$

$$\Rightarrow (2 \cos^2 2\theta - 1) (2 \cos 2\theta + 1) = 0$$

either

$$2 \cos^2 2\theta - 1 = 0 \quad \text{or} \quad \Rightarrow 2 \cos 2\theta + 1 = 0$$

$$\Rightarrow \cos 4\theta = 0 \quad \text{or} \quad \Rightarrow \cos 2\theta = -\frac{1}{2}$$

$$\Rightarrow 4\theta = (2n+1) \frac{\pi}{2} \quad \text{or} \quad \Rightarrow \cos 2\theta = \cos 2 \frac{\pi}{3}$$

$$\Rightarrow \theta = (2n+1) \frac{\pi}{8} \quad \text{or} \quad \Rightarrow 2\theta = 2m\pi \pm 2 \frac{\pi}{3}$$

$$\Rightarrow \theta = m\pi \pm \frac{\pi}{3}$$

Thus,

$$\theta = (2n+1) \frac{\pi}{8} \quad \text{or} \quad \theta = m\pi \pm \frac{\pi}{3}, m, n \in \mathbb{Z}$$

Q4(v)

We have,

$$\cos \theta + \sin \theta = \cos 2\theta + \sin 2\theta$$

$$\Rightarrow \cos \theta - \cos 2\theta = \sin 2\theta - \sin \theta$$

$$\Rightarrow 2 \sin \frac{3\theta}{2} \cdot \sin \frac{\theta}{2} = 2 \cos \frac{3\theta}{2} \cdot \sin \frac{\theta}{2}$$

$$\Rightarrow 2 \sin \frac{\theta}{2} \left(\sin \frac{3\theta}{2} - \cos \frac{3\theta}{2} \right) = 0$$

$$\Rightarrow 2 \sin \frac{\theta}{2} \left(\sin \frac{3\theta}{2} - \cos \frac{3\theta}{2} = 0 \right)$$

either

$$\sin \frac{\theta}{2} = 0 \quad \text{or} \quad \sin \frac{3\theta}{2} - \cos \frac{3\theta}{2} = 0$$

$$\Rightarrow \frac{\theta}{2} = n\pi, n \in \mathbb{Z} \quad \text{or} \quad \tan \frac{3\theta}{2} = 1 = \tan \frac{\pi}{4}$$

$$\Rightarrow \theta = 2n\pi, n \in \mathbb{Z} \quad \text{or} \quad \frac{3\theta}{2} = n\pi + \frac{\pi}{4}$$

$$\text{or} \quad \theta = 2n\frac{\pi}{3} + \frac{\pi}{3.2}, n \in \mathbb{Z}$$

Thus,

$$\Rightarrow \theta = 2n\pi \quad \text{or} \quad 2n\frac{\pi}{3} + \frac{\pi}{6}, n \in \mathbb{Z}$$

Q4(vi)

We have,

$$\sin \theta + \sin 2\theta + \sin 3\theta = 0$$

$$\Rightarrow \sin 2\theta + 2 \sin 2\theta \cdot \cos \theta = 0$$

$$\Rightarrow \sin 2\theta + (1 + 2 \cos \theta) = 0$$

$\Rightarrow$ either

$$\sin 2\theta = 0 \quad \text{or} \quad 1 + 2 \cos \theta = 0$$

$$\Rightarrow 2\theta = n\pi, n \in \mathbb{Z} \quad \text{or} \quad \cos \theta = -\frac{1}{2} = \cos \left(\pi - \frac{\pi}{3} \right)$$

$$\Rightarrow \theta = \frac{n\pi}{2}, n \in \mathbb{Z} \quad \text{or} \quad \theta = 2m\pi \pm \frac{2\pi}{3}, m \in \mathbb{Z}$$

Thus,

$$\theta = \frac{n\pi}{2}, n \in \mathbb{Z} \quad \text{or} \quad \theta = 2m\pi \pm \frac{2\pi}{3}, m \in \mathbb{Z}$$

Q4(vii)

Given , $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$

$$(\sin 4x + \sin 2x) + (\sin 3x + \sin x) = 0$$

Using , $(\sin A + \sin B)$ formula $\Rightarrow$

$$2\sin\left[\frac{(4x+2x)}{2}\right]\cos\left[\frac{4x-2x}{2}\right] + 2\sin\left[\frac{(3x+x)}{2}\right]\cos\left[\frac{(3x-x)}{2}\right] = 0$$

$$2\sin 3x \cos x + 2\sin 2x \cos x = 0$$

$$2\cos x (\sin 3x + \sin 2x) = 0$$

$$2\cos x \left(2\sin\left[\frac{(3x+2x)}{2}\right]\cos\left[\frac{(3x-2x)}{2}\right] \right) = 0$$

$$4\cos x \sin \frac{5x}{2} \cos \frac{x}{2} = 0$$

$$\cos x = 0 ; \sin \frac{5x}{2} = 0 ; \cos \frac{x}{2} = 0$$

$$x = \frac{(2n+1)\pi}{2} ; \frac{5x}{2} = m\pi ; \frac{x}{2} = \frac{(2r+1)\pi}{2}$$

$$x = \frac{(2n+1)\pi}{2} ; x = \frac{2m\pi}{5} ; x = (2r+1)\pi, m, r, n \in \mathbb{Z}$$

Q4(viii)

We have,

$$\sin 3\theta - \sin \theta = 4\cos^2 \theta - 2$$

$$\Rightarrow 2\cos 2\theta \cdot \sin \theta = 2(2\cos^2 \theta - 1)$$

$$\Rightarrow 2\cos 2\theta \cdot \sin \theta = 2\cos 2\theta \quad \left[\because \cos 2\theta = 2\cos^2 \theta - 1 \right]$$

$$\Rightarrow 2\cos 2\theta (\sin \theta - 1) = 0$$

either

$$\cos 2\theta = 0 \quad \text{or} \quad \sin \theta - 1 = 0$$

$$\Rightarrow 2\theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \quad \text{or} \quad \sin \theta = 1 = \sin \frac{\pi}{2}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{4}, n \in \mathbb{Z} \quad \text{or} \quad \theta = m\pi + (-1)^m \frac{\pi}{2}, m \in \mathbb{Z}$$

Thus,

$$\theta = (2n+1)\frac{\pi}{4}, n \in \mathbb{Z} \quad \text{or} \quad m\pi + (-1)^m \frac{\pi}{2}, m \in \mathbb{Z}$$

Q5(i)

$$\tan x + \tan 2x + \frac{(\tan x + \tan 2x)}{1 - \tan x \tan 2x} = 0$$

$$[\tan x + \tan 2x] \left[1 + \frac{1}{1 - \tan x \tan 2x} \right] = 0$$

$$\tan x + \tan 2x (2 - \tan x \tan 2x) = 0$$

$$\tan x = \tan(-2x) \text{ or } \tan x \tan 2x = 2$$

$$x = n\pi - 2x \text{ or } \tan x \cdot \frac{2\tan x}{1 - \tan^2 x} = 2$$

$$3x = n\pi \text{ or } \frac{2\tan^2 x}{1 - \tan^2 x} = 2$$

$$3x = n\pi \text{ or } 2\tan^2 x = 2 - 2\tan^2 x$$

$$3x = n\pi \text{ or } 4\tan^2 x = 2$$

$$x = \frac{n\pi}{3} \text{ or } \tan^2 x = 1/2$$

$$x = \frac{n\pi}{3} \text{ or } x = m\pi \pm \tan^{-1}\left(\frac{1}{\sqrt{2}}\right), \quad n, m \in \mathbb{Z}$$

Q5(ii)

$$\tan \theta + \tan 2\theta = \tan(\theta + 2\theta)$$

$$\tan \theta + \tan 2\theta - \frac{\tan \theta + \tan 2\theta}{1 - \tan \theta \tan 2\theta} = 0$$

$$[\tan \theta + \tan 2\theta] \left[1 - \frac{1}{1 - \tan \theta \tan 2\theta} \right] = 0$$

$$[\tan \theta + \tan 2\theta] \left[\frac{1 - \tan \theta \tan 2\theta - 1}{1 - \tan \theta \tan 2\theta} \right] = 0$$

$$[\tan \theta + \tan 2\theta] \left[\frac{-\tan \theta \tan 2\theta}{1 - \tan \theta \tan 2\theta} \right] = 0$$

$$\tan \theta = 0 \text{ or } \tan 2\theta = 0 \text{ or } \tan \theta + \tan 2\theta = 0$$

$$\theta = n\pi \text{ or } \frac{n\pi}{2} \text{ or } \tan \theta \left[\frac{1 - \tan^2 \theta + 2}{1 - \tan^2 \theta} \right] = 0$$

$$\theta = n\pi \text{ or } \frac{n\pi}{2} \text{ or } \tan \theta = \pm \sqrt{3}$$

$$\theta = m\pi \text{ or } \frac{n\pi}{3}, \quad m, n \in \mathbb{Z}$$

Q5(iii)

We have,

$$\begin{aligned} & \tan 3\theta + \tan \theta = 2 \tan 2\theta \\ \Rightarrow & \tan 3\theta - \tan 2\theta = \tan 2\theta - \tan \theta \\ \Rightarrow & \tan 3\theta - \tan 2\theta = \tan 2\theta - \tan \theta \\ \Rightarrow & 2 \sin^2 \theta \sin 2\theta = 0 \\ \Rightarrow & \text{either} \\ & \sin \theta = 0 \quad \text{or} \quad \sin 2\theta = 0 \\ \Rightarrow & \theta = n\pi, n \in \mathbb{Z} \quad \text{or} \quad 2\theta = m\pi, m \in \mathbb{Z} \\ \Rightarrow & \theta = n\pi, n \in \mathbb{Z} \quad \text{or} \quad \theta = m \frac{\pi}{2}, m \in \mathbb{Z} \end{aligned}$$

Q6(i)

We have,

$$\begin{aligned} & \sin \theta + \cos \theta = \sqrt{2} \\ \Rightarrow & \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = 1 \\ \Rightarrow & \sin \frac{\pi}{4} \sin \theta + \cos \frac{\pi}{4} \cos \theta = 1 \quad \left[\because \cos \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \right] \\ \Rightarrow & \cos \left(\theta - \frac{\pi}{4} \right) = \cos 0^\circ \\ \Rightarrow & \theta - \frac{\pi}{4} = 2n\pi, n \in \mathbb{Z} \\ \Rightarrow & \theta = 2n\pi + \frac{\pi}{4}, n \in \mathbb{Z} \\ \therefore & \theta = (8n+1) \frac{\pi}{4}, n \in \mathbb{Z} \end{aligned}$$

Q6(ii)

$$\sqrt{3} \cos \theta + \sin \theta = 1$$

Divide both side by 2, we get

$$\begin{aligned} & \frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta = \frac{1}{2} \\ \Rightarrow & \cos \frac{\pi}{6} \cos \theta + \sin \frac{\pi}{6} \sin \theta = \frac{1}{2} \quad \left[\because \sin \frac{\pi}{6} = \frac{1}{2}, \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \right] \\ \Rightarrow & \cos \left(\theta - \frac{\pi}{6} \right) = \cos \frac{\pi}{3} \\ \Rightarrow & \theta - \frac{\pi}{6} = 2n \pm \frac{\pi}{3}, n \in \mathbb{Z} \\ \Rightarrow & \theta = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6}, n \in \mathbb{Z} \\ \Rightarrow & \theta = (4n+1) \frac{\pi}{2} \quad \text{or} \quad (12m-1) \frac{\pi}{6}, n, m \in \mathbb{Z} \end{aligned}$$

Q6(iii)

We have,

$$\sin \theta + \cos \theta = 1$$

divide both side by $\sqrt{2}$, we get,

$$\begin{aligned} \Rightarrow \quad & \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = \frac{1}{\sqrt{2}} \\ \Rightarrow \quad & \sin \frac{\pi}{4} \sin \theta + \cos \frac{\pi}{4} \cos \theta = \frac{1}{\sqrt{2}} \\ \Rightarrow \quad & \cos \left(\theta - \frac{\pi}{4} \right) = \cos \frac{\pi}{4} \\ \Rightarrow \quad & \theta = \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, n \in \mathbb{Z} \\ \Rightarrow \quad & \theta = 2n\pi + \frac{\pi}{2} \quad \text{or} \quad 2n\pi, n \in \mathbb{Z} \end{aligned}$$

Q6(iv)

We have,

$$\sec \theta = 1 + \cot \theta$$

$$\Rightarrow \quad \frac{1}{\sin \theta} = 1 + \frac{\cos \theta}{\sin \theta}$$

$$\Rightarrow \quad 1 = \sin \theta + \cos \theta$$

Divide both side by $\sqrt{2}$, we get,

$$\begin{aligned} \Rightarrow \quad & \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = \frac{1}{\sqrt{2}} \\ \Rightarrow \quad & \sin \frac{\pi}{4} \sin \theta + \cos \frac{\pi}{4} \cos \theta = \frac{1}{\sqrt{2}} \\ \Rightarrow \quad & \cos \left(\theta - \frac{\pi}{4} \right) = \cos \frac{\pi}{4} \\ \Rightarrow \quad & \theta = \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, n \in \mathbb{Z} \\ \therefore \quad & \theta \left(2n\pi + \frac{\pi}{2} \right) \quad \text{or} \quad 2n\pi, n \in \mathbb{Z} \end{aligned}$$

Q6(v)

$$(\sqrt{3}-1)\cos\theta + (\sqrt{3}+1)\sin\theta = 2$$

Divide on both sides by $2\sqrt{2}$

$$\frac{(\sqrt{3}-1)}{2\sqrt{2}}\cos\theta + \frac{(\sqrt{3}+1)}{2\sqrt{2}}\sin\theta = \frac{1}{\sqrt{2}}$$

$$\sin\left(\theta + \tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)\right) = \sin\frac{\pi}{4}$$

$$\theta = 2n\pi + \frac{\pi}{3} \text{ or } 2n\pi - \frac{\pi}{6} \quad n \in \mathbb{Z}$$

Q7(i)

$$\cot x + \tan x = 2$$

$$2\sin x \cos x = 1$$

$$\sin 2x = 1$$

$$2x = \frac{(2n+1)}{2}\pi$$

$$x = \frac{(2n+1)}{4}\pi, n \in \mathbb{Z}$$

Q7(ii)

$$2\sin^2\theta = 3\cos\theta$$

$$2-2\cos^2\theta = 3\cos\theta$$

$$2\cos^2\theta + 3\cos\theta - 2 = 0$$

$$2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$$

$$(\cos\theta + 2)(2\cos\theta - 1) = 0$$

$$\cos\theta = -2 \text{ or } \cos\theta = 0.5$$

$$\cos\theta = -2, \text{ never possible}$$

$$\cos\theta = 0.5, \theta = 60^\circ, 300^\circ$$

Q7(iii)

$$\sec x \cos 5x + 1 = 0$$

$$\frac{\cos 5x + \cos x}{\cos x} = 0 \Rightarrow \cos x \neq 0$$

$$2 \cos 3x \cos 2x = 0$$

$$\cos 3x = 0 \text{ or } \cos 2x = 0$$

$$3x = \frac{\pi}{2} \text{ or } 2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{\pi}{6}$$

Q7(iv)

$$2 \sin^2 \theta + 5 - 6 = 0$$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \pm \frac{1}{\sqrt{2}}$$

$$\theta = n\pi \pm \frac{\pi}{4}$$

Q7(v)

$$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$$

$$(\sin x + \sin 3x) - 3\sin 2x - (\cos x + \cos 3x) + 3\cos 2x = 0$$

$$2\sin 2x \cos x - 3\sin 2x - 2\cos 2x \cos x + 3\cos 2x = 0$$

$$\sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0$$

$$(2\cos x - 3)(\sin 2x - \cos 2x) = 0$$

$$\cos x = \frac{3}{2} \text{ or } \sin 2x - \cos 2x = 0$$

$$\text{but } \cos x \in [-1, 1] \Rightarrow \cos x \neq \frac{3}{2}$$

$$\sin 2x = \cos 2x$$

$$\tan 2x = 1$$

$$2x = n\pi + \frac{\pi}{4}$$

$$x = \frac{n\pi}{2} + \frac{\pi}{8}$$