

Ex 13.1

Derivatives as a Rate Measurer Ex 13.1 Q1

Let total surface area of the cylinder be A

$$A = 2\pi r(h + r)$$

Differentiating it with respect to r as r varies

$$\begin{aligned}\frac{dA}{dr} &= 2\pi r(0 + 1) + (h + r)2\pi \\ &= 2\pi r + 2\pi h + 2\pi r\end{aligned}$$

$$\frac{dA}{dr} = 4\pi r + 2\pi h$$

Derivatives as a Rate Measurer Ex 13.1 Q2

Let D be the diameter and r be the radius of sphere,

So, volume of sphere = $\frac{4}{3}\pi r^3$

$$v = \frac{4}{3}\pi \left(\frac{D}{2}\right)^3$$

$$v = \frac{4}{24}\pi D^3$$

Differentiating it with respect to D .

$$\frac{dv}{dD} = \frac{12}{24}\pi D^2$$

$$\frac{dv}{dD} = \frac{\pi D^2}{2}$$

Derivatives as a Rate Measurer Ex 13.1 Q3

Given, radius of sphere (r) = 2cm.

We know that,

$$v = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dr} = 4\pi r^2 \quad \text{--- (i)}$$

And $A = 4\pi r^2$

$$\frac{dA}{dr} = 8\pi r \quad \text{--- (ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{dv}{dr}}{\frac{dA}{dr}} = \frac{4\pi r^2}{8\pi r}$$

$$\frac{dv}{dA} = \frac{r}{2}$$

$$\left(\frac{dv}{dA}\right)_{r=2} = 1$$

Derivatives as a Rate Measurer Ex 13.1 Q4

Let r be two radius of circular disc.

We know that,

$$\text{Area } A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r \quad \text{--- (i)}$$

Circumference $C = 2\pi r$

$$\frac{dC}{dr} = 2\pi \quad \text{--- (ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{dA}{dr}}{\frac{dC}{dr}} = \frac{2\pi r}{2\pi}$$

$$\frac{dA}{dC} = r$$

$$\left(\frac{dA}{dC}\right)_{r=3} = 3$$

Derivatives as a Rate Measurer Ex 13.1 Q5

Let r be the radius, v be the volume of cone and h be height

$$v = \frac{1}{3}\pi r^2 h$$

$$\frac{dv}{dr} = \frac{2}{3}\pi r h$$

Derivatives as a Rate Measurer Ex 13.1 Q6

Let r be radius and A be area of circle, so

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\left(\frac{dA}{dr}\right)_{r=5} = 2\pi (5)$$

$$\left(\frac{dA}{dr}\right)_{r=5} = 10\pi$$

Derivatives as a Rate Measurer Ex 13.1 Q7

Here, $r = 2$ cm

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\left(\frac{dV}{dr}\right)_{r=2} = 4\pi(2)^2$$

$$\left(\frac{dV}{dr}\right)_{r=2} = 16\pi$$

Derivatives as a Rate Measurer Ex 13.1 Q8

Marginal cost is the rate of change of total cost with respect to output.

$$\therefore \text{Marginal cost (MC)} = \frac{dC}{dx} = 0.007(3x^2) - 0.003(2x) + 15$$

$$= 0.021x^2 - 0.006x + 15$$

$$\text{When } x = 17, \text{ MC} = 0.021(17^2) - 0.006(17) + 15$$

$$= 0.021(289) - 0.006(17) + 15$$

$$= 6.069 - 0.102 + 15$$

$$= 20.967$$

Hence, when 17 units are produced, the marginal cost is Rs. 20.967

Derivatives as a Rate Measurer Ex 13.1 Q9

Marginal revenue is the rate of change of total revenue with respect to the number of units sold.

$$\therefore \text{Marginal Revenue (MR)} = \frac{dR}{dx} = 13(2x) + 26 = 26x + 26$$

When $x = 7$,

$$\text{MR} = 26(7) + 26 = 182 + 26 = 208$$

Hence, the required marginal revenue is Rs 208.

Derivatives as a Rate Measurer Ex 13.1 Q10

$$R(x) = 3x^2 + 36x + 5$$

$$\frac{dR}{dx} = 6x + 36$$

$$\left.\frac{dR}{dx}\right|_{x=5} = 6 \times 5 + 36$$

$$= 30 + 36$$

$$= 66$$

This, as per the question, indicates the money to be spent on the welfare of the employees, when the number of employees is 5.

Ex 13.2

Derivatives as a Rate Measurer Ex 13.2 Q1

Let x be the side of square.

Given, $\frac{dx}{dt} = 4 \text{ cm/min}$, $x = 8 \text{ cm}$

We know that

$$\text{Area } (A) = x^2$$

$$\frac{dA}{dt} = 2x \frac{dx}{dt}$$

$$\left(\frac{dA}{dt}\right)_{8 \text{ cm}} = 2 \times (8) (4)$$

$$\frac{dA}{dt} = 64 \text{ cm}^2/\text{min}$$

Area increases at a rate of $64 \text{ cm}^2/\text{min}$.

Derivatives as a Rate Measurer Ex 13.2 Q2

Let edge of the cube is $x \text{ cm}$.

$$\frac{dx}{dt} = 3 \text{ cm/sec}, x = 10 \text{ cm}$$

Let V be volume of cube,

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$= 3 (10)^2 \times (3)$$

$$= 900 \text{ cm}^3/\text{sec}$$

So,

Volume increases at a rate of $900 \text{ cm}^3/\text{sec}$.

Derivatives as a Rate Measurer Ex 13.2 Q3

Let x be the side of the square.

Here, $\frac{dx}{dt} = 0.2$ cm/sec.

$$P = 4x$$

$$\begin{aligned}\frac{dP}{dt} &= 4 \frac{dx}{dt} \\ &= 4 \times (0.2)\end{aligned}$$

$$\frac{dP}{dt} = 0.8 \text{ cm/sec}$$

So, perimeter increases at the rate of 0.8 cm/sec.

Derivatives as a Rate Measurer Ex 13.2 Q4

The circumference of a circle (C) with radius (r) is given by

$$C = 2\pi r.$$

Therefore, the rate of change of circumference (C) with respect to time (t) is given by,

$$\frac{dC}{dt} = \frac{dC}{dr} \cdot \frac{dr}{dt} \text{ (By chain rule)}$$

$$= \frac{d}{dr}(2\pi r) \cdot \frac{dr}{dt}$$

$$= 2\pi \cdot \frac{dr}{dt}$$

It is given that $\frac{dr}{dt} = 0.7$ cm/s.

Hence, the rate of increase of the circumference is $2\pi(0.7) = 1.4\pi$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q5

Let r be the radius of the spherical soap bubble.

Here, $\frac{dr}{dt} = 0.2$ cm/sec, $r = 7$ cm

$$\text{Surface Area } (A) = 4\pi r^2$$

$$\frac{dA}{dt} = 4\pi (2r) \frac{dr}{dt}$$

$$\left(\frac{dA}{dt}\right)_{r=7} = 4\pi (2 \times 7) \times 0.$$

$$= 11.2\pi \text{ cm}^2/\text{sec}.$$

So, area of bubble increases at the rate of $11.2\pi \text{ cm}^2/\text{sec}$.

Derivatives as a Rate Measurer Ex 13.2 Q6

The volume of a sphere (V) with radius (r) is given by,

$$V = \frac{4}{3}\pi r^3$$

$\therefore$ Rate of change of volume (V) with respect to time (t) is given by,

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} \text{ [By chain rule]}$$

$$= \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \cdot \frac{dr}{dt}$$

$$= 4\pi r^2 \cdot \frac{dr}{dt}$$

It is given that $\frac{dV}{dt} = 900 \text{ cm}^3/\text{s}$.

$$\therefore 900 = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{900}{4\pi r^2} = \frac{225}{\pi r^2}$$

Therefore, when radius = 15 cm,

$$\frac{dr}{dt} = \frac{225}{\pi(15)^2} = \frac{1}{\pi}$$

Hence, the rate at which the radius of the balloon increases when the radius is 15 cm is $\frac{1}{\pi}$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q7

Let r be the radius of the air bubble.

Here, $\frac{dr}{dt} = 0.5$ cm/sec, $r = 1$ cm

$$\text{Volume } (V) = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi (3r^2) \frac{dr}{dt}$$

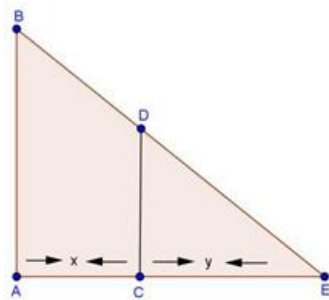
$$= 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi (1)^2 \times (0.5)$$

$$\frac{dV}{dt} = 2\pi \text{ cm}^3/\text{sec.}$$

So, volume of air bubble increases at the rate of $2\pi \text{ cm}^3/\text{sec.}$

Derivatives as a Rate Measurer Ex 13.2 Q8



Let AB be the lamp-post. Suppose at time t , the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE .

Here, $\frac{dx}{dt} = 5$ km/hr
 $CD = 2$ m, $AB = 6$ m

Here, $\triangle ABE$ and $\triangle CDE$ are similar, so

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{6}{2} = \frac{x+y}{y}$$

$$3y = x + y$$

$$2y = x$$

$$2 \frac{dy}{dt} = \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{5}{2} \text{ km/hr}$$

So, the length of his shadow increases at the rate of $\frac{5}{2}$ km/hr.

Derivatives as a Rate Measurer Ex 13.2 Q9

The area of a circle (A) with radius (r) is given by $A = \pi r^2$.

Therefore, the rate of change of area (A) with respect to time (t) is given by,

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \frac{d}{dr}(\pi r^2) \frac{dr}{dt} = 2\pi r \frac{dr}{dt} \text{ [By chain rule]}$$

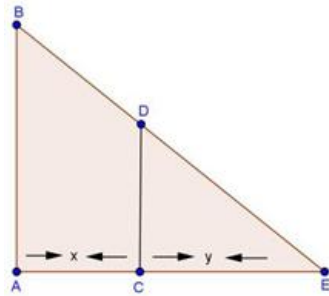
It is given that $\frac{dr}{dt} = 4 \text{ cm/s}$.

Thus, when $r = 10 \text{ cm}$,

$$\frac{dA}{dt} = 2\pi(10)(4) = 80\pi$$

Hence, when the radius of the circular wave is 8 cm, the enclosed area is increasing at the rate of $80\pi \text{ cm}^2/\text{s}$

Derivatives as a Rate Measurer Ex 13.2 Q10



Let AB be the height of pole. Suppose at time t , the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE , then

$$\frac{dx}{dt} = 1.1 \text{ m/sec}$$

$\triangle ABE$ is similar to $\triangle CDE$,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{600}{160} = \frac{x+y}{y}$$

$$\frac{15}{4} = \frac{x+y}{y}$$

$$15y = 4x + 4y$$

$$11y = 4x$$

$$11 \frac{dy}{dx} = 4 \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{4}{11}(1.1)$$

$$\frac{dy}{dt} = 0.4 \text{ m/sec}$$

Rate of increasing of shadow = 0.4 m/sec.

Derivatives as a Rate Measurer Ex 13.2 Q11

Let AB be the height of source of light. Suppose at time t , the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE , then

$$\frac{dx}{dt} = 2 \text{ m/sec}$$

$\triangle ABE$ is similar to $\triangle CDE$,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{900}{180} = \frac{x+y}{y}$$

$$5y = x + y$$

$$4y = x$$

$$4 \frac{dy}{dt} = \frac{dx}{dt}$$

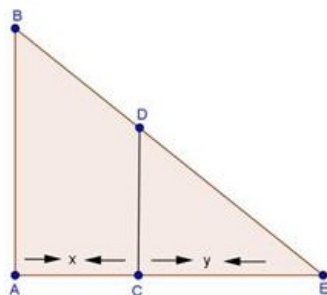
$$\frac{dy}{dt} = \frac{2}{4}$$

$$= \frac{1}{2}$$

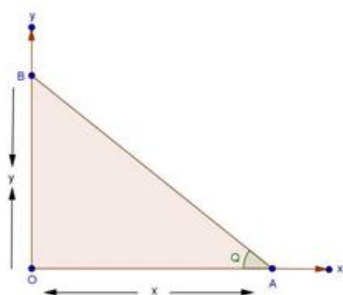
$$\frac{dy}{dt} = 0.5 \text{ m/sec}$$

So, rate of increase of shadow is 0.5 m/sec.

The diagram of the problem is shown below



Derivatives as a Rate Measurer Ex 13.2 Q12



Let AB be the position of the ladder, at time t , such that $OA = x$ and $OB = y$

Here,

$$OA^2 + OB^2 = AB^2$$

$$x^2 + y^2 = (13)^2$$

$$x^2 + y^2 = 169 \quad \text{---(i)}$$

And $\frac{dx}{dt} = 1.5 \text{ m/sec}$

From figure, $\tan \theta = \frac{y}{x}$

Differentiating equation (i) with respect to t ,

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(1.5)x + 2y \frac{dy}{dt} = 0$$

$$3x + 2y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{3x}{2y}$$

Differentiating equation (ii) with respect to t ,

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{d \frac{dy}{dt} - y \frac{dx}{dt}}{x^2}$$

$$= \frac{x \times \left(-\frac{3x}{2y}\right) - y(1.5)}{x^2}$$

$$= \frac{-1.5x^2 - 1.5y^2}{yx^2}$$

$$\frac{d\theta}{dt} = \frac{-1.5(x^2 + y^2)}{x^2 y \sec^2 \theta}$$

$$= \frac{-1.5(x^2 + y^2)}{x^2 y (1 + \tan^2 \theta)}$$

$$\frac{d\theta}{dt} = \frac{-1.5(x^2 + y^2)}{x^2 y \left(1 + \frac{y^2}{x^2}\right)}$$

$$= \frac{-1.5(x^2 + y^2) \times x^2}{x^2 y (x^2 + y^2)}$$

$$= \frac{-1.5}{y}$$

$$= \frac{-1.5}{\sqrt{169 - x^2}}$$

$$= \frac{-1.5}{\sqrt{169 - 144}}$$

$$= \frac{-1.5}{5}$$

$$= -0.3 \text{ radian/sec}$$

So, angle between ladder and ground is decreasing at the rate of 0.3 radian/sec.

Here, curve is

$$y = x^2 + 2x$$

And $\frac{dy}{dt} = \frac{dx}{dt}$ ---(i)

$$y = x^2 + 2x$$

$$\Rightarrow \frac{dy}{dt} = 2x \frac{dx}{dt} + 2 \frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{dx}{dt} (2x + 2)$$

Using equation (i),

$$2x + 2 = 1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

So, $y = x^2 + 2x$

$$= \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)$$

$$= \frac{1}{4} - 1$$

$$y = -\frac{3}{4}$$

So, required points is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$.

Derivatives as a Rate Measurer Ex 13.2 Q14

Here,

$$\frac{dx}{dt} = 4 \text{ units/sec, and } x = 2$$

And, $y = 7x - x^3$

$$\text{Slope of the curve (S)} = \frac{dy}{dx}$$

$$S = 7 - 3x^2$$

$$\frac{ds}{dt} = -6x \frac{dx}{dt}$$

$$= -6(2)(4)$$

$$= -48 \text{ units/sec}$$

So, slope is decreasing at the rate of 48 units/sec.

Derivatives as a Rate Measurer Ex 13.2 Q15

Here,

$$\frac{dy}{dt} = 3 \frac{dx}{dt} \quad \text{---(i)}$$

And, $y = x^3$

$$\frac{dy}{dt} = 3x^2 \frac{dx}{dt}$$

$$3 \frac{dx}{dt} = 3x^2 \frac{dx}{dt} \quad [\text{Using equation (i)}]$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

$$\text{Put } x = 1 \Rightarrow y = (1)^3 = 1$$

$$\text{Put } x = -1 \Rightarrow y = (-1)^3 = -13$$

So, the required points are (1,1) and (-1,-1).

Derivatives as a Rate Measurer Ex 13.2 Q16(i)

Here,

$$2 \frac{d(\sin \theta)}{dt} = \frac{d\theta}{dt}$$

$$2 \times \cos \theta \frac{d\theta}{dt} = \frac{d\theta}{dt}$$

$$2 \cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}.$$

Derivatives as a Rate Measurer Ex 13.2 Q16(ii)

$$\frac{d\theta}{dt} = -2 \frac{d}{dt}(\cos \theta)$$

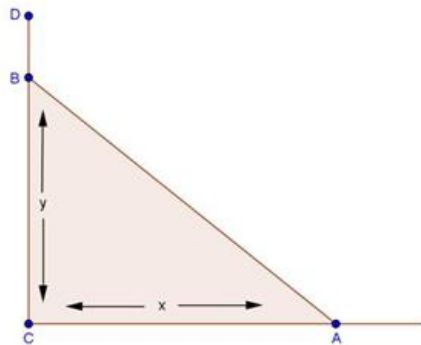
$$\frac{d\theta}{dt} = -2(-\sin \theta) \frac{d\theta}{dt}$$

$$1 = 2 \sin \theta$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

Derivatives as a Rate Measurer Ex 13.2 Q17



Let CD be the wall and AB is the ladder

Here, $AB = 6$ meter and $\left(\frac{dx}{dt}\right)_{x=4} = 0.5$ m/sec.

From figure,

$$AB^2 = x^2 + y^2$$

$$(6)^2 = x^2 + y^2$$

$$36 = x^2 + y^2$$

Differentiating it with respect to t ,

$$0 = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

---(i)

$$\left(\frac{dy}{dt}\right)_{x=4} = \frac{4(0.5)}{\sqrt{36-x^2}}$$

$$= -\frac{2}{\sqrt{36-16}}$$

$$= -\frac{2}{2\sqrt{5}}$$

$$= -\frac{1}{\sqrt{5}} \text{ m/sec.}$$

So, ladder top is sliding at the rate of $\frac{1}{\sqrt{5}}$ m/sec.

Now, to find x when $\frac{dx}{dt} = -\frac{dy}{dt}$

From equation (i),

$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$-\frac{dx}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$x = y$$

Now,

$$36 = x^2 + y^2$$

$$36 = x^2 + x^2$$

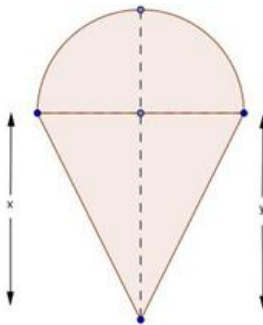
$$2x^2 = 36$$

$$x^2 = 18$$

$$x = 3\sqrt{2} \text{ m}$$

When foot and top are moving at the same rate, foot of wall is $3\sqrt{2}$ meters away from the wall

Derivatives as a Rate Measurer Ex 13.2 Q18



Let height of the cone is x cm . and radius of sphere is r cm .

Here given,

$$x = 2r \quad \text{---(i)}$$

$$h = x + r$$

$$h = 2r + r$$

$$h = 3r \quad \text{---(ii)}$$

v = volume of cone + volume of hemisphere

$$= \frac{1}{3} \pi r^2 x + \frac{2}{3} \pi r^3$$

$$= \frac{1}{3} \pi r^2 (2r) + \frac{2}{3} \pi r^3 \quad \text{[Using equation (ii)]}$$

$$v = \frac{2}{3} \pi r^3 + \frac{2}{3} \pi r^3$$

$$= \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \pi \left(\frac{h}{3}\right)^3$$

$$v = \frac{4}{81} \pi h^3$$

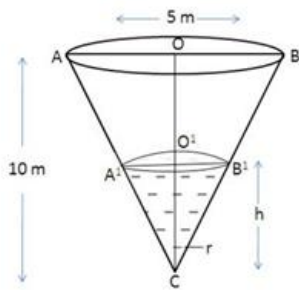
$$\frac{dv}{dh} = \frac{4}{81} \pi \times 3h^2$$

$$\left(\frac{dv}{dh}\right)_{h=9} = \frac{12}{81} \pi (9)^2$$

$$\left(\frac{dv}{dh}\right)_{h=9} = 12\pi \text{ cm}^2$$

Volume is changing at the rate $12\pi \text{ cm}^2$ with respect to total height.

Derivatives as a Rate Measurer Ex 13.2 Q19



Let α be the semi vertical angle of the cone CAB whose height CO is 10 m and radius $OB = 5$ m.

Now,

$$\begin{aligned}\tan \alpha &= \frac{OB}{CO} \\ &= \frac{5}{10} \\ \tan \alpha &= \frac{1}{2}\end{aligned}$$

Let V be the volume of the water in the cone, then

$$\begin{aligned}V &= \frac{1}{3} \pi (O'B')^2 (CO') \\ &= \frac{1}{3} \pi (h \tan \alpha)^2 (h) \\ V &= \frac{1}{3} \pi h^3 \tan^2 \alpha\end{aligned}$$

$$V = \frac{\pi}{12} h^2 \quad \left[\because \tan \alpha = \frac{1}{2} \right]$$

$$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$$

$$\pi = \frac{\pi}{4} h^2 \frac{dh}{dt} \quad \left[\because \frac{dV}{dt} = \text{m}^3/\text{min} \right]$$

$$\frac{dh}{dt} = \frac{4}{h^2}$$

$$\left(\frac{dh}{dt} \right)_{2.5} = \frac{4}{(2.5)^2} \quad [\because h = 10 - 7.5 = 2.5 \text{ m}]$$

$$\begin{aligned}&= \frac{4}{6.25} \\ &= 0.64 \text{ m/min}\end{aligned}$$

So, water level is rising at the rate of 0.64 m/min.

Derivatives as a Rate Measurer Ex 13.2 Q20

Let AB be the lamp-post. Suppose at time t , the man CD is at a distance x m. from the lamp-post and y m. be the length of the shadow CE .

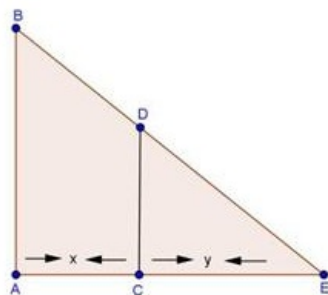
Here, $\frac{dx}{dt} = 6 \text{ km/hr}$
 $CD = 2 \text{ m}, AB = 6 \text{ m}$

Here, $\triangle ABE$ and $\triangle CDE$ are similar

So, $\frac{AB}{CD} = \frac{AE}{CE}$
 $\frac{6}{2} = \frac{x+y}{y}$
 $3y = x+y$
 $2y = x$
 $2 \frac{dy}{dt} = \frac{dx}{dt}$
 $2 \frac{dy}{dt} = 6$
 $\frac{dy}{dt} = 3 \text{ km/hr}$

So, length of his shadow increases at the rate of 3 km/hr.

The diagram of the problem is shown below



Derivatives as a Rate Measurer Ex 13.2 Q21

Here, $\frac{dA}{dt} = 2 \text{ cm}^2/\text{sec}$

To find $\frac{dV}{dt}$ at $r = 6 \text{ cm}$

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$2 = 8\pi r \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r} \text{ cm/sec}$$

Now, $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi r^2 \left(\frac{1}{4\pi r} \right)$$

$$= r$$

$$\frac{dV}{dt} = 6 \text{ cm}^3/\text{sec}$$

So, volume of bubble is increasing at the rate of $6 \text{ cm}^3/\text{sec}$.

Derivatives as a Rate Measurer Ex 13.2 Q22

Here, $\frac{dr}{dt} = 2 \text{ cm/sec}$, $\frac{dh}{dt} = -3 \text{ cm/sec}$

To find $\frac{dV}{dt}$ when $r = 3 \text{ cm}$, $h = 5 \text{ cm}$

Now, $V = \text{volume of cylinder}$

$$V = \pi r^2 h$$

$$\begin{aligned}\frac{dV}{dt} &= \pi \left[2r \frac{dr}{dt} \times h + r^2 \frac{dh}{dt} \right] \\ &= \pi [2(3)(2)(5) + (3)^2(-3)] \\ &= \pi [60 - 27] \\ \frac{dV}{dt} &= 33\pi \text{ cm}^3/\text{sec}\end{aligned}$$

So, volume of cylinder is increasing at the rate of $33\pi \text{ cm}^3/\text{sec}$.

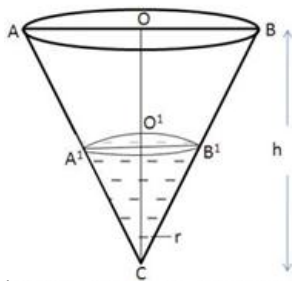
Derivatives as a Rate Measurer Ex 13.2 Q23

Let V be volume of sphere with inner radius r and outer radius R , then

$$\begin{aligned}V &= \frac{4}{3}\pi(R^3 - r^3) \\ \frac{dV}{dt} &= \frac{4}{3}\pi \left(3R^2 \frac{dR}{dt} - 3r^2 \frac{dr}{dt} \right) \\ 0 &= \frac{4\pi}{3} \left(R^2 \frac{dR}{dt} - r^2 \frac{dr}{dt} \right) && [\text{Since volume } V \text{ is constant}] \\ R^2 \frac{dR}{dt} &= r^2 \frac{dr}{dt} \\ (8)^2 \frac{dR}{dt} &= (4)^2 (1) \\ \frac{dR}{dt} &= \frac{16}{64} \\ \frac{dR}{dt} &= \frac{1}{4} \text{ cm/sec}\end{aligned}$$

Rate of increasing of outer radius = $\frac{1}{4} \text{ cm/sec}$.

Derivatives as a Rate Measurer Ex 13.2 Q24



Let α be the semi vertical angle of the cone CAB whose height CO is half of radius OB .

Now,

$$\begin{aligned}\tan \alpha &= \frac{OB}{CO} \\ &= \frac{OB}{2OB} \quad [\because CO = 2OB] \\ \tan \alpha &= \frac{1}{2}\end{aligned}$$

Let V be the volume of the sand in the cone

$$\begin{aligned}V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi \left(\frac{h}{2} \right)^2 h \\ &= \frac{\pi}{12} h^3\end{aligned}$$

$$\frac{dV}{dt} = \frac{3\pi}{12} h^2 \frac{dh}{dt}$$

$$50 = \frac{3\pi}{12} h^2 \frac{dh}{dt} \quad \left[\because \frac{dV}{dt} = 50 \text{ cm}^3/\text{min} \right]$$

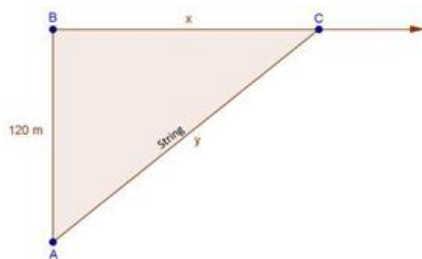
$$\frac{dh}{dt} = \frac{200}{\pi h^2}$$

$$= \frac{200}{\pi (5)^2}$$

$$\frac{dh}{dt} = \frac{8}{3.14} \text{ cm/min}$$

$$\text{Rate of increasing of height} = \frac{8}{\pi} \text{ cm/min}$$

Derivatives as a Rate Measurer Ex 13.2 Q25



Let C be the position of kite and AC be the string.

$$\text{Here, } y^2 = x^2 + (120)^2 \quad \text{---(i)}$$

$$2y \frac{dy}{dt} = 2x \frac{dx}{dt}$$

$$y \frac{dy}{dt} = x \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y} (52) \quad \text{---(ii)} \quad \left[\because \frac{dx}{dt} = 52 \text{ m/sec} \right]$$

From equation (i),

$$y^2 = x^2 + (120)^2$$

$$(130)^2 = x^2 + (120)^2$$

$$x^2 = 16900 - 14400$$

$$x^2 = 2500$$

$$x = 50$$

Using equation (ii),

$$\frac{dy}{dt} = \frac{x}{y} (52)$$

$$= \frac{50}{130} (52)$$

$$= 20 \text{ m/sec}$$

So, string is being paid out at the rate of 20 m/sec.

Derivatives as a Rate Measurer Ex 13.2 Q26

Here,

$$\frac{dy}{dt} = 2 \frac{dx}{dt} \quad \text{---(i)}$$

$$\text{and } y = \left(\frac{2}{3}\right)x^3 + 1$$

$$\frac{dy}{dt} = \frac{2}{3} \times 3x^2 \frac{dx}{dt}$$

$$2 \frac{dx}{dt} = 2x^2 \frac{dx}{dt} \quad \text{[Using equation (i)]}$$

$$2 = 2x^2$$

$$\Rightarrow x = \pm 1$$

$$y = \left(\frac{2}{3}\right)x^3 + 1$$

$$\text{Put } x = 1, \quad y = \frac{2}{3} + 1 = \frac{5}{3}$$

$$\text{Put } x = -1, \quad y = \frac{2}{3}(-1) + 1 = \frac{1}{3}$$

So, required point $\left(1, \frac{5}{3}\right)$ and $\left(-1, \frac{1}{3}\right)$.

Derivatives as a Rate Measurer Ex 13.2 Q27

Here,

$$\frac{dx}{dt} = \frac{dy}{dt} \quad \text{---(i)}$$

and curve is

$$y^2 = 8x$$

$$2y \frac{dy}{dt} = 8 \frac{dx}{dt}$$

$$2y = 8 \quad \text{[using equation (i)]}$$

$$y = 4$$

$$\Rightarrow (4)^2 = 8x$$

$$\Rightarrow x = 2$$

So, required point = (2, 4).

Derivatives as a Rate Measurer Ex 13.2 Q28

Let edge of cube be x cm

Here,

$$\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$$

To find $\frac{dA}{dt}$ when $x = 10$ cm

We know that

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \left(\frac{dx}{dt} \right)$$

$$9 = 3(10)^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{100} \text{ cm/sec}$$

Now, $A = 6x^2$

$$\frac{dA}{dt} = 12x \frac{dx}{dt}$$

$$= 12(10) \left(\frac{3}{100} \right)$$

$$\frac{dA}{dt} = 3.6 \text{ cm}^2/\text{sec}.$$

Derivatives as a Rate Measurer Ex 13.2 Q29

Given, $\frac{dV}{dt} = 25 \text{ cm}^3/\text{sec}$

To find $\frac{dA}{dt}$ when $r = 5$ cm

We know that,

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi (3r^2) \frac{dr}{dt}$$

$$25 = 4\pi (5)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi} \text{ cm/sec}$$

Now, $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8\pi (5) \left(\frac{1}{4\pi} \right)$$

$$\frac{dA}{dt} = 10 \text{ cm}^2/\text{sec}.$$

Derivatives as a Rate Measurer Ex 13.2 Q30

Given,

$$\frac{dx}{dt} = -5 \text{ cm/min}$$

$$\frac{dy}{dt} = 4 \text{ cm/min}$$

(i) To find $\frac{dP}{dt}$ when $x = 8 \text{ cm}, y = 6 \text{ cm}$

$$P = 2(x + y)$$

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$= 2(-5 + 4)$$

$$\frac{dP}{dt} = -2 \text{ cm/min}$$

(ii) To find $\frac{dA}{dt}$ when $x = 8 \text{ cm}$ and $y = 6 \text{ cm}$

$$A = xy$$

$$\frac{dA}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt}$$

$$= (8)(4) + (6)(-5)$$

$$= 32 - 30$$

$$\frac{dA}{dt} = 2 \text{ cm}^2/\text{min}.$$

Derivatives as a Rate Measurer Ex 13.2 Q31

Let r be the radius of the given disc and A be its area.

$$\text{Then, } A = \pi r^2$$

$$\therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad [\text{by chain rule}]$$

Now, the approximate increase of radius = $dr = \frac{dr}{dt} \Delta t = 0.05 \text{ cm/sec}$

$\therefore$ the approximate rate of increase in area is given by

$$dA = \frac{dA}{dt} (\Delta t) = 2\pi r \left(\frac{dr}{dt} \Delta t \right) = 2\pi (3.2) (0.05) = 0.320\pi \text{ cm}^2/\text{s}$$